Design Environment for Low-amplitude Thermoacoustic Energy Conversion

DeltaEC
Version 6.2

Users Guide

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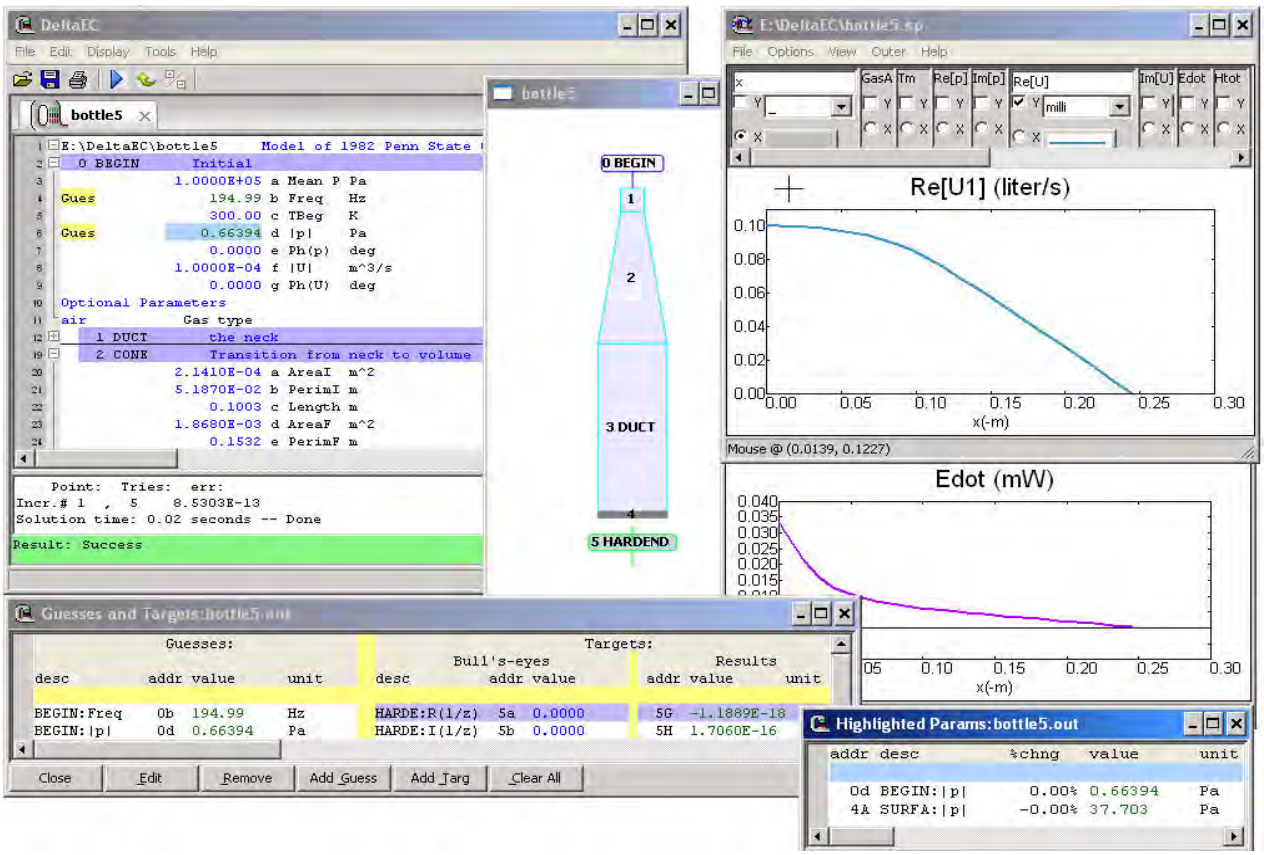

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1. Introduction

DeltaEC—Design Environment for Low-Amplitude ThermoAcoustic Energy Conversion—is a computer program that can calculate details of how thermoacoustic equipment performs, or can help the user to design equipment to achieve desired performance. Input data can be modified or entered via DeltaEC's user interface or by using any text editor. Results can be examined via the user interface, built-in graphics displays, the operating system's text utilities, or any spreadsheet or graphics software.

For good portability, the computational core of DeltaEC is compiled from FORTRAN-77 and the user interface is built on Python. The current code for IBM-compatibles requires at least a 386 processor, because it uses a DOS extender to create a flat 32-bit memory environment. All calculations are performed in double precision. Version 6 is currently running on Windows-based PCs, and a Macintosh version is due in 2008. We hope that our use of Python will soon yield true operating-system independence, so Linux and Unix variants will be easily supported, too.

The transition from DeltaE version 5 to DeltaEC version 6 marks the inclusion of thermoacoustic mixture separation and steady-flow effects, as well as the transition from a DOS-keyboard user interface to a Python graphical, keyboard-and-mouse user interface with a built-in plotter. "Transition Guide for Experienced Users" (<`TransitionGuide.pdf`>, in your DeltaEC installation folder) outlines these recent changes more fully.

1.1. What DeltaEC does

DeltaEC numerically integrates in one spatial dimension using a low-amplitude, "acoustic" approximation and sinusoidal time dependence. It integrates the wave equation and sometimes other equations such as the energy equation, in a gas (or a very compressible, thermodynamically active liquid), in a geometry given by the user as a sequence of segments (no more than 200) such as ducts, compliances, transducers, and thermoacoustic stacks or regenerators. A glance through the figures below will orient the new user to the broad range of situations that DeltaEC can handle.

DeltaEC always assumes a time dependence of $e^{i\omega t}$, so its "wave" equation is essentially the second-order Helmholtz differential equation for the complex pressure amplitude $p_1(x)$, which can be regarded as two coupled first-order differential equations for $p_1(x)$ and the complex volume flow rate amplitude $U_1(x)$. An integration of these differential equations is performed for each segment, with pressures, volume flow rates, and some other variables matched at the junctions between segments. In stacks and regenerators, the acoustic solution for pressures and volume flow rates is found simultaneously with the solution of the energy-flow equation in order to obtain the mean-temperature profile as well. The energy flow

Figure 1.1: Driven, lossy plane-wave resonator.

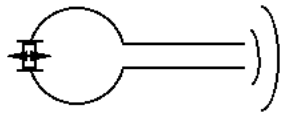


Figure 1.2: Driven, radiating Helmholtz resonator.

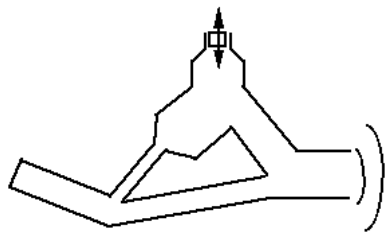


Figure 1.3: Driven duct network.

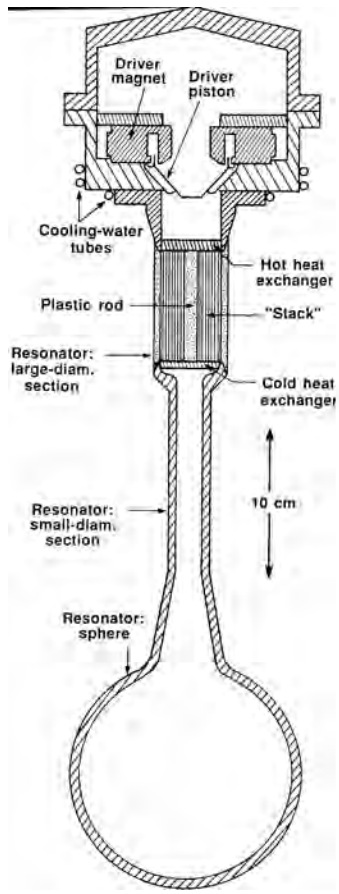


Figure 1.4: Standing-wave thermoacoustic refrigerator (Hofler style [1, 2]).

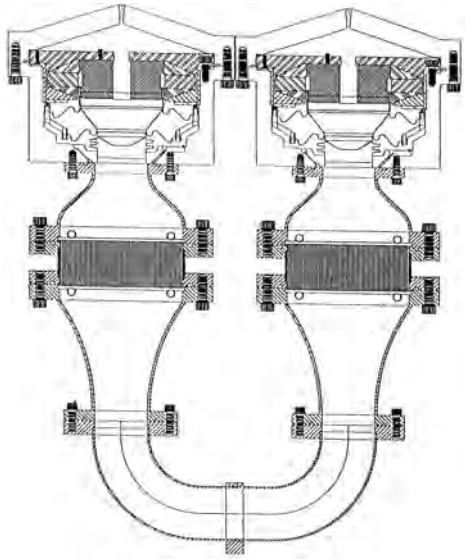


Figure 1.5: Standing-wave thermoacoustic refrigerator (TALSR style [3]).

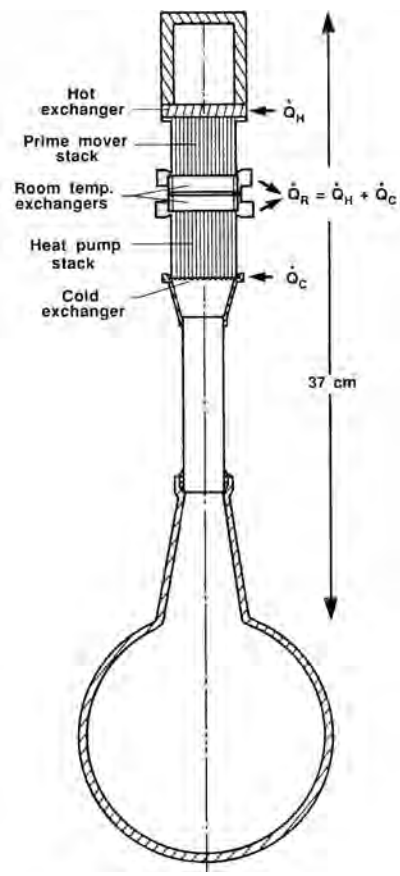


Figure 1.6: John Wheatley's heat-driven refrigerator, affectionately called "the beer cooler" in the early 1980s [4].

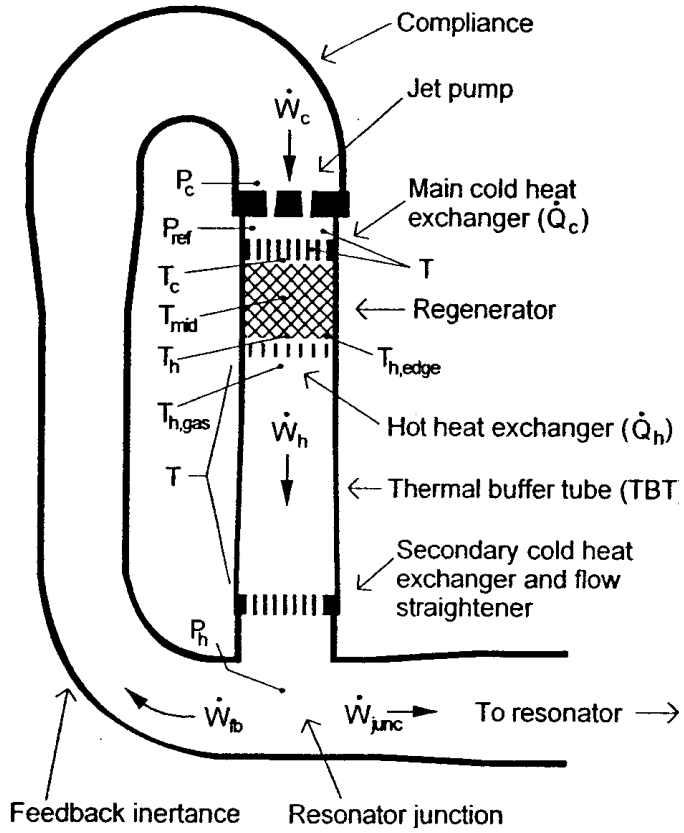


Figure 1.7: Thermoacoustic-Stirling hybrid engine [5].

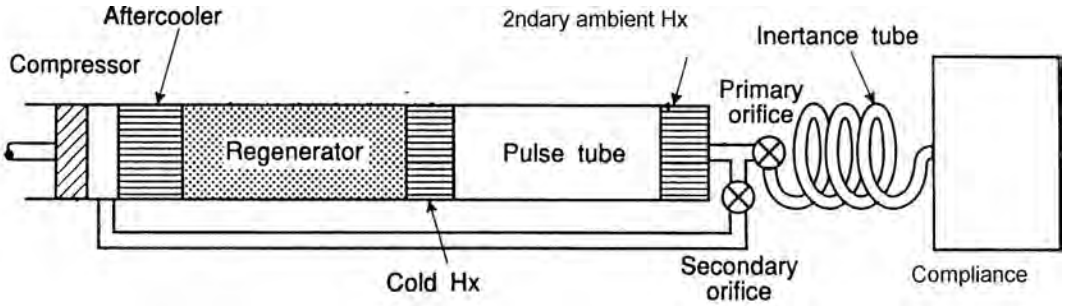


Figure 1.8: Double-inlet pulse-tube refrigerator [6, 7].

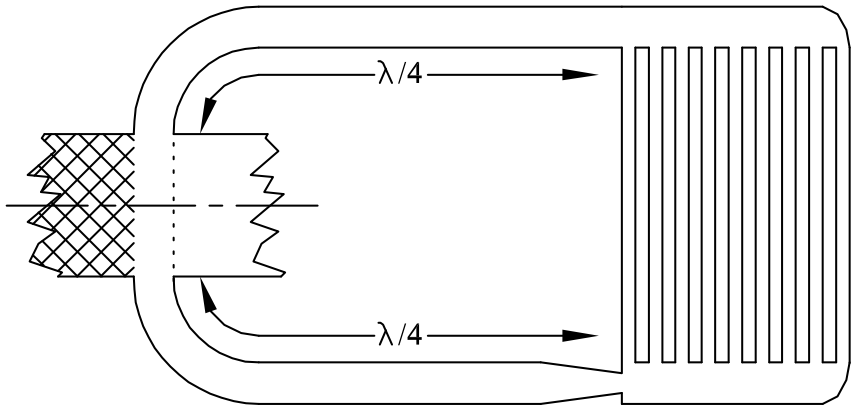


Figure 1.9: Resonant self-circulating heat exchanger with one gas diode. [8, 9]

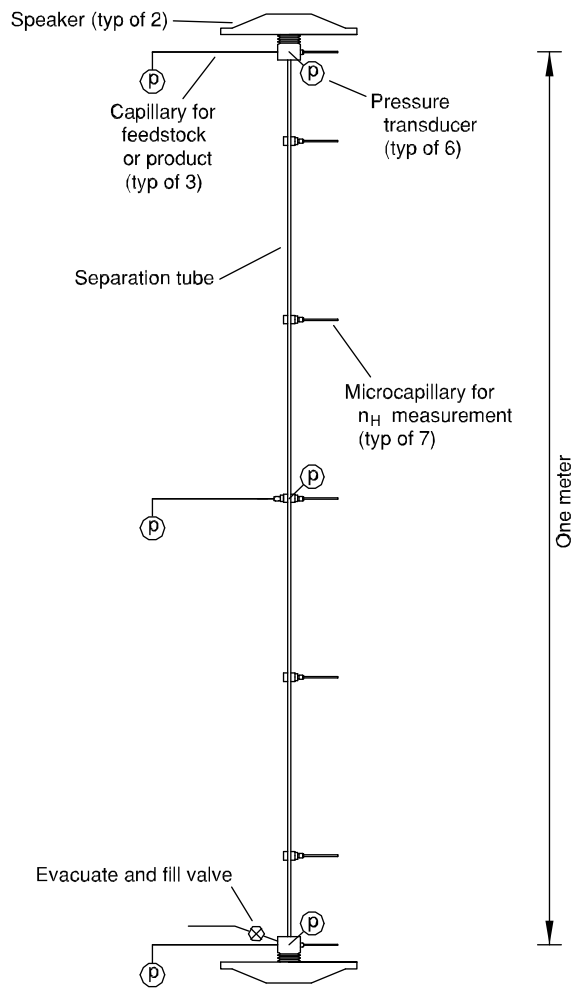


Figure 1.10: Half-wavelength helium-argon mixture separator [10].

through stacks and regenerators is controlled by temperatures and/or heat flows at adjacent heat exchangers. With binary gas mixtures in mixture-separation channels, the solution of the wave equation is found simultaneously with that of the equation describing the mole fluxes of the components of the mixture, inverted to find the light mole fraction as a function of position. Modeling of acoustics superimposed on steady flows is also supported.

