Erratum, Table in Section 6.1, Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators

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Professor Robert Keolian (Graduate Program in Acoustics, Penn State University) kindly pointed out two mistakes in one line in the Table at the bottom of page 139, in the entry for $\dot{W}_{\rm lost}$ for thermal mixing. The correct result is

$$\begin{aligned} \dot{W}_{\rm lost} &= \dot{M} c_p T_0 \log[(x + \tau(1-x))/\tau^{1-x}], && (1)\\ \text{where } \dot{M} &= \dot{M}_1 + \dot{M}_2, && (2)\\ x &= \dot{M}_1/\dot{M} \text{ and } \tau = T_2/T_1, && (3) \end{aligned}$$

with log being the natural logarithm. Note that line (3) above differs from the corresponding expressions on page 139 in *two* ways.

Here is a derivation of this correct result, in the slightly more general case of two ideal gases with possibly nonequal specific heats.

Consider two streams of ideal gases flowing at mass-flow rates $\dot{M}_A$ and $\dot{M}_B$, whose temperatures are T_A and T_B, respectively, which mix to form a single stream

$$\dot{M} = \dot{M}_A + \dot{M}_B \tag{4}$$

at temperature T. The first law of thermodynamics shows that

$$T = \frac{\dot{M}_A c_{p,A} T_A + \dot{M}_B c_{p,B} T_B}{\dot{M}_A c_{p,A} + \dot{M}_B c_{p,B},} \tag{5}$$

where $c_{p,j}$ is the isobaric specific heat per unit mass of the j'th ideal gas.

The (rate of) lost work in this irreversible mixing process is given by

$$\dot{W}_{\rm lost} = \dot{M}_A e_{x,A} + \dot{M}_B e_{x,B} - \dot{M} e_{x,{\rm mix}}, \tag{6}$$

where $e_{x,j}$ and $e_{x,{\rm mix}}$ are the ideal-gas flow exergies per unit mass of the j'th gas and of the mixture, given by Eq. (3.51) in A. Bejan, *Advanced Engineering Thermodynamics* (Wiley, 2nd edition, 1997):

$$e_{x,j} = c_{p,j} T_0 \left(\frac{T_j}{T_0} - 1 - \log \frac{T_j}{T_0} \right) \tag{7}$$

plus a pressure term that we ignore. Only the logarithmic terms survive when Eq. (7) is substituted into Eq. (6), yielding

$$\begin{aligned} \dot{W}_{\rm lost} = {} & -\dot{M}_A c_{p,A} T_0 \log \frac{T_A}{T_0} - \dot{M}_B c_{p,B} T_0 \log \frac{T_B}{T_0} \\ & + \left(\dot{M}_A c_{p,A} + \dot{M}_B c_{p,B} \right) T_0 \log \frac{\dot{M}_A c_{p,A} T_A + \dot{M}_B c_{p,B} T_B}{\dot{M}_A c_{p,A} T_0 + \dot{M}_B c_{p,B} T_0}. \end{aligned} \tag{8}$$

To simplify this expression, let

$$x = \frac{\dot{M}_A c_{p,A}}{\dot{M}_A c_{p,A} + \dot{M}_B c_{p,B}}, \tag{9}$$

$$\tau_A = T_A/T_0, \quad \tau_B = T_B/T_0, \tag{10}$$

and find

$$\dot{W}_{\text{lost}} = \left(\dot{M}_A c_{p,A} + \dot{M}_B c_{p,B}\right) T_0 \log \frac{x + (1-x)\ \tau_B/\tau_A}{(\tau_B/\tau_A)^{(1-x)}}. \tag{11}$$

Setting $c_{p,A} = c_{p,B}$ and letting $\tau = \tau_B/\tau_A$ yields Eqs. (1)–(3), where $A \to 1$ and $B \to 2$ to match the notation in the Table in the book.

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