With its multi-parameter shooting method to satisfy a variety of mixed boundary conditions, DeltaEC gives the user considerable freedom in choosing which variables are computed as "solutions." For example, in a simple acoustic resonator (the first example below), DeltaEC can compute the input impedance as a function of frequency, or the resonance frequency for a given geometry and gas, or the length required to give a desired resonance frequency, or even the concentration in a binary gas mixture required to give a desired resonance frequency in a given geometry.

DeltaEC includes few of the nonlinear effects that arise at high amplitudes, so be cautious using it when Mach numbers or Reynolds numbers are high. The principal exceptions to this rule are the turbulence algorithm in ducts and cones (introduced briefly in Chapter 2 and described in detail in Section 10.1.1) and the minor-loss segment, which models lumped-element dissipation where pressure difference is proportional to the square of the velocity (introduced briefly in Chapter 6 and described in detail in Section 10.2.4).

1.2. The Users Guide

This Users Guide is a <`.pdf`> document, compatible with Adobe Acrobat 6.0 and later versions. Its internal hyperlinking is convenient for online navigating. Its overall organization makes it suitable for studying a printed copy, and a few blank pages have been inserted so that double-sided printing yields a copy with all chapters starting on odd-numbered pages, so chapter tabs in a three-ring binder can be used. If some of the figures are garbled or missing from your printed version, your printer may not be postscript-compatible, and you should find the Advanced options button in Adobe Acrobat's print dialog, click it, and select "Print as image" at 300 dpi.

In Part I, "Tutorial," we teach the use of DeltaEC by increasingly complicated examples in Chapters 2–7. Chapter 2 is just acoustics, without thermoacoustics. It serves to introduce DeltaEC's input/output formats and editing, running, and plotting features. Chapter 5 gives the most complete discussion of the overall principles behind the thermoacoustics computations, and the simplest thermoacoustic engine and refrigerator examples. The agreement of such examples with published experimental data serves as validation of the code. Chapter 6 introduces the effects of superimposed steady flows, and Chapter 7 introduces separation mole fluxes and concentration gradients in binary-gas mixtures.

Part II, "Reference," includes segment-by-segment reference sections for the experienced user, documenting the assumptions built into the computations for each segment and the data format for each segment. "Reference" also includes chapters with calculation formulas for thermophysical properties. It is our hope that experienced users can quickly find the information they need in Part II, especially using the index and the internal hyperlinks, while new users will find the wordier explanations of the Tutorial chapters helpful.

The examples we've included are simpler than DeltaEC files we use in our own research.

We've maintained this simplicity in the Users Guide to avoid clutter. Experienced users will find that the number of segments in their DeltaEC files grows and grows, as small effects are included and non-standard results are displayed with `RPN` segments.

Some of the examples here were run in MS-DOS, others on a Mac, others with Windows. Many of the examples were created using earlier versions of DeltaE and DeltaEC, so there may be some formatting irregularities. In Chapter 2, screen shots of the user interface are often shown, but in later chapters most examples are displayed as they appear in a text editor after DeltaEC has saved them to the disk. This keeps the Users Guide <`.pdf`> file size smaller.

We assume that the reader of the Users Guide is very comfortable with basic linear acoustics [11] and reasonably familiar with thermoacoustics. The choices of which variables to regard as independent and integrable reflects the philosophy of Ref. [12], extended here to accommodate mixture separation and superimposed steady flow. We use variables as defined, for example, in the lists of symbols in Refs. [12] and [4], and Chapter 15.

1.3. Obtaining DeltaEC

The latest version of DeltaEC is freely available for noncommercial, educational, and evaluation use from `www.lanl.gov/thermoacoustics/`. DeltaEC is under continual development, so regular users should update their copies occasionally. Users contemplating commercial use of the software should contact Bill Ward (`ww@lanl.gov`) for an update on the current policy.

There is no formal registration for DeltaEC, no fee, and no formal support or warranty of any kind (please read the copyright notice and disclaimer in Section 1.6). We are interested in hearing from users so that we can fix any bugs that are found. Please report any bugs related to platform compatibility and the user interface to Bill Ward (`ww@lanl.gov`), with a cc to Greg Swift (`swift@lanl.gov`); please report any bugs related to thermoacoustic computations to Greg with a cc to Bill.

News of your successes using this code will encourage us and our sponsors to consider this effort worthwhile and will enable us to continue DeltaEC's development and freely-shared status. We are especially grateful when you acknowledge DeltaEC in publications and reports and when you mention it to individuals at agencies that support acoustics research. This improves our chances to create and distribute improvements to DeltaEC in the future.

1.4. Installing DeltaEC

Download the installation file from the Los Alamos thermoacoustics website, and run it. DeltaEC should automatically install itself in <`d:\Program Files\DeltaEC`> or some other sensible default folder. This Users Guide (<`UsersGuide.pdf`>) appears in the folder <`DeltaEC`>, as does the two-page <`TransitionGuide.pdf`> for users with considerable experience with earlier versions of DeltaE. A subfolder includes the example files discussed in the rest of the Users Guide. In Windows, this all appears as a program group accessible in the usual way through the Start button, and a desktop icon for launching the program should appear. The file extension <`.out`> should be automatically associated with DeltaEC,

so double clicking any <`.out`> file should also launch the program (after performing the standard rebooting of the computer that Windows always requires for establishing such file association after program installation).

DeltaEC can be uninstalled via the Windows Control Panel, Add or Remove Programs; or by running <`d:\Program Files\DeltaEC\DE-uninst.exe`>. To install a new version of DeltaEC, it shouldn't be necessary to un-install a previous version, but uninstalling the previous version (or manually deleting all the files and subfolders in <`d:\Program Files\DeltaEC`>) will help clean up unwanted junk such as test files that you might have created in the <`Examples`> folder. If a re-installation of DeltaEC does not work as it should, even after uninstalling the previous version, look for ghost processes named <`DeltaEC.exe`> in the Windows task manager, and kill them; or reboot your computer to kill such ghosts.

1.5. Acknowledgments

The development of DeltaE and DeltaEC has been supported in part by many agencies and entities: Tektronix Corporation, SPAWAR, the Naval Postgraduate School, ONR, Praxair, and, most importantly, by several offices of the Department of Energy: Advanced Industrial Concepts, Materials Science (a part of Basic Energy Sciences in the Office of Science), the Technology Transfer Initiative Office, and the Office of Fossil Energy. Local support at Los Alamos has also been provided, via our Industrial Partnership Office (now called the Technology Transfer Division), our Locally Directed R&D program, and our group management.

In the 1990s, a long discussion with Pat Arnott helped us define the initial scope of this work, and comparisons with the results of parallel-plate-stack codes (written by Al Migliori and Dick Martin) were useful in the early stages. Suggestions by Kim Godshalk, Charles Jin, Tom Hofler, Jeff Olson, Scott Backhaus, Vince Kotsubo, and Jalal Zia led to significant improvements in capability and usability through the years. A project-kickoff meeting at Penn State motivated the development of the `RPN` segment. Charles Jin, Ray Radebaugh, and the code REGEN3.1 were indispensable in development of the stacked-screen algorithm. Scott Backhaus, David Gardner, Matt Poese, and Steve Garrett have helped shape the DeltaEC user interface.

1.6. Copyright

This Software was produced under a U.S. Government contract (DE-AC52-06NA25396) by Los Alamos National Laboratory, which is operated by the Los Alamos National Security, LLC (LANS) for the U.S. Department of Energy's National Nuclear Security Administration. The U.S. Government is licensed to use, reproduce, and distribute this Software. Permission is granted to the public to copy and use this Software without charge, provided that this Notice and any statement of authorship are reproduced on all copies. Neither the Government nor the LANS makes any warranty, express or implied, or assumes any liability or responsibility for the user of this Software.

Part I: Tutorial

2. Acoustics & user interface

In this Chapter we use the simplest acoustic segments, especially ducts and cones, to introduce DeltaEC's most basic acoustics and user-interface features.

2.1. Acoustics in DeltaEC

DeltaEC deals with one-dimensional sequences of acoustic and thermoacoustic elements, called *segments*, so DeltaEC's "wave" equation is one-dimensional. We always assume a time dependence of $\mathrm{Re}[e^{i\omega t}]$, so the "wave" equation can be taken as the second-order Helmholtz differential equation for the complex pressure amplitude $p_1(x)$. In its most familiar form, for an x-independent cross sectional area A, without viscous or thermal-hysteresis losses, it is

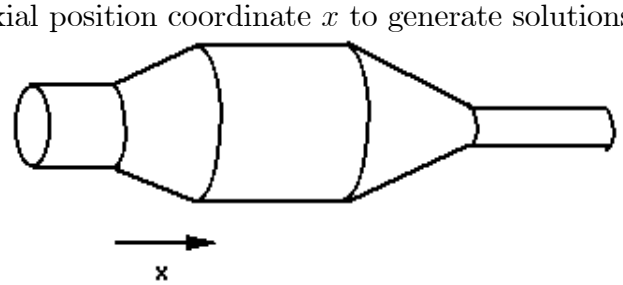
$$p_1 + \frac{a^2}{\omega^2}\frac{d^2 p_1}{dx^2} = 0. \tag{2.1}$$

It is often easier to think of this second-order equation as two coupled first-order equations in pressure p_1 and volume flow rate U_1 :

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{A}U_1, \tag{2.2}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A}{\rho_m a^2}p_1. \tag{2.3}$$

This point of view is taken in Ref. [12]. The dp_1/dx equation is derived from the momentum equation of fluid mechanics and the dU_1/dx equation is derived from the continuity equation of fluid mechanics. In this form, the equations are ready for simultaneous numerical integration along the axial position coordinate x to generate solutions $p_1(x)$ and $U_1(x)$.



DeltaEC uses more complicated momentum and continuity equations that include additional effects such as dissipation of acoustic power along the sides of ducts. It uses different equations in different segments to suit local circumstances. For example, in boundary-layer

approximation in large-diameter ducts and shallow cones, the governing equations are

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{A}\left[1-\frac{1-i}{2}\frac{\Pi}{A}\delta_\nu\right]^{-1} U_1, \tag{2.4}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A}{\rho_m a^2}\left[1+\frac{1-i}{2}\frac{\Pi}{A}\frac{\gamma-1}{1+\epsilon_s}\delta_\kappa\right] p_1. \tag{2.5}$$

where A is the cross-sectional area, Π is the perimeter, δ_ν and δ_κ are the viscous and thermal penetration depths, respectively, γ is the ratio of isobaric and isochoric specific heats, and ϵ_s is a correction for thermal properties of the solid wall that is usually negligible. The equations for each DeltaEC segment are given in Chapters 10 and 11.

DeltaEC uses continuity of p_1 and U_1 to pass from the end of one segment to the beginning of the next. Within each segment, wave propagation depends on local parameters such as area and perimeter as well as on global parameters such as frequency. Although DeltaEC uses analytic solutions to the governing equations for some of the simplest segment types, it usually must integrate the equations numerically, so it is generally correct to imagine DeltaEC beginning at the beginning of a series of segments and numerically integrating "a momentum equation" $\mathcal{F}_{\text{momentum}}$, such as Eq. (2.2) or (2.4), and "a continuity equation" $\mathcal{F}_{\text{continuity}}$, such as Eq. (2.3) or (2.5), through each segment, sequentially, to the end of the series of segments, always using the local values of variables such as area, perimeter, and p_1 and U_1 themselves:

$$\frac{dp_1}{dx} = \mathcal{F}_{\text{momentum}}(p_1, U_1, T_m, p_m, \omega, \text{geometry, gas properties, solid properties, etc.}), \tag{2.6}$$

$$\frac{dU_1}{dx} = \mathcal{F}_{\text{continuity}}(p_1, U_1, T_m, p_m, \omega, \text{geometry, gas properties, solid properties, etc.}), \tag{2.7}$$

where most variables on the right-hand sides of these equations are functions of x.

The solution $p_1(x)$, $U_1(x)$ is only determined uniquely if four real boundary conditions are imposed, because the governing equations can be expressed as two coupled first-order equations in two complex variables or as four coupled first-order equations in four real variables. This is true whether considering a single segment or a one-dimensional series of segments with each joined to its neighbor(s) by continuity of p_1 and U_1. If all four boundary conditions are given at the initial end of the apparatus (i.e., if we know the magnitude and phase of p_1 and the magnitude and phase of U_1 at the initial end) then the integration is straightforward, proceeding from the initial end to the final end. But usually one or more boundary conditions are instead given at the final end. In such circumstances DeltaEC uses a shooting method,[1] by *guess*ing any unknowns among the four numbers defining p_1

[1] Precisely speaking, DeltaEC forms a system of nonlinear equations from the model using the guesses and targets that the user selects, and manipulates the guesses to make the integration results match the targets. The routine incorporated in the code is called DNSQ, and it is part of the SLATEC Common Mathematical Library, which is freely available through the internet software repository at "http://www.netlib.org." Its algorithm is a modification of the Powell hybrid method. See Chapter 8 for details.

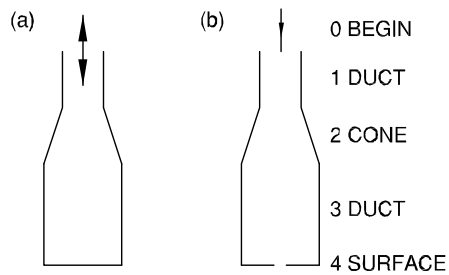


Figure 2.1: A simple open-topped bottle for the first example, <`bottle0.out`>. (a) Conventional representation. (b) DeltaEC representation.

and U_1 at the initial end of the integration, integrating to the other end, comparing the results with the *target* boundary conditions imposed at that other end or elsewhere, and adjusting its guesses until the integration results meet the targets.

One of DeltaEC's most powerful features is that the guesses are not limited to the conventional choices consisting of real and imaginary parts of p_1 and U_1 (or, equivalently, magnitudes and phases) at the initial end of the numerical integration along x. Any variables that have an effect on the downstream target variables can be used. This enables DeltaEC to calculate a resonance frequency, a geometrical dimension, a temperature, or even the concentration in a binary gas mixture in order to satisfy given boundary conditions.

2.2. I/o file structure

We begin this tutorial with an open-topped bottle. This example introduces the basic input/output file structure of DeltaEC and how to "run" the numerical integration.

DeltaEC recognizes all of its own output files as valid input files. In practice, users rarely make a new file from scratch; it is much more common to start with an existing file and modify it as needed. Thus, we begin with a file called <`bottle0.out`> (included in the <`Examples\Acoustics`> directory or folder that DeltaEC installs, usually in <`d:\ProgramFiles\DeltaEC\Examples\Acoustics`>), which we created earlier from DeltaEC's user interface. Figure 2.1 shows the acoustician's usual cartoon of this bottle and a pictorial representation of how we modeled it for DeltaEC. For now, examine this file in a text editor. A DeltaEC model file must always be a plain text file, in the native text format of the machine on which it is running.

```
TITLE     Model of 1982 Penn State Championship Bottle
!->E:\deltaec\TEXFILES\examples\acoust\BOTTLE0.OUT
!Created@10:24:32  01-Jun-2007 with DeltaEC Vers. 6.0a0 under win32
!Using Win 5.1.2600 (Service Pack 2) under Python DeltaEC.
!--------------------------------  0 ---------------------------------
BEGIN      The mouth
 1.0000E+05 a Mean P Pa
  300.00    b Freq   Hz
  300.00    c TBeg   K
    1.0000  d |p|    Pa
     0.0000 e Ph(p)  deg
 1.0000E-04 f |U|    m^3/s
     0.0000 g Ph(U)  deg
air           Gas type
!--------------------------------  1 ---------------------------------
DUCT       the neck
 2.1410E-04 a Area   m^2                    0.0000 A |p|    Pa
 5.1870E-02 b Perim  m                      0.0000 B Ph(p)  deg
```

```
 1.7780E-02 c Length m                      0.0000 C |U|    m^3/s
                                            0.0000 D Ph(U)  deg
                                            0.0000 E Htot   W
ideal           Solid type                  0.0000 F Edot   W
!--------------------------------  2 --------------------------------
CONE       Transition from neck to volume
 2.1410E-04 a AreaI  m^2                    0.0000 A |p|    Pa
 5.1870E-02 b PerimI m                      0.0000 B Ph(p)  deg
    0.1003  c Length m                      0.0000 C |U|    m^3/s
 1.8680E-03 d AreaF  m^2                    0.0000 D Ph(U)  deg
    0.1532  e PerimF m                      0.0000 E Htot   W
ideal           Solid type                  0.0000 F Edot   W
!--------------------------------  3 --------------------------------
DUCT       Bottle volume
 1.8680E-03 a Area   m^2                    0.0000 A |p|    Pa
    0.1532  b Perim  m                      0.0000 B Ph(p)  deg
    0.1270  c Length m                      0.0000 C |U|    m^3/s
                                            0.0000 D Ph(U)  deg
                                            0.0000 E Htot   W
ideal           Solid type                  0.0000 F Edot   W
!--------------------------------  4 --------------------------------
SURFACE     Bottom End
 1.8680E-03 a Area   m^2                    0.0000 A |p|    Pa
                                            0.0000 B Ph(p)  deg
                                            0.0000 C |U|    m^3/s
                                            0.0000 D Ph(U)  deg
                                            0.0000 E Htot   W
ideal           Solid type                  0.0000 F Edot   W
```

Several features of DeltaEC model files are illustrated here. Model files should be named <`something.out`>. These files consist of a set of "segments" whose order and format are important. After a `TITLE` (with an optional heading where the user can type notes), the initial (or "zeroth") segment is always a `BEGIN` segment. This is one of the "logistical" segments of DeltaEC, usually having no actual geometrical correspondence to parts of the hardware being modeled. Subsequent segments describe the geometry and other properties of the "physical" hardware parts of the acoustic engine—here, `DUCT`s, a `CONE`, and a `SURFACE`. All units are MKS. The number and order of data in each segment is crucial. Within each line, the first number (e.g., "`1.e5`" or "`100.`") or word (e.g., "`helium`" or "`BEGIN`") is the most important to DeltaEC as input.[2] Subsequent fields on each line give the units or name of the variable whose value appears. Lines that begin with "!" are comments with no impact on the integrations. Numbers can be in fixed or exponential format.[3] Segment names are stored uppercase, and only the first five characters are interpreted.

The column of zeroes on the right side shows where DeltaEC's calculated results will appear. At the moment, they are all zero because DeltaEC has not yet calculated anything.

`BEGIN` sets the stage, in this case, with 1-bar room-temperature air being driven at 300 Hz with a pressure amplitude of 1 Pa and a volume flow rate amplitude of 0.0001 m^3/s, in phase with the pressure oscillation. A few of these variables, such as frequency, extend unchanged through all subsequent segments. Other variables, such as pressure amplitude, are initial conditions for DeltaEC's numerical integration through subsequent segments.

A lossy duct segment, `DUCT`, comes next. Here, we have made the perimeter equal to $\sqrt{4\pi \times \text{area}}$, to make this a circular duct. A `CONE` and another `DUCT` come next, completing the shape of the bottle. The file ends with a `SURFACE` to account for oscillatory-pressure thermal-hysteresis dissipation at the bottom.

[2] Exceptions: The file reader looks beyond the first field of characters to resolve ambiguity about optional parameters, described later.

[3] In some computers, integers must be followed by a decimal point. Also, some computers require the last line in the file to be followed by an end-of-line character, before the end-of-file character occurs.

2.3. Running DeltaEC

As children, most of us learn that blowing across the mouth of a bottle produces a pleasant audible tone. With this example, the new user of DeltaEC will obtain the frequency of that tone for a bottle of specified dimensions, while exploring the user interface and learning to "run" the numerical integration.

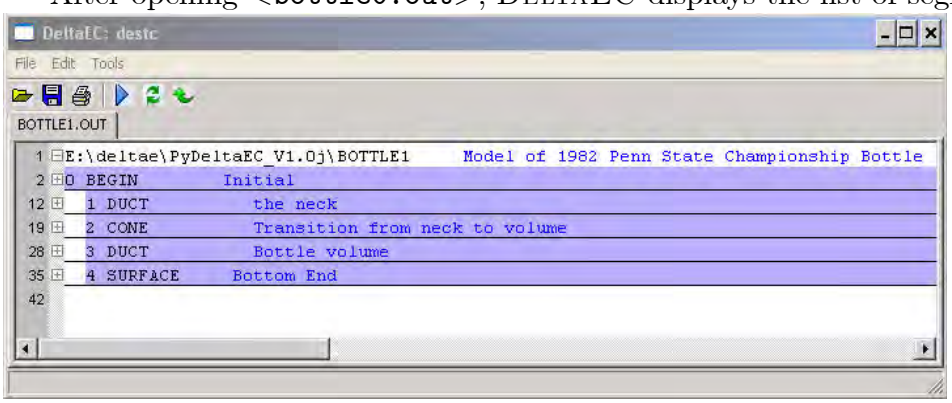
Open the file <`bottle0.out`>, using whichever of these three methods you prefer:

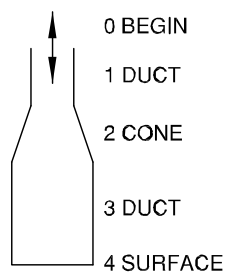
1. Open the folder <`d:\ProgramFiles\DeltaEC\Examples\Acoustics`> (or whatever other path your installation has created), and double click on the file <`bottle0.out`>.

2. In Windows, find DeltaEC through the Start button at the bottom-left corner of your desktop, via Programs and the DeltaEC group. Double click on the program to launch it. Then open the file <`bottle0.out`> via the DeltaEC Files pulldown menu or by clicking the icon that looks like a manila folder. In either case, find your way to the <`Examples\Acoustics`> folder in the usual way.

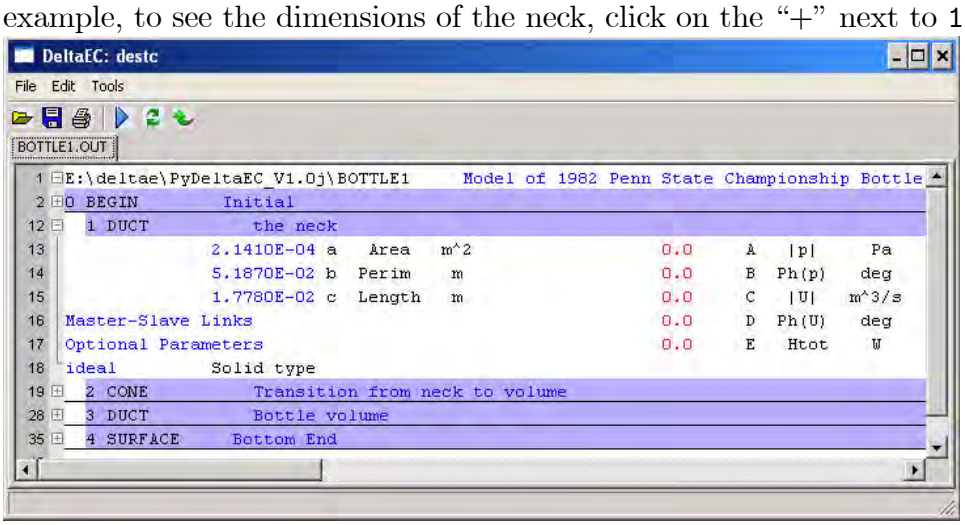
3. Double click on the Desktop DeltaEC icon to launch DeltaEC, and open the file <`bottle0.out`> as described in method 2.

After opening <`bottle0.out`>, DeltaEC displays the list of segments, like this:





Clicking on the "+" next to any segment's name exposes that segment's details. For example, to see the dimensions of the neck, click on the "+" next to `1 DUCT`:



Blue text (numbers or words) means you can double click or right click to make a change or see more options. Italic red numbers indicate what will be a result, as soon as the calculation is run, but at the moment these numbers are probably inconsistent with the blue input numbers. Green numbers that you will see shortly indicate good results: calculated by DeltaEC and consistent with all the other numbers in the model.

You can change the size of the font in DeltaEC's main display by spinning your mouse wheel while holding down the Ctrl key, or by using Ctrl n, Ctrl b, and Ctrl /. (Be sure the computer's focus is on the DeltaEC main display—click it if necessary.)

Now show all the segments in the model, by clicking on the "+" for each segment or on the big "+/−" on the toolbar. What you see should look like this:[4]

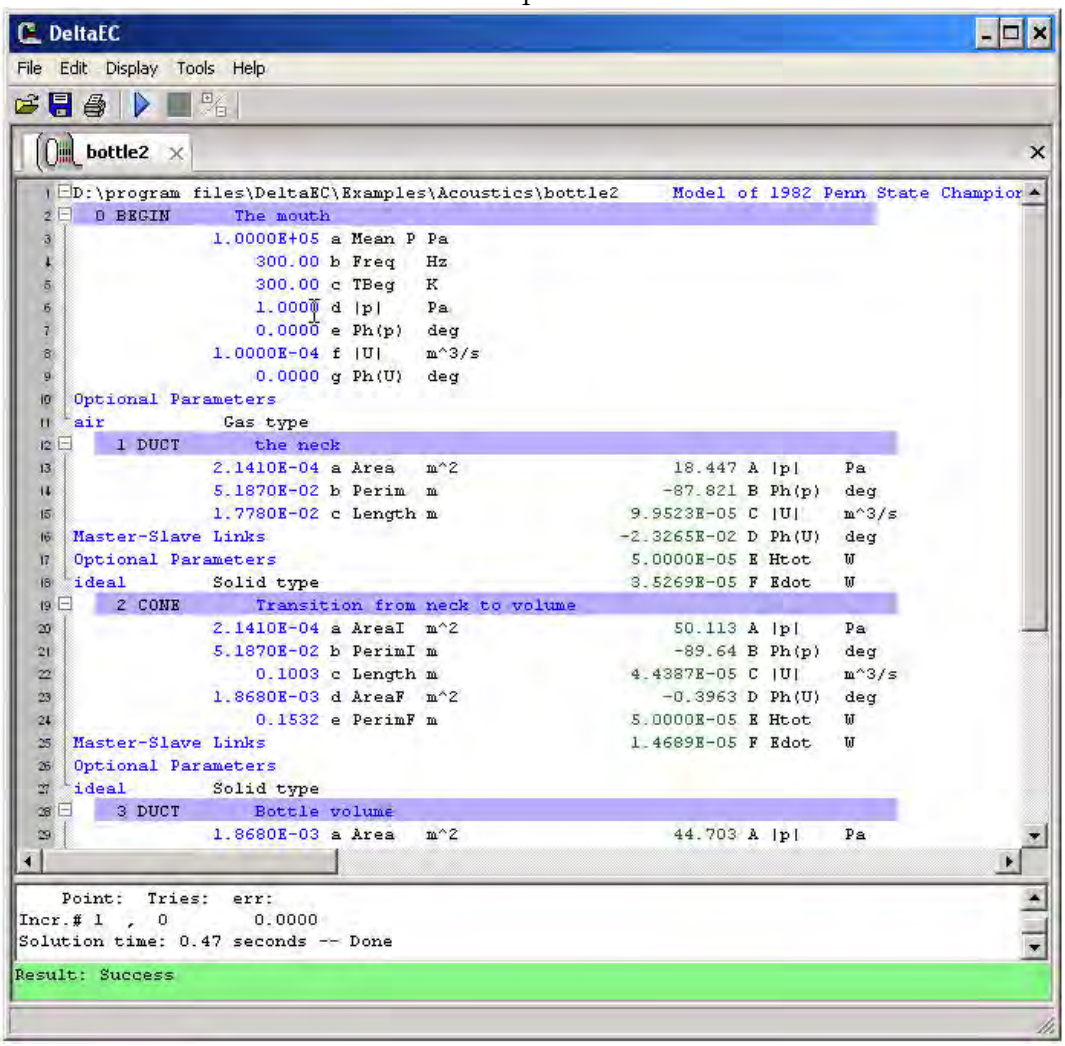
```
TITLE     Model of 1982 Penn State Championship Bottle
!---------------------------------  0 ---------------------------------
 BEGIN      The mouth
  1.0000E+05 a Mean P    Pa
   300.00    b Freq      Hz
   300.00    c TBeg      K
     1.0000  d  |p|      Pa
     0.0000  e Ph(p)     deg
  1.0000E-04 f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 air        Gas type
!---------------------------------  1 ---------------------------------
 DUCT       the neck
  2.1410E-04 a Area      m^2               0.0000   A |p|       Pa
  5.1870E-02 b Perim      m                0.0000   B Ph(p)     deg
  1.7780E-02 c Length     m                0.0000   C |U|     m^3/s
                                           0.0000   D Ph(U)     deg
                                           0.0000   E Hdot      W
 ideal    Solid type                       0.0000   F Edot      W
!---------------------------------  2 ---------------------------------
 CONE       Transition from neck to volume
 2.1410E-04  a AreaI     m^2               0.0000   A |p|       Pa
 5.1870E-02  b PerimI     m                0.0000   B Ph(p)     deg
     0.1003  c Length     m                0.0000   C |U|     m^3/s
 1.8680E-03  d AreaF     m^2               0.0000   D Ph(U)     deg
     0.1532  e PerimF     m                0.0000   E Hdot      W
  ideal      Solid type                    0.0000   F Edot      W
!---------------------------------  3 ---------------------------------
 DUCT       Bottle volume
 1.8680E-03  a Area      m^2               0.0000   A |p|       Pa
     0.1532  b Perim      m                0.0000   B Ph(p)     deg
     0.1270  c Length     m                0.0000   C |U|     m^3/s
                                           0.0000   D Ph(U)     deg
                                           0.0000   E Hdot      W
 ideal      Solid type                    0.0000   F Edot      W
(etc.)
```

The `BEGIN` segment establishes that the bottle is full of air at atmospheric pressure and room temperature, and it launches a 300-Hz wave into the bottle with a pressure amplitude of 1 Pa and a volume-flow-rate amplitude of 10^{-4} m^3/s. (We will examine its "optional" parameters in later chapters, when we encounter them.)

Now, run the file, by typing 'r', by clicking the blue "Run" button, or by accessing "Run" in the Tools pulldown menu. (To encourage users to get in the habit of using the keyboard shortcuts, the Users Guide summarizes all such options in the briefest possible way, with the shortcut letter underlined, e.g., run. Infrequently used operations whose shortcuts the user has not memorized can be found by browsing the pulldown menus.)

[4] Please ignore minor formatting differences between what you see on your computer screen and what is present in the Users Guide. Much of the Users Guide was written before the user interface was complete, and it's much easier for us to let text-formatted results page-break automatically in the Users Guide than to manage large figures that show the actual computer-screen display. Text-formatted results also minimize the Users Guide file size.

Upon running, the Run Monitor appears at the bottom of the display, indicating with its cheerful green color and the word "Success" that the run has succeeded, in this case taking only a fraction of a second. (Typing "b" toggles the Run Monitor display off and on, and resets it to default size.) Now the results in the right column of the model display are nonzero, and they have changed from red italic to green, indicating that they are consistent with one another and with the blue input numbers:



DELTAEC has integrated through the model and calculated some results, in particular $p_1(x)$ and $U_1(x)$. The results in the right column of the <`.out`> file show values for $p_1(x)$ and $U_1(x)$ and other results at the *end* of each segment. If a few numbers are all you want to see, you are done!

One of the other results is the acoustic power `Edot`,

$$\dot{E} = \frac{1}{2}\mathrm{Re}\left[p_1\widetilde{U}_1\right], \tag{2.8}$$

which is always listed at line `F`.

The total power `Htot`, listed at line `E`, is introduced in Chapter 5; it can safely be ignored in this chapter, and for all purely acoustic problems in which heat exchangers and time-averaged temperature gradients in the x direction are of no concern.

2.4. State plots

State plots allow the user to view graphs showing the distribution of temperature, pressure, volume flow rate, energy, and many other variables as functions of x along the entire length of a model or any subset of that length. After each successful run, DeltaEC creates a state-plot file, <`.sp`>, with this information in tabular format. You can open this file from your hard disk folder with any text editor or spreadsheet program, or display it with DeltaEC's plotter, using "Plot SP file" in the Display pulldown menu. The example below, <`bottle2.sp`> (<`bottle2`> is identical to <`bottle0`>, except that it includes the <`.ssv`> file, described below), does not show all of the available columns of data; a text editor or spreadsheet would let you scroll across to view more columns to the right.

```
 ->bottle2.sp
!Created@15:28:56  7-Apr-07 with DeltaE Vers. 6.0g7 for the IBM/PC-Compatible
 -= Model of 1982 Penn State Championship Bottle                              =-
Leg:Seg x(m) GasA(m^2)   T(K)   Re[p](Pa)   Im[p](Pa)   Re[U](m3/s) Im[U](m3/s) Edot(W)     Htot(W)     Xdot(W)     H2k(W)
 0:  1  0.000 2.1410E-04  300.0     1.0000      0.0000   1.0000E-04     0.0000   5.0000E-05  5.0000E-05  5.0000E-05  5.0000E-05
 0:  1  0.002 2.1410E-04  300.0     0.9704     -1.8463   9.9995E-05 -5.0520E-09  4.8524E-05  5.0000E-05  4.8524E-05  5.0000E-05
 0:  1  0.004 2.1410E-04  300.0     0.9408     -3.6924   9.9981E-05 -9.8803E-09  4.7049E-05  5.0000E-05  4.7049E-05  5.0000E-05
 0:  1  0.005 2.1410E-04  300.0     0.9111     -5.5381   9.9957E-05 -1.4485E-08  4.5574E-05  5.0000E-05  4.5574E-05  5.0000E-05
 0:  1  0.007 2.1410E-04  300.0     0.8813     -7.3833   9.9924E-05 -1.8864E-08  4.4099E-05  5.0000E-05  4.4099E-05  5.0000E-05
 0:  1  0.009 2.1410E-04  300.0     0.8514     -9.2278   9.9881E-05 -2.3019E-08  4.2625E-05  5.0000E-05  4.2625E-05  5.0000E-05
 0:  1  0.011 2.1410E-04  300.0     0.8215    -11.071    9.9828E-05 -2.6949E-08  4.1152E-05  5.0000E-05  4.1152E-05  5.0000E-05
 0:  1  0.012 2.1410E-04  300.0     0.7915    -12.914    9.9766E-05 -3.0653E-08  3.9680E-05  5.0000E-05  3.9680E-05  5.0000E-05
 0:  1  0.014 2.1410E-04  300.0     0.7615    -14.755    9.9695E-05 -3.4132E-08  3.8208E-05  5.0000E-05  3.8208E-05  5.0000E-05
 0:  1  0.016 2.1410E-04  300.0     0.7314    -16.595    9.9614E-05 -3.7385E-08  3.6738E-05  5.0000E-05  3.6738E-05  5.0000E-05
 0:  1  0.018 2.1410E-04  300.0     0.7013    -18.434    9.9523E-05 -4.0412E-08  3.5269E-05  5.0000E-05  3.5269E-05  5.0000E-05
 0:  2  0.018 2.1410E-04  300.0     0.7013    -18.434    9.9523E-05 -4.0412E-08  3.5269E-05  5.0000E-05  3.5269E-05  5.0000E-05
 0:  2  0.028 3.0593E-04  300.0     0.5705    -27.065    9.8705E-05 -5.6940E-08  2.8928E-05  5.0000E-05  2.8928E-05  5.0000E-05
 0:  2  0.038 4.1411E-04  300.0     0.4906    -33.188    9.7222E-05 -7.4046E-08  2.5078E-05  5.0000E-05  2.5078E-05  5.0000E-05
 0:  2  0.048 5.3864E-04  300.0     0.4382    -37.707    9.4923E-05 -9.2334E-08  2.2540E-05  5.0000E-05  2.2540E-05  5.0000E-05
 0:  2  0.058 6.7951E-04  300.0     0.4021    -41.129    9.1662E-05 -1.1242E-07  2.0741E-05  5.0000E-05  2.0741E-05  5.0000E-05
 0:  2  0.068 8.3673E-04  300.0     0.3762    -43.761    8.7296E-05 -1.3494E-07  1.9373E-05  5.0000E-05  1.9373E-05  5.0000E-05
 0:  2  0.078 1.0103E-03  300.0     0.3570    -45.799    8.1690E-05 -1.6053E-07  1.8260E-05  5.0000E-05  1.8260E-05  5.0000E-05
 0:  2  0.088 1.2002E-03  300.0     0.3425    -47.373    7.4714E-05 -1.8986E-07  1.7293E-05  5.0000E-05  1.7293E-05  5.0000E-05
 0:  2  0.098 1.4065E-03  300.0     0.3313    -48.576    6.6246E-05 -2.2360E-07  1.6404E-05  5.0000E-05  1.6404E-05  5.0000E-05
(etc., etc.)
```

State-plot files have the following features:

- Numbers are lined up in columns. The initial columns are leg number (relevant when `TBRANCH` and/or multiple `BEGIN` segments are used; described in Section 11.2.1), segment number, and x.
- DeltaEC writes `Nint` + 1 lines of data for each segment that it integrates (e.g., ducts, cones, and stacks). `Nint` is the number of Runge-Kutta integration steps—see Section 8.1 for details.
- DeltaEC writes two lines of data for lumped elements (e.g., `SURFACE`) and other short segments that do direct end-to-end calculations (e.g., most heat exchangers): one line before and one line after the segment is computed.
- Logistical segments (e.g., `BEGIN`) that do not correspond to any hardware parts generate nothing in the <`.sp`> file.

The file <`bottle2.ssv`> instructed DeltaEC's plotter that we would like to see graphical plots of $p_1(x)$ and $U_1(x)$ after each run. These graphs, shown in Fig. 2.2, should have appeared on your computer after the run, and will automatically refresh after each new run, as the user makes changes to the model.

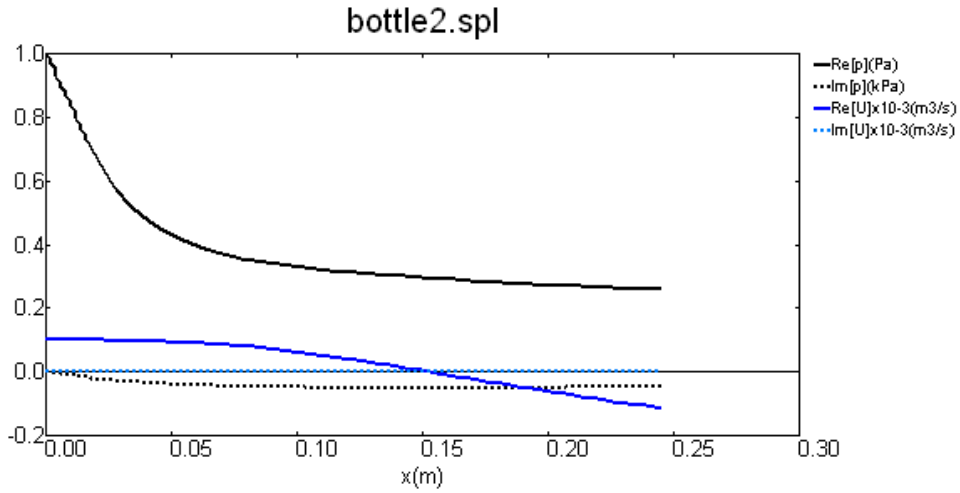


Figure 2.2: State plots of p_1 and U_1 for <`bottle2`>.

The real parts of p_1 and U_1 show the values of oscillating pressure and oscillating volumetric flow at times $\omega t = 0$, 2π, 4π, ... and the imaginary parts show their values at times $\omega t = -\pi/2$, $2\pi - \pi/2$, $4\pi - \pi/2$, ...

DeltaEC's plotter lets the user select other plots easily. The time-averaged product of p_1 and U_1 is acoustic power $\dot{E}$, which is also interesting. To change the display for this model to $\dot{E}$ vs x, check the `y` box under `Edot`, and uncheck the `Re[p]`, `Im[p]`, `Re[U]`, and `Im[U]` boxes, to obtain Fig. 2.3. Chapter 3 explains much more about DeltaEC's plot capabilities.

Now use Save Copy As to save this file with a new name, <`bottle3.out`>. Examine the resulting file with a text editor. Note that the output results giving p_1 and U_1 at the ends of the `DUCT`, `CONE`, `DUCT`, and `SURFACE` are now part of the file. The fact that we want to plot $\dot{E}$ vs x from now on has also been included in the associated <`bottle3.ssv`> file.

2.5. Guesses and targets

The astute reader may have noticed that U_1 was not zero at the `SURFACE` above. (In fact, we left a hole in the bottom of Fig. 2.1(b) to indicate the possibility of this leakage.) This shows that our choice of initial values for p_1 and U_1 at the `BEGIN` was incompatible with a bottle that is supposed to be sealed at the bottom. In this section we introduce DeltaEC's shooting method to resolve this issue, allowing boundary conditions at the end of a model (or anywhere else in a model) to be "targeted" to desired values.

Start DeltaEC again, opening file <`bottle2.out`>. As a matter of good habit, run it, and check some plots for sanity (your own sanity, as well as DeltaEC's). Now open the "Guesses and Targets" summary display box. You can open this box through the pull-down menus by clicking on Display, then Guesses Targets. Alternatively, just type "g"(the "g" stands for "guesses"). The guess–target summary should look like this:

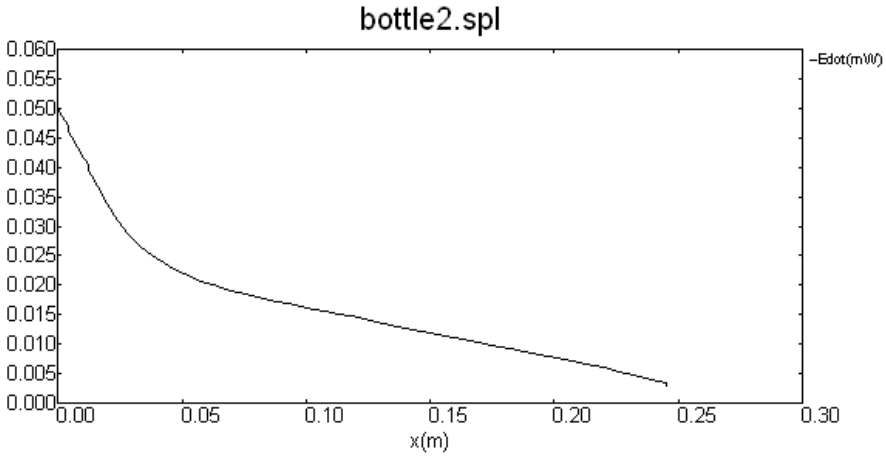
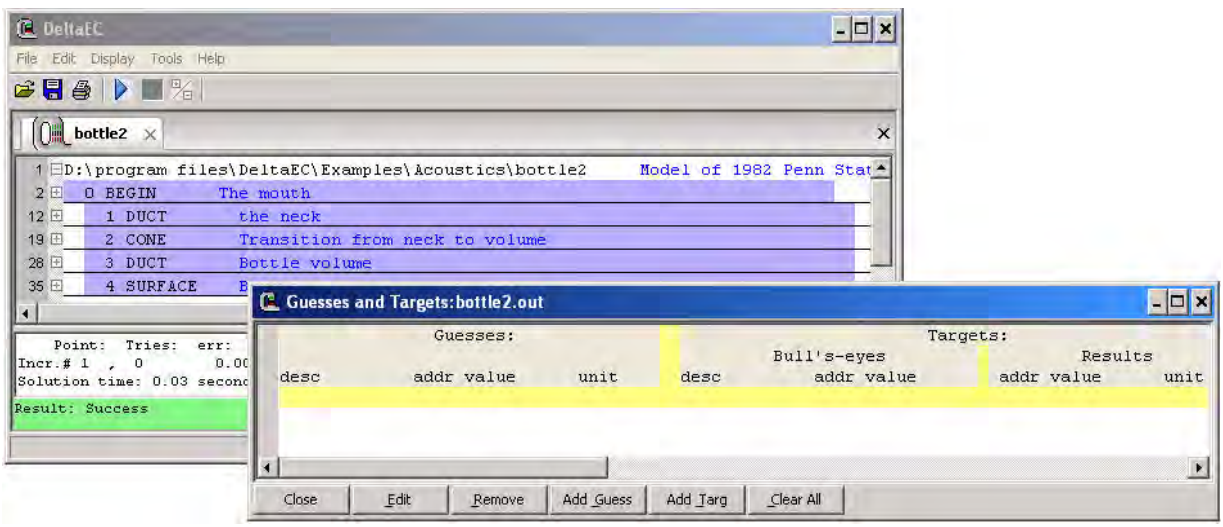


Figure 2.3: $\dot{E}$ vs x for <`bottle2`>.



The table is blank: At this point DELTAEC has no guesses and no targets. Each time we have run DELTAEC thus far in the tutorial, it has integrated along x only once, starting from the values of p_1 and U_1 that we gave it in the `BEGIN` segment.

To set up this model with appropriate targets, we need to insert an additional segment at the end of the model. To insert a segment, pull down the Edit menu and click on `Insert Seg`, or type "i", or right click below the last segment and select `Append`. Whichever way you choose to do it, insert a `HARDEND`, at the end of the model, after the `SURFACE`.

Now establish `5a` and `5b` in the `HARDEND` as targets by clicking on `Possible targets` in the `HARDEND` itself or through Add Targ in the guess–target summary. Inspect your work thus far by examining the guess–target summary:

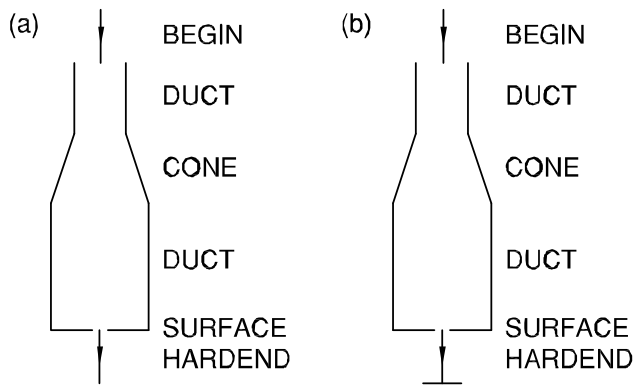
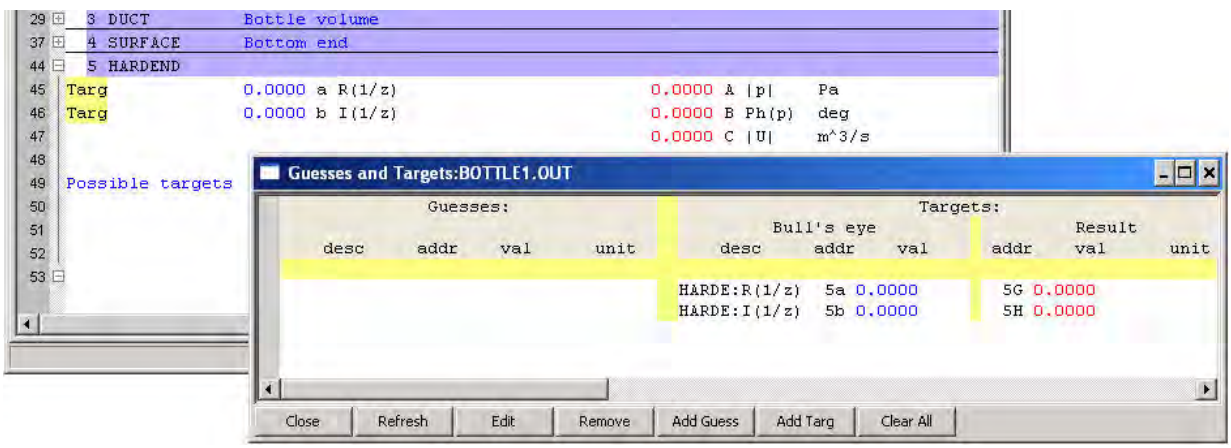


Figure 2.4: The bottle. (a) HARDEND untargeted, so U_1 can be nonzero at the bottom. (b) HARDEND targeted to (0.00, 0.00), to seal the bottom.



Also notice how the targets are flagged with yellow markers in the main display.

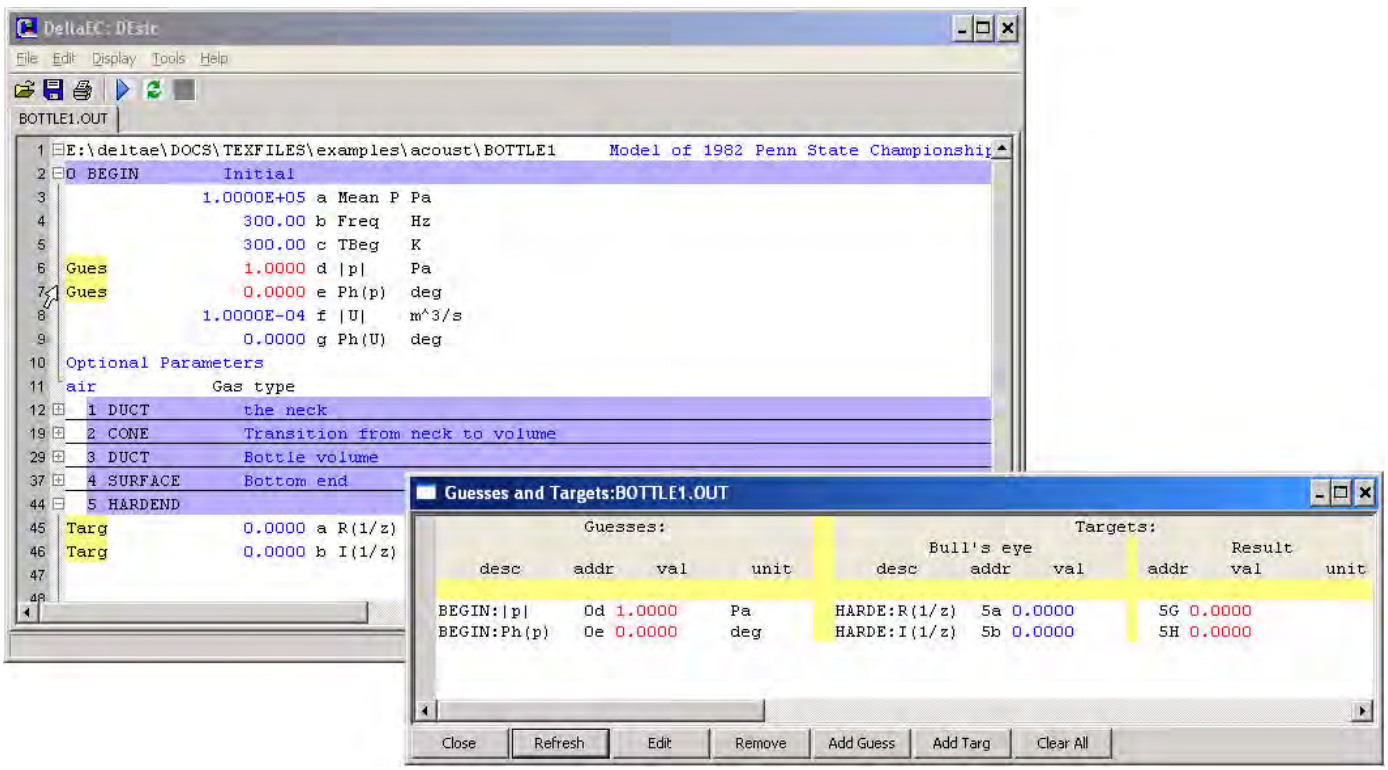
(We could have programmed HARDEND segments with the real and imaginary parts of U_1 as our targets, but experience has shown that the DeltaEC shooting-method algorithm usually works without extra attention from us if we normalize the targeted U_1 by dividing by area and p_1. The rare occasions when more attention is needed are described in Section 8.3.2.)

We have two targets, which will soon enforce complex $U_1 = 0$ at the bottom end of the model, appropriate for the sealed end of the bottle. We must choose two guesses that DeltaEC can adjust in order to meet those targets. Basically, DeltaEC integrates from BEGIN to HARDEND. DeltaEC will refine whatever two guess variables we allow it to vary, to find a solution to this acoustics problem that arrives at the HARDEND with zero complex volume flow rate.[5] What to choose as these guesses depends on exactly what situation you

[5] Users sometimes ask why we need *two* targets and *two* guesses when we are really only trying to impose *one* constraint at the HARDEND, namely $|U_1| = 0$. The answer to this question can be appreciated by imagining a blindfolded archer trying to hit a bull's-eye. If an observer tells the archer "Your last attempt missed by 2.5 meters," which way is the archer supposed to revise her aim for her next attempt to hit the target? More useful information is conveyed if the observer tells the archer "Your last attempt struck 1.5 meters above the bull's-eye and 2.0 meters to the left of the bull's-eye." Like the blindfolded archer's target, DeltaEC's HARDEND is a point target in a *two*-dimensional space, namely the complex plane of U_1. DeltaEC needs to

are trying to model. Do you have in mind a resonator driven by a fixed volume flow rate amplitude, or one that is driven by a fixed pressure amplitude? Neither answer is wrong; it depends on the circumstances.

In the present example, we arbitrarily choose to keep $|U_1|$ fixed at 0.0001 m^3/s, and let DELTAEC adjust p_1. Begin to accomplish this by adding `0d` and `0e` as guesses, by double clicking or right clicking on their values and selecting "guess," or through Add Guess in the guess–target summary. Inspect the resulting situation by looking at the summary.

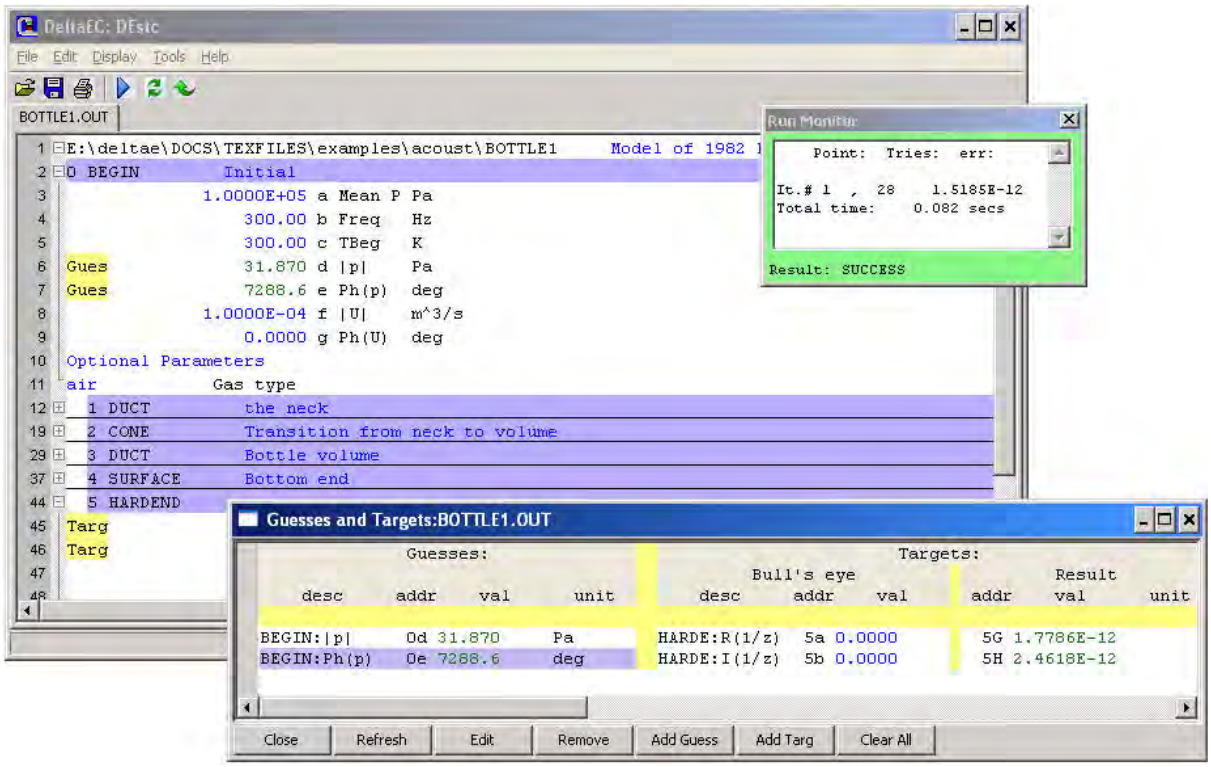


The target values are red italic, indicating that DELTAEC does not know whether results are consistent with input variables; it knows that it has not yet run with the most recent changes of input variables. The guesses (1 Pa, 0 degrees) are still just the values that are left over from before. DELTAEC has not yet adjusted the guesses to try to hit the targets, but it is ready to do so, because there are equal numbers of guesses and targets: two each in this case.

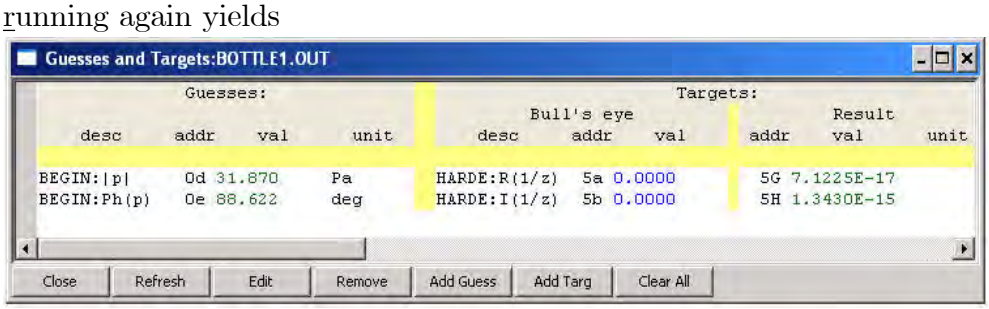
With an equal number of guesses and targets, let's run. After running, DELTAEC's Run Monitor at the bottom of the display reports that it integrated along x 28 times while adjusting guesses and coming ever closer to the targets, and it spent 0.08 seconds doing so. (The numbers that are reported by the Run Monitor may vary from computer to computer and version to version. How close to the targets is judged to be "close enough" is described in Section 8.3.2.)

Inspect the results:

adjust its aim (its guesses) in *two* dimensions to hit the target, and DELTAEC can hit the target in a small number of attempts if it gets *two*-dimensional information about how far off-target it is.



The value of the phase shown in `0e`, 7288.59°, might seem strange until we realize that DeltaEC does not understand that integer multiples of 360° can be subtracted from any phase angle without consequence. With the initial guessed value of `0d` so far from reality, DeltaEC's shooting method wandered far, looking for a solution; under normal circumstances, we might have given it a more plausible starting point (such as the result of a previous run). Subtracting 7200° from the value of `0e` for the sake of our human sanity and running again yields



The targets are accurately met, because DeltaEC has adjusted the two guesses to be 31.870 Pa, 88.622 degrees. The state plots also show that the targets have been met, because they show that $|U_1| = 0$ at the end.

Save the <`.out`> file with a new filename of your choosing, and examine it with a text editor to see how DeltaEC has indicated which variables are guesses and targets, next to the guesses and targets themselves and in the restart table at the bottom of the file.

Most of the thought required to successfully use DeltaEC occurs while trying to figure out which of the variables are appropriate targets and guesses, i.e., what *constraints* must be imposed on *results* of a single DeltaEC integration pass, and what variables are needed as *inputs* by a single DeltaEC integration pass but are in fact *unknown* to us when we

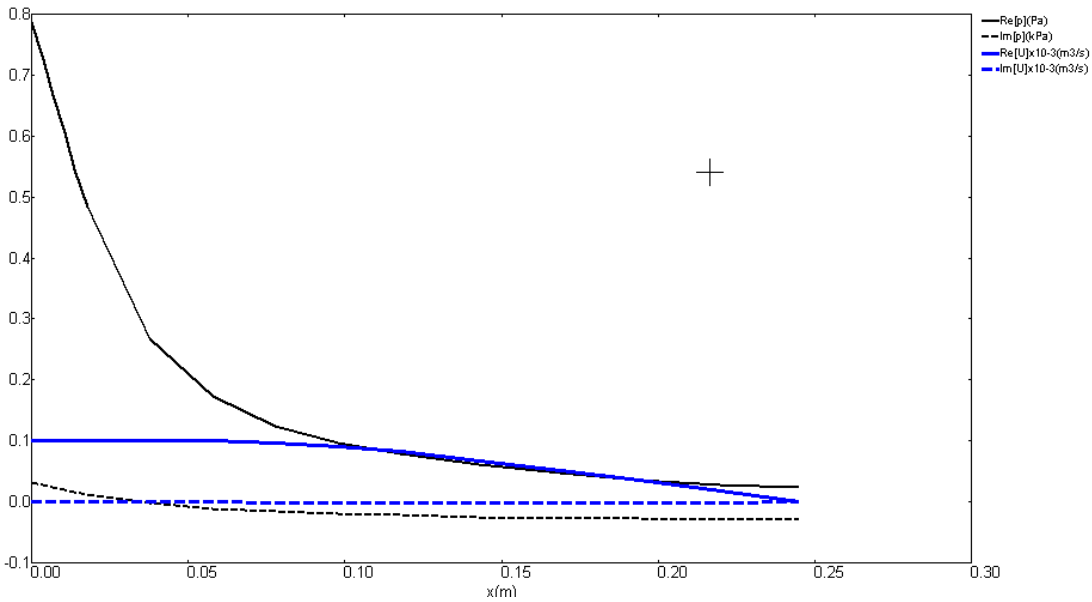


Figure 2.5: State plots for the bottle, with `HARDEND` targets met so $U_1 = 0$ at the bottom.

ask DeltaEC to run. While many choices are possible, as long as the number of guesses equals the number of targets, some choices are physically nonsensical and cannot succeed. For example, asking DeltaEC to try to achieve resonance in a given bottle by guessing the mean pressure of the air would be futile, because the speed of sound in an ideal gas is independent of pressure. In the examples in subsequent Chapters, and in advanced use of DeltaEC, the choice of good target and guess variables is not always as easy as it is in this simple example.

Notice the use of columnar location and color in the display: Columns organize variables according to DeltaEC's most short-sighted internal needs, while colors organize variables according to users' needs. The numbers in the left column (ordinary blue numbers, and the guesses, and the bull's-eye target values) are variables needed by DeltaEC in each pass of its integration, as it integrates repeatedly, adjusting the guesses to try to hit the bull's-eyes. Variables in the right column are computed as results in each pass, and are displayed green when targets are met. We users, however, often wish to forget that DeltaEC must adjust guesses and hit targets. From our perspective, blue numbers are what we give DeltaEC as known quantities, and green numbers are the results that we want DeltaEC to give us. If the green numbers are indeed green, DeltaEC feels confident of its results, but when those numbers are red DeltaEC is not displaying a consistent set of results, for any of a number of reasons—usually because we have not yet run it or because it has missed the targets.

Now let's find the resonance frequency f, which is the frequency at which the bottle sings when we blow across its top. Look at the guess–target summary to get oriented.

```
Guess-Target Summary:
GUESS      0d          0e
name  BEGIN: |p|  BEGIN:Ph(p)
units     Pa          deg
value  31.870        88.587
TARGET      5a          5b
name  HARDE:R(1/z HARDE:I(1/z
```

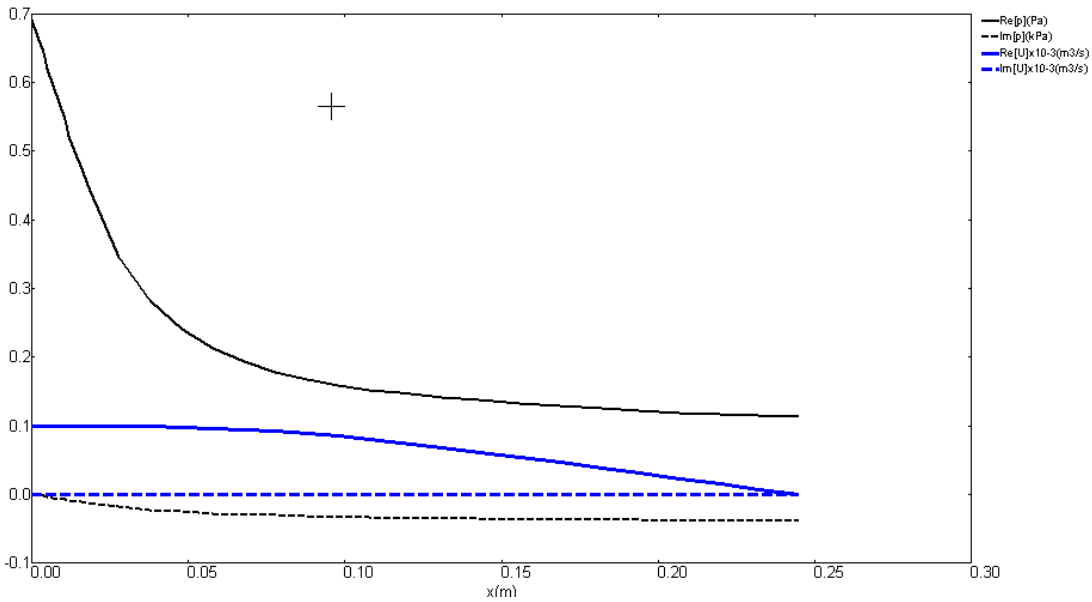


Figure 2.6: State plot with `HARDEND` targeted and resonance achieved, so $U_1 = 0$ at the bottom and p_1 and U_1 are in phase at the mouth.

```
value     0.0000      0.0000
result -5.6504E-13  4.1801E-14
```

First, think about the physics long enough to realize that having p_1 and U_1 in phase at the mouth is a suitable, approximate condition for resonance.[6] Now make the corresponding changes in the DeltaEC model: modify `0e` to be 0° instead of 41.5° and remove `0e` from the guess list so DeltaEC cannot change it any more. Next, add `0b`, the frequency, in the guess list. Inspect the guess–target summary again. Now, "run" the calculation. Inspect the results in the main display or the guess–target summary, finding that the resonance frequency is 194.99 Hz. Save this version as `bottle5`, for use later in this Chapter.

As acousticians, we know that this resonator has more than one resonance mode. To find another one, modify `0b` to be 600 Hz, and run again. With this starting guess, DeltaEC converges on the 2nd mode, resembling 3/4 of a wavelength from end to end, at 1029.6 Hz.

DeltaEC can use any physically appropriate input variable as a guess. We could have learned what temperature makes the system resonate at 180.00 Hz, by using `0c` instead of `0b` as a guess. (The answer is 255 Kelvin.) Or, by using `1c` as a guess, we could have found out what length the neck needs to be to put the resonance at 180 Hz at 300 K.

DeltaEC's gas library contains three binary mixtures of gases: He-Xe, He-Ar, and Ne-Xe. These introduce another real variable in `BEGIN`: the mole fraction of helium in the mixture (for example, HeAr with $n_L = 0.889$ indicates 88.9% helium and 11.1% argon).

[6] That short sentence is an example of where novice users of DeltaEC who have insufficient acoustics knowledge can have a great deal of trouble. Please do not expect to learn "everything you need to know" about acoustics from DeltaEC. Take a university acoustics course or study an acoustics textbook! Remember that DeltaEC cannot "understand" physical acoustics better than you do. It merely calculates, accurately and quickly. N.b.: we said "approximate condition for resonance" because some users might want to include radiation impedance. See Section 10.4.1.

The helium fraction in one of these mixtures can be used as a guess variable for resonance. Simply add `0j` as a guess. Thus, we could learn the helium concentration that would have to be added to argon in the bottle to make the resonance equal 180.00 Hz.

2.6. Other acoustics features

Here we briefly describe some additional DeltaEC features that are relevant to a simple acoustic apparatus for which $\mathcal{F}_{\text{momentum}}$ and $\mathcal{F}_{\text{continuity}}$ in Eqs. (2.6) and (2.7) are the only equations that we want DeltaEC to integrate.

2.6.1. Simple segment types

A list of the most commonly used purely acoustic segment types (including those introduced above) is given below. More detailed descriptions and the calculations performed by each are given in Chapter 10 and Chapter 11.

`TITLE` Required at the top.

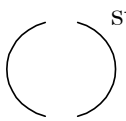
`BEGIN` Required immediately after `TITLE`. The "zeroth" segment must be `BEGIN`. It contains global variables such as mean pressure, gas type, and frequency, and initial conditions for variables that can vary with x, including p_1, U_1, and mean temperature (whose x dependence is introduced in Chapter 5). See details in Section 11.1.2.

`SURFACE` A surface area with pressure-hysteresis loss in its thermal penetration depth. Usually used at ends of ducts. See details in Section 10.2.2.

`DUCT` A duct, with viscous and thermal losses at the wall if the perimeter is realistically nonzero. Separate entry of area and perimeter accommodates ducts of any cross-sectional shape. Laminar or turbulent. See details in Section 10.1.1.

`CONE` A cone, often used to adapt between ducts of different sizes. Uses lossy Webster horn equation. Laminar or turbulent. See details in Section 10.1.2.

`COMPLIANCE` An acoustic compliance, i.e., a volume full of gas. Pressure-hysteresis losses on surface. See details in Section 10.2.1.



`IMPEDANCE` A lumped-parameter, series impedance. Resistance and inertial reactance are specified. See details in Section 10.2.3.

`IDUCER` and `VDUCER` Current-driven and voltage-driven transducers, with parameters independent of frequency, connected "from the side" of the `DUCT`s or other segments. See Fig. 2.7. See details in Section 10.4.2.

`ISPEAKER` and `VSPEAKER` Current-driven and voltage-driven electrodynamic transducers, parameterized by mass, B-l product, etc., so that impedance coefficients depend on frequency. Connected "from the side" of a sequence of `DUCT`s or other segments. See Fig. 2.7. See details in Section 10.4.2.

`IEDUCER` and `VEDUCER`, `IESPEAKER` and `VESPEAKER` The four transducers that were described above, which do not have "`E`" as the second letter in their names, are connected in *side-branched* configurations, where the "back side" of the transducer hangs outside of DeltaEC's computation space. The four `E`nclosed versions are their *series* counterparts, with one side of the transducer facing the previous segment and the other side facing the subsequent segment, so that the volume flow rate remains constant across the segment. See Fig. 2.8. See details in Section 10.3.1.

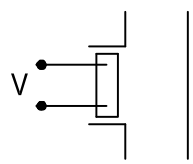


Figure 2.7: `IDUCER`, `VDUCER`, `ISPEAKER`, and `VSPEAKER` are attached as side branches to the series of DeltaEC segments.

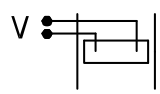
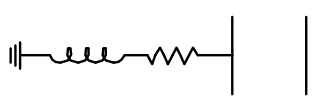


Figure 2.8: `IEDUCER`, `VEDUCER`, `IESPEAKER`, and `VESPEAKER` are acoustically in series with the adjacent DeltaEC segments.

`BRANCH` A frequency-independent side-branch impedance. See details in Section 10.4.1.



`OPNBRANCH` and `PISTBRANCH` Frequency-dependent side-branch impedances with the frequency characteristics of the radiation impedance of an opening into 4π or 2π solid angle, respectively. See details in Section 10.4.1.

`HARDEND` A logistical final segment, used to enforce $U_1 = 0$ through use of the inverse of the acoustic impedance in the target list. See details in Section 11.1.3.

`SOFTEND` A logistical final segment. Can be used to set $p_1 = 0$ through use of the acoustic impedance in the target list. Useful for defining a mirror-image plane in symmetric apparatus with a pressure node at a center of symmetry. Also used for loops and

networks of segments, to mark a location that will be reconnected elsewhere in the model. See details in Section 11.1.3.

2.6.2. Gases and solids

DeltaEC recognizes many gases[7] and solids, and routinely calculates their properties such as density, specific heat, and viscosity. Gases are specified in `BEGIN` segments, and solids are specified in each physically realistic segment. As described in detail in Chapter 12, DeltaEC includes the following "gases": dry air, humid air and fog, helium, neon, He-Xe mixtures, He-Ar mixtures, Ne-Xe mixtures, hydrogen, deuterium, nitrogen, carbon dioxide, natural-gas combustion products (i.e., flue gas), liquid sodium, and eutectic liquid sodium-potassium. The ideal-gas equation of state is used for the gases, while sodium and eutectic sodium-potassium have their own equations of state.

Solids, described in detail in Chapter 13, include Kapton, Mylar, stainless steel, molybdenum, tungsten, copper, nickel, celcor, and ideal. An ideal solid has extremely high heat capacity, density, and thermal conductivity.

DeltaEC also allows users to specify "external," user-defined gases or solids that are not part of its own internal library. Properties are calculated, according to local T_m and p_m, from coefficients read from a user-written text file. Up to five user-defined gases and five user-defined solids can be used at one time. Details of user-defined pure gases and binary mixtures are described in Sections 12.15.1 and 12.15.2, respectively. Details of user-defined solids are described in Section 13.10.

To illustrate the use of a single-component user-defined gas, consider the example below. To replace DeltaEC's internal "ideal" helium gas (see Section 12.1) with a more accurate representation that calculates density and sound speed including the first coefficient of the virial expansion for helium, we can create the following file in a text editor, call it <`helium.tpf`>, and put it in the same folder as our model:

```
! user-defined gas; He with first virial coeff for (B=12cc/mole)
! Equation is:
! C0 + C1*pm/(T+C2*pm) + C3*T + C4*T^2 + C5*T^C6 + pm^2 *C7*T^C8 + pm*C9
! Density, rho (m^3):
  0.  4.814e-4 1.44e-6
! isobaric heat capacity, cp (J/kg-K):
  5192.
! Thermal conductivity, k (W/m-K):
 0. 0. 0. 0. 0. 0.0025672  0.716
! Square of sound speed, a^2 (m^2/s^2):
  0.  0.  0.  3461.92  0. 0. 0. 0.  0. .0100
! Viscosity, mu (kg/m-s):
 0. 0. 0. 0. 0. 0.412e-6  0.68014
```

The coefficients for density in this file are used by DeltaEC in Eq. (12.1) and were determined by a user who knew that

$$\rho_m = \frac{p_m m}{R_{\text{univ}}(T_m + Bp_m/R_{\text{univ}})}, \tag{2.9}$$

[7] A "fluid" is a gas or a liquid. Gases are the most commonly used fluids in thermoacoustics and DeltaEC. However, liquid sodium and eutectic liquid sodium-potassium are included in DeltaEC because they are thermodynamically powerful, with non-negligible thermal expansion coefficients and non-negligible $(\partial s/\partial p)_T$, when close enough to their critical points. For further details on the use of these liquids in thermoacoustics, see Ref. [13]. Perhaps we should refer to the "gases" used in DeltaEC as "fluids," but for simplicity we always call them gases despite the fact that some liquids are included.

where $R_{\text{univ}} = 8.314$ J/mole-K, the molar mass $m = 0.0040026$ kg/mole, and the first virial coefficient $B = 1.2 \times 10^{-5}$ m^3/mole. Thus, the user set $C_1 = m/R_{\text{univ}}$ and $C_2 = B/R_{\text{univ}}$ in the density line in the <`.tpf`> file. For squared sound speed, he knew that

$$a^2 = \frac{\gamma R_{\text{univ}} T}{M} \left(1 + 2 \frac{B p_m}{R_{\text{univ}} T_m} \right), \tag{2.10}$$

so he set $C_3 = \gamma R_{\text{univ}}/m$, and $C_9 = 2B\gamma/m$ in the square-of-sound-speed line in the <`.tpf`> file, where $\gamma = 5/3$.

User-defined gas mixtures (described in Section 12.15.2) and user-defined solids (described in Section 13.10) follow similar formats.

It is a good idea to check your implementation of a new user-defined gas or solid very carefully by using one of the two ways of displaying or plotting thermophysical properties described in Section 2.7.7.

2.6.3. Turbulence

A turbulence algorithm can be enabled in `DUCT` and `CONE`, by double clicking on "Optional parameters." Checking the Turbulence box brings up parameter `d` in `DUCT` and parameter `f` in `CONE`, which specify the relative roughness (roughness height divided by pipe diameter). Set the roughness equal to zero for smooth walls, or set it to a small value greater than zero for rough walls. (We usually use 5×10^{-4}, even if this is larger than the actual relative roughness of the channel.) To ensure a laminar calculation, check the laminar box for the segment (which causes the roughness parameter to be hidden from view in the <`.out`> file).

The turbulence algorithm, which is described in detail in Section 10.1.1, follows the quasi-steady approximation, (the spirit of the assumptions of Iguchi *et al.*, Ref. [14]), assuming that oscillatory-flow losses can be calculated by using the steady-flow Moody friction factor at each instant of time during the oscillatory flow, with the surface roughness used as an experimentally determined fitting parameter. The quasi-steady assumption has only a little experimental validation in the range of Reynolds number and R/δ_ν of interest in thermoacoustics (where R is radius), but we believe it provides a useful estimate, much better than no estimate at all.

2.7. Other useful features

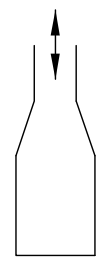
2.7.1. Keeping parameters equal

The `sameas nl` feature ties an input parameter in one segment to a value in another segment. This helps prevent typographical errors in the <`.out`> file, and is especially useful in linking dimensions of adjacent segments that you might want to vary all together while modifying or plotting, such as areas of adjacent segments when increasing the size of the apparatus. Specify the segment number and line letter (e.g., "`sameas 3a`").

The following example is for the bottle we modeled above:

```
!---------------  1 ------------------------
DUCT       The neck
2.141e-4    a Area     m
```

```
5.187E-2    b Perim    m
1.778E-2    c Length   m
ideal      Solid type
!---------------  2 ------------------------
CONE     transition from neck to volume
sameas 1a   a AreaI   m^2
sameas 1b   b PerimI   m
   .100     c Length   m
sameas 3a   d AreaF   m^2
sameas 3b   e PerimF   m
ideal      Solid type
!---------------  3 ------------------------
DUCT      Bottle volume
1.868e-3  a Area      m^2
0.1532    b Perim      m
0.1270    c Length     m
ideal      Solid type
!---------------  4 ------------------------
SURFACE     Second End
sameas  3a  a Area    m^2
ideal      Solid type
```



When you add a parameter specified by `sameas` to the guess list, you must first sever the `sameas` relationship. This is required because the value at this point will be controlled by DeltaEC's shooting method. But if a variable that is the root of several `sameas` references is caused to change by DeltaEC itself in any ways, e.g., as a guess, all `sameas` references to this root within the model will change with it.

2.7.2. Master–slave links

When geometric variables are changed, whether by the user, by DeltaEC's shooting method, or by DeltaEC's incremental plot routine, there are often geometric relationships with other parameters that should be maintained. For example, if the area of a duct increases, we should usually increase the associated perimeter as well. Another common wish is to lengthen one segment while simultaneously shortening another segment to keep overall length constant. Master–slave links (formerly known as "special modes") are available in some segments for just these purposes; a Master-Slave Links option appears in a segment whenever a built-in parameter-linking capability is possible for variables in that segment.

For example, consider a `DUCT` whose cross section is a half circle attached to a square, as shown in Fig. 2.9. The cross-sectional area is

$$A = \frac{1}{2}\frac{\pi}{4}d^2 + d^2, \tag{2.11}$$

and the perimeter is

$$\Pi = \frac{\pi}{2}d + 3d. \tag{2.12}$$

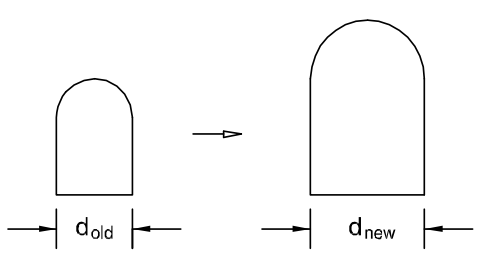


Figure 2.9: When the area of a `DUCT` is changed and perimeter–area linking is in use, the perimeter is automatically recalculated to maintain the `DUCT`'s shape.

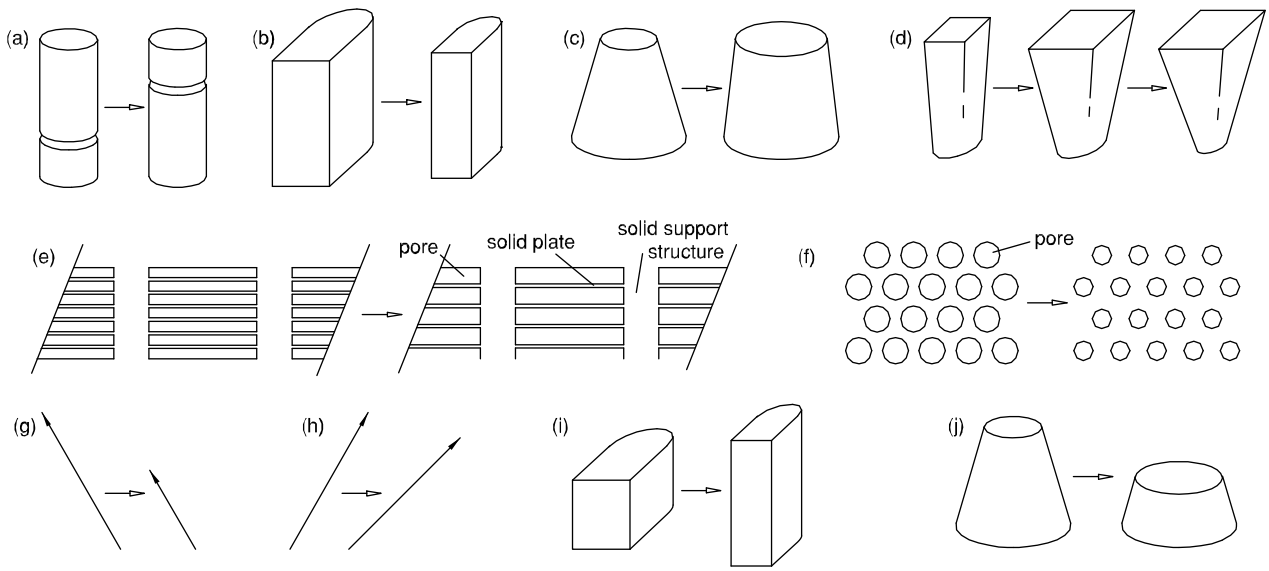


Figure 2.10: Illustration of some master–slave linking. (a) Two lengths can be linked so that the total length remains constant when one length is changed. (b), (c), (d) Perimeters can be linked to areas so that shapes remain constant when areas are changed. (e) In `STKSLAB`, porosity can be linked to pore size and solid thickness, to keep the fraction of the stack that's devoted to support structure constant. The plane of this illustration is perpendicular to x. (f) In `STKCIRC` and `TX`, porosity can be linked to hole radius, to keep the number of holes constant. The plane of this illustration is perpendicular to x. (g), (h) In segments with impedances, the imaginary part can be linked to the real part so that either phase or magnitude remains constant when the real part is changed. (i) In `STKDUCT`, maintain shape and volume when length is changed. (j) In `CONE`, adjust length to maintain taper angle when areas are changed.

Notice that

$$c = \frac{\Pi^2}{A} = \frac{(\pi + 6)^2}{\pi/2 + 4} \simeq 15.001 \tag{2.13}$$

is a constant, independent of the size d of the `DUCT`, but depending on the half-circle-plus-square character of its shape. Additional examples are $c = 4$ for a circular duct, $c = 16$ for a square duct. When perimeter–area linking is established in a `DUCT`, DeltaEC remembers the value of this constant c, and uses it thereafter in

$$\Pi = \sqrt{cA} \tag{2.14}$$

to update the perimeter based on the area, thereby maintaining the cross-sectional shape. Thus, as area is changed, this link keeps a circular duct circular and maintains the aspect ratio of a rectangular duct.

Below is a list of all parameter-link options and the segment types for which they are available, including some for segments that have not yet been introduced in this tutorial. The numbers in parentheses at the end indicate how the link is recorded in the restart table at the bottom of the <`.out`> file. Figure 2.10 illustrates these links.

Adjust length in segment n when the length here changes, to keep their sum constant: All segments with length. (In the restart table, this is indicated as $n > 0$.)

Adjust porosity (i.e., `GasA/A`) when gas gap or solid thickness changes in `STKSLAB`, keeping constant area and constant fraction of the area devoted to support ribs. Adjust porosity (i.e., `GasA/A`) when hole radius or area changes in `STKCIRC` or `TX`, keeping constant number of holes. Set porosity (i.e., `GasA/A`) equal to $ab/(a+l)(b+l)$ in `STKRECT`. For details, see Section 10.5.1. (In the restart table, this is indicated as -1.)

Adjust perimeter when area changes, to maintain shape: `DUCT` and `STKDUCT`. Adjust initial perimeter when initial area changes, to maintain initial end's shape: `CONE` and `STKCONE`. (In the restart table, this is indicated as -2.)

Adjust final perimeter when final area changes, to maintain shape of final end: `CONE` and `STKCONE`. (In the restart table, this is indicated as -3.)

Adjust length and both perimeters, to maintain end shapes and taper angle: `CONE` and `STKCONE`. (In the restart table, this is indicated as -4.)

Adjust surface area when volume changes, to maintain shape: `COMPLIANCE`. (In the restart table, this is indicated as -5.)

Adjust area and perimeter when length changes, to maintain constant volume and cross-sectional shape: `STKDUCT`. (In the restart table, this is indicated as -6.)

Adjust imaginary part when real part changes, to maintain constant magnitude (when possible): `IMPEDANCE`, `BRANCH`, and `TBRANCH`. (In the restart table, this is indicated as -7.)

Adjust imaginary part when real part changes, to maintain constant phase angle: `IMPEDANCE`, `BRANCH`, and `TBRANCH`. (In the restart table, this is indicated as -8.)

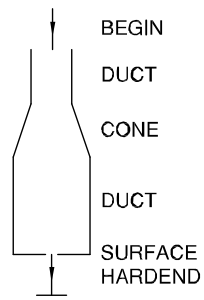
Adjust initial and final perimeters in `CONE` and `STKCONE` as initial and final areas change, maintaining the shapes on both ends. (In the restart table, this is indicated as -9.)

Master–slave links can be tracked by status fields in the center of the <`.out`> file, between the input and output columns. The master parameter—the one through which the mode is controlled—has the form `Mstr`. The slave parameter, i.e., the one that cannot be modified independently when it is controlled by a link, has a status indicator of the form *nn*`p` to indicate its master's address (e.g., `2a`).

Newly inserted `DUCT` and `CONE` segments automatically initialize with their perimeters linked to their areas on the basis of a circular shape, with a radius of 1 meter.

With each of the <`bottle`> model's perimeters slaved to its local area, and adjacent areas that must be identical connected by `sameas`, the <`bottle`> model looks like this:

```
 2 0 BEGIN        Initial
12   1 DUCT         the neck
13              2.1410E-04 a Area   m^2      Mstr          12.044 A |p|     Pa
14              5.1870E-02 b Perim  m        1a           -87.986 B Ph(p)   deg
15              1.7780E-02 c Length m                  9.9797E-05 C |U|     m^3/s
16 Master-Slave Links                                 -9.4266E-03 D Ph(U)   deg
17 Optional Parameters                                 3.3197E-05 E Htot    W
18 ideal        Solid type                             2.1219E-05 F Edot    W
19   2 CONE         Transition from neck to volume
20     Same  1a 2.1410E-04 a AreaI  m^2                 33.945 A |p|     Pa
21     Same  1b 5.1870E-02 b PerimI m                  -89.794 B Ph(p)   deg
22                  0.1003 c Length m                  7.5895E-05 C |U|     m^3/s
23     Same  3a 1.8680E-03 d AreaF  m^2               -3.4905E-02 D Ph(U)   deg
24     Same  3b     0.1532 e PerimF m                  3.3197E-05 E Htot    W
25 Master-Slave Links                                  5.4209E-06 F Edot    W
26 Optional Parameters
27 ideal        Solid type
28   3 DUCT         Bottle volume
29              1.8680E-03 a Area   m^2      Mstr          37.703 A |p|     Pa
30                  0.1532 b Perim  m        3a           -89.859 B Ph(p)   deg
31                  0.1270 c Length m                  2.3620E-08 C |U|     m^3/s
32 Master-Slave Links                                   -89.859 D Ph(U)   deg
33 Optional Parameters                                 3.3197E-05 E Htot    W
34 ideal        Solid type                             4.4527E-07 F Edot    W
35   4 SURFACE      Bottom End
36     Same  3a 1.8680E-03 a Area   m^2                 37.703 A |p|     Pa
37                                                      -89.859 B Ph(p)   deg
```



so it is apparent that the bottle has only five independent geometrical dimensions—neck area, bulb area, and the lengths of the neck, cone, and bulb—whose independence is indicated by their blue color. All other dimensions depend on those five. The user who is interested in studying the resonance frequency as a function of geometry can now change those five dimensions at will, confident that self-consistent geometry will be maintained by the master–slave and `sameas` links.

2.7.3. Schematic display

A schematic display of the geometry of a DeltaEC model can be turned on by clicking on "view schematic" in the Display pulldown menu. As an example, the schematic of the <`bottle`> model is shown in Fig. 2.11. The schematic sketch is to scale: Dimensions in the x direction are proportional to the lengths of segments with length, and dimensions perpendicular to x are proportional to $\sqrt{4A/\pi}$ in segments having area and length. A glance at this display can help the user catch order-of-magnitude typographical errors in a model and keep track of the topology of large models. Left click and drag up to zoom in for a closer view, and right click to bring up a menu of segment operations similar to the right-click menu on each segment in the main model.

New and still evolving in DeltaEC version 6.2: When both a schematic display and a state plot are active, phasor information from the state plot can be displayed on the schematic, based on the position of the mouse cursor within the schematic. Click Alt when the cursor is over the desired position in a highlighted physical segment in the schematic, to create a small phasor plot showing both p_1 and U_1 at that position. The magnitude of each phasor is scaled by the maximum magnitude found in the model. The colors are the same as the default colors in the state plot: black for p_1, blue for U_1. Drag the phasor circle or its anchor point with the mouse to reposition them. In integrable segments like `DUCT`s, `Nint` unique x locations are available; nonintegrable segments like `COMPLIANCE` have only one location.

Figure 2.11: The schematic display of the <`bottle`> model. Dimensions are to scale, assuming that areas are circular. Right-clicking on any segment brings up a menu of options, similar to right-clicking on a segment in the main model display.

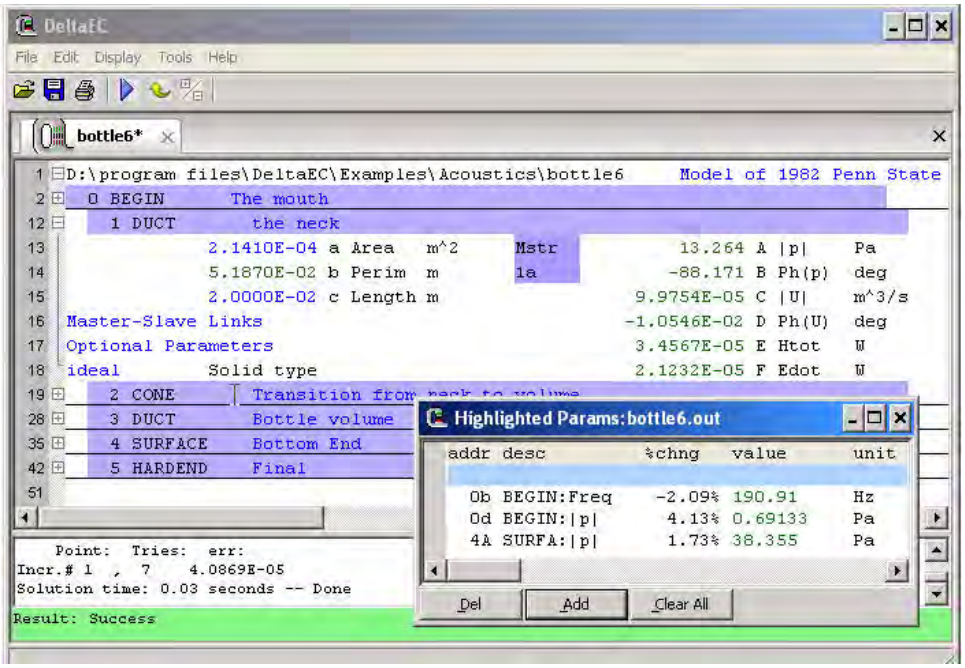
To delete a phasor circle, move the mouse over the anchor point and press the Alt or Delete key. To delete all phasor circles, close and reopen the schematic window.

2.7.4. Highlights display

DeltaEC models often have many dozens of segments. The user is usually not interested in examining *every* numerical result after every run—typically, we examine the guesses (and perhaps targets) to ensure that DeltaEC has not gotten completely lost, and we examine a small number of other results of particular interest. The highlights display shows a compact, user-customized list of such variables of particular interest, which is updated after every run.

Any input or output variables can be added to the highlight list.

Returning to the bottle as an example, we might be particularly interested in the pressure amplitude at the mouth of the bottle and at the bottom of the bottle, and the resonance frequency, as we explore different dimensions of the neck by modifying its area and/or length and running. A convenient setup for such an investigation looks something like the figure below, just after lengthening the neck and running. In ordinary use, we would type 'm' to modify one or more parameters, then 'r' to run, examine the results, and repeat.



2.7.5. Structured branches, loops, and networks

Although `BRANCH` and `OPNBRANCH` can be useful, they are often inadequate for describing the variations in branch impedance with operating conditions. For example, the branch might be a Helmholtz resonator whose impedance changes significantly with frequency. Further, `BRANCH` and `OPNBRANCH` are wholly inadequate when networks include reconnecting topology, as in Fig. 1.3. The `TBRANCH` and `UNION` segments overcome these inadequacies.

When it encounters a `TBRANCH`, DeltaEC treats subsequent segments as the sequential members of the branch, integrating until it reaches a `HARDEND` or `SOFTEND`. It then "returns to the trunk," treating further segments as trunk members. The logic of this is shown in Fig. 2.12. At the `TBRANCH`, the branch impedance determines how the (complex) volume flow rate splits up. Usually we use the branch impedance as a pair of guesses that DeltaEC adjusts in its usual way to hit two targets elsewhere. `TBRANCH`ed models tend to have many guesses and targets, since every `**END` contributes two targets (and a few more targets are almost always needed for temperatures and other variables). Of course, branches can have sub-branches of their own.

Networks with loops, like Fig. 1.3, can also be handled by DeltaEC, through use of `TBRANCH` and `UNION`. The `UNION` segment is used to tell DeltaEC to "connect" a `TBRANCH`'s `SOFTEND` back to the trunk at the location of the `UNION` segment. The branch's `SOFTEND` potential targets are not used. However, at least two input variables (`b` and `c`) of the `UNION` segment should always be used as targets. As DeltaEC processes the `UNION` segment, it assigns the current values of the complex pressure at the `SOFTEND` referenced by the number in parameter `a` of the `UNION` segment. These values are compared to the local complex pressure result, at this `UNION`, in the trunk, and DeltaEC's shooting-method iteration should run until their difference is zero. This assures that the end of the branch and the trunk share the same complex p_1 where they meet at the `UNION`. As before, a guessed branch

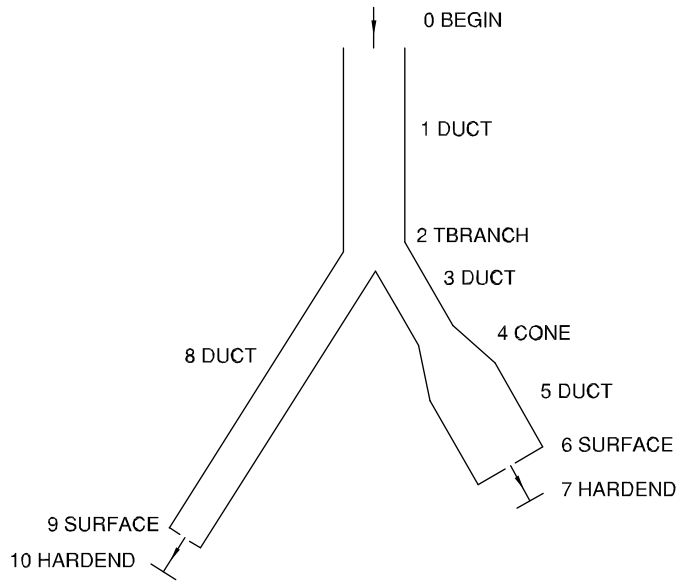


Figure 2.12: TBRANCH allows DeltaEC to integrate in branched geometries like this.

impedance usually determines how the (complex) volume flow rate splits up at the TBRANCH. Volume flow rates are summed at the UNION. (Note that the UNION targets are somewhat different from HARDEND targets discussed above, because "target" numerical values in UNION are not specified by the user—they vary from run to run, depending on what is happening at the associated SOFTEND.)

Figure 2.13 and the DeltaEC models below show three ways to use TBRANCH and UNION to model a parallel LRC network. The reader is encouraged to invent one or more additional ways to model this network, and try them.

Here is the DeltaEC model corresponding to Fig. 2.13(b):

```
TITLE     Example LRC1
! lrc1.out
!Created@19:30:01  9-Jul-06 with DeltaE Vers. 5.5b6 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
 BEGIN      Initialize things
  1.0000E+05 a Mean P    Pa              6.1460E-03 A  |U|   G(  0f)     P
    60.000   b Freq      Hz                80.940   B Ph(U)  G(  0g)     P
   300.00    c TBeg      K               3.7260E+05 C Re(Zb) G(  1a)     P
  2000.0     d  |p|      Pa             -2.7012E+05 D Im(Zb) G(  1b)     P
     0.0000  e Ph(p)     deg
  6.1460E-03 f  |U|     m^3/s    G
    80.940   g Ph(U)     deg     G
 air        Gas type
!--------------------------------  1 --------------------------------
 TBRANCH    the split
  3.7260E+05 a Re(Zb) Pa-s/m^3   G        2000.0     A |p|       Pa
 -2.7012E+05 b Im(Zb) Pa-s/m^3   G           0.0000  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                           35.940   D Ph(U)     deg
                                            3.5185  E Hdot      W
                                            3.5185  F Edot      W
                                           -2.5508  G EdotTr    W
!--------------------------------  2 --------------------------------
 IMPEDANCE  pure inertance
     0.0000  a Re(Zs) Pa-s/m^3           2282.4     A |p|       Pa
  1.0000E+05 b Im(Zs) Pa-s/m^3             -8.8682  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                           35.940   D Ph(U)     deg
                                            3.5185  E Hdot      W
                                            3.5185  F Edot      W
```

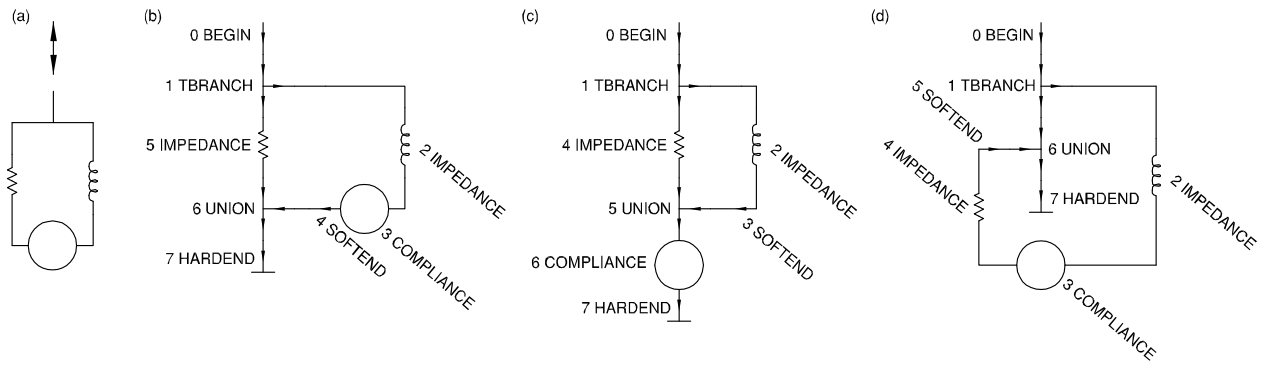


Figure 2.13: A driven, parallel LRC network. (a) Schematic. (b) One way to model the LRC network in DeltaEC, with the L and C in the `TBRANCH` and the R in the trunk, rejoined at the `UNION`, whose downstream end must be sealed by a targeted `HARDEND`. (c) Another way to model the LRC, with the L in the `TBRANCH` and the R in the trunk, rejoined at the `UNION`, whose downstream end feeds the C, whose exit must be sealed by a targeted `HARDEND`. (d) A third way, with the L, R, and C all in the `TBRANCH`. Returning to the trunk, DeltaEC encounters no segment before the `UNION`, so the `UNION` effectively connects the initial and final ends of the branch to each other. The `UNION`'s exit must be sealed by a targeted `HARDEND`.

```
!--------------------------------  3 --------------------------------
 COMPLIANCE      a compliance tank
  4.8360E-02 a SurfAr    m^2   Fn(  3b)   2282.4     A |p|       Pa
  1.0000E-03 b Volume    m^3     S= -5      -8.8682  B Ph(p)     deg
                                          4.3459E-03 C |U|     m^3/s
                                           -54.060   D Ph(U)     deg
                                             3.4951  E Hdot      W
 ideal      Solid type                      3.4951  F Edot      W
!--------------------------------  4 --------------------------------
 SOFTEND    this will reconnect at the UNION
     0.0000  a Re(z)             (t)      2282.4     A |p|       Pa
     0.0000  b Im(z)             (t)        -8.8682  B Ph(p)     deg
                                          4.3459E-03 C |U|     m^3/s
                                           -54.060   D Ph(U)     deg
                                             3.4951  E Hdot      W
                                             3.4951  F Edot      W
                                            44.392   G Re(z)
                                            44.690   H Im(z)
!--------------------------------  5 --------------------------------
 IMPEDANCE  pure resistance
  1.0000E+05 a Re(Zs) Pa-s/m^3            2282.4     A |p|       Pa
     0.0000  b Im(Zs) Pa-s/m^3              -8.8682  B Ph(p)     deg
                                          4.3459E-03 C |U|     m^3/s
                                            125.94   D Ph(U)     deg
                                             -3.4951  E Hdot      W
                                             -3.4951  F Edot      W
!--------------------------------  6 --------------------------------
 UNION      close the loop here
     4.0000  a SegNum                     2282.4     A |p|       Pa
  2282.4     b |p|End    Pa    =  6A?       -8.8682  B Ph(p)     deg
    -8.8682  c Ph(p)E    deg   =  6B?     7.0901E-17 C |U|     m^3/s
                                           -82.266   D Ph(U)     deg
                                          2.3118E-14 E Hdot      W
!--------------------------------  7 --------------------------------
 HARDEND    seal the final end of the resonator
     0.0000  a R(1/z)          =  7G?     2282.4     A |p|       Pa
     0.0000  b I(1/z)          =  7H?       -8.8682  B Ph(p)     deg
                                          7.0901E-17 C |U|     m^3/s
                                           -82.266   D Ph(U)     deg
                                          2.3118E-14 E Hdot      W
                                          2.3118E-14 F Edot      W
                                          7.4001E-17 G R(1/z)
```

And here is the DeltaEC model corresponding to Fig. 2.13(c):

```
TITLE     Example LRC2
```

```
 lrc2.out
!Created@19:31:13  9-Jul-06 with DeltaE Vers. 5.5b6 for the IBM/PC-Compatible
!---------------------------------  0 ---------------------------------
 BEGIN      Initialize things
  1.0000E+05 a Mean P    Pa              6.1460E-03 A  |U|   G(  0f)     P
    60.000   b Freq      Hz                80.940   B Ph(U)  G(  0g)     P
   300.00    c TBeg      K               3.7260E+05 C Re(Zb) G(  1a)     P
  2000.0     d  |p|      Pa             -2.7012E+05 D Im(Zb) G(  1b)     P
     0.0000  e Ph(p)     deg
  6.1460E-03 f  |U|     m^3/s    G
    80.940   g Ph(U)     deg     G
 air        Gas type
!---------------------------------  1 ---------------------------------
 TBRANCH    the split
  3.7260E+05 a Re(Zb) Pa-s/m^3   G        2000.0     A |p|       Pa
 -2.7012E+05 b Im(Zb) Pa-s/m^3   G           0.0000  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                           35.940   D Ph(U)     deg
                                            3.5185  E Hdot      W
                                            3.5185  F Edot      W
                                           -2.5508  G EdotTr    W
!---------------------------------  2 ---------------------------------
 IMPEDANCE  pure inertance
     0.0000  a Re(Zs) Pa-s/m^3            2282.4     A |p|       Pa
  1.0000E+05 b Im(Zs) Pa-s/m^3              -8.8682  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                           35.940   D Ph(U)     deg
                                            3.5185  E Hdot      W
                                            3.5185  F Edot      W
!---------------------------------  3 ---------------------------------
 SOFTEND    this will reconnect at the UNION
     0.0000  a Re(z)             (t)      2282.4     A |p|       Pa
     0.0000  b Im(z)             (t)        -8.8682  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                           35.940   D Ph(U)     deg
                                            3.5185  E Hdot      W
                                            3.5185  F Edot      W
                                           44.690   G Re(z)
!---------------------------------  4 ---------------------------------
 IMPEDANCE  pure resistance
  1.0000E+05 a Re(Zs) Pa-s/m^3            2282.4     A |p|       Pa
     0.0000  b Im(Zs) Pa-s/m^3              -8.8682  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                          125.94    D Ph(U)     deg
                                           -3.4951  E Hdot      W
                                           -3.4951  F Edot      W
!---------------------------------  5 ---------------------------------
 UNION      close the loop here
     3.0000  a SegNum                    2282.4     A |p|       Pa
  2282.4     b |p|End    Pa    =  5A?       -8.8682  B Ph(p)     deg
    -8.8682  c Ph(p)E    deg   =  5B?     6.1460E-03 C |U|     m^3/s
                                           80.940   D Ph(U)     deg
                                         2.3429E-02 E Hdot      W
                                         2.3429E-02 F Edot      W
                                          300.00    G End-T     K
!---------------------------------  6 ---------------------------------
 COMPLIANCE a compliance tank
  4.8360E-02 a SurfAr    m^2   Fn(  6b)   2282.4     A |p|       Pa
  1.0000E-03 b Volume    m^3     S= -5      -8.8682  B Ph(p)     deg
                                         2.8686E-14 C |U|     m^3/s
                                          119.89    D Ph(U)     deg
                                        -2.0495E-11 E Hdot      W
 ideal      Solid type                  -2.0495E-11 F Edot      W
!---------------------------------  7 ---------------------------------
 HARDEND    seal the final end of the resonator
     0.0000  a R(1/z)          =  7G?     2282.4     A |p|       Pa
     0.0000  b I(1/z)          =  7H?       -8.8682  B Ph(p)     deg
                                         2.8686E-14 C |U|     m^3/s
                                          119.89    D Ph(U)     deg
                                        -2.0495E-11 E Hdot      W
                                        -2.0495E-11 F Edot      W
                                        -6.5606E-14 G R(1/z)
```

And finally, here is the DELTAEC model corresponding to Fig. 2.13(d):

```
TITLE     Example 1:  Plane-wave resonator
 lrc3.out
!Created@19:32:21  9-Jul-06 with DeltaE Vers. 5.5b6 for the IBM/PC-Compatible
!---------------------------------  0 ---------------------------------
 BEGIN      Initialize things
  1.0000E+05 a Mean P    Pa              6.1460E-03 A  |U|   G(  0f)     P
    60.000   b Freq      Hz                80.940   B Ph(U)  G(  0g)     P
   300.00    c TBeg      K               3.7260E+05 C Re(Zb) G(  1a)     P
  2000.0     d  |p|      Pa             -2.7012E+05 D Im(Zb) G(  1b)     P
```

```
    0.0000  e Ph(p)     deg
  6.1460E-03 f  |U|     m^3/s    G
    80.940   g Ph(U)     deg     G
 air        Gas type
!---------------------------------  1 ---------------------------------
 TBRANCH    the split
  3.7260E+05 a Re(Zb) Pa-s/m^3   G        2000.0     A |p|       Pa
 -2.7012E+05 b Im(Zb) Pa-s/m^3   G           0.0000  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                           35.940   D Ph(U)     deg
                                            3.5185  E Hdot      W
                                            3.5185  F Edot      W
                                           -2.5508  G EdotTr    W
!---------------------------------  2 ---------------------------------
 IMPEDANCE  pure inertance
    0.0000  a Re(Zs) Pa-s/m^3            2282.4     A |p|       Pa
  1.0000E+05 b Im(Zs) Pa-s/m^3              -8.8682  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                           35.940   D Ph(U)     deg
                                            3.5185  E Hdot      W
                                            3.5185  F Edot      W
!---------------------------------  3 ---------------------------------
 COMPL      a compliance tank
  4.8360E-02 a SurfAr    m^2   Fn(  3b)   2282.4     A |p|       Pa
  1.0000E-03 b Volume    m^3     S= -5      -8.8682  B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                          -54.060   D Ph(U)     deg
                                            3.4951  E Hdot      W
 ideal      Solid type                     3.4951  F Edot      W
!---------------------------------  4 ---------------------------------
 IMPEDANCE      pure resistance
  1.0000E+05 a Re(Zs) Pa-s/m^3            2000.0     A |p|       Pa
    0.0000  b Im(Zs) Pa-s/m^3           -3.4849E-13 B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                          -54.060   D Ph(U)     deg
                                            2.5508  E Hdot      W
                                            2.5508  F Edot      W
!---------------------------------  5 ---------------------------------
 SOFTEND    this will reconnect at the UNION
    0.0000  a Re(z)             (t)      2000.0     A |p|       Pa
    0.0000  b Im(z)             (t)     -3.4849E-13 B Ph(p)     deg
                                         4.3459E-03 C |U|     m^3/s
                                          -54.060   D Ph(U)     deg
                                            2.5508  E Hdot      W
                                            2.5508  F Edot      W
                                           32.398   G Re(z)
!---------------------------------  6 ---------------------------------
 UNION      close the loop here
    5.0000  a SegNum                     2000.0     A |p|       Pa
  2000.0    b |p|End    Pa    =  6A?        0.0000  B Ph(p)     deg
 -3.4849E-13 c Ph(p)E    deg   =  6B?     1.3999E-16 C |U|     m^3/s
                                          -73.811   D Ph(U)     deg
                                         3.9031E-14 E Hdot      W
!---------------------------------  7 ---------------------------------
 HARDEND    seal the final end of the resonator
    0.0000  a R(1/z)          =  7G?     2000.0     A |p|       Pa
    0.0000  b I(1/z)          =  7H?        0.0000  B Ph(p)     deg
                                         1.3999E-16 C |U|     m^3/s
                                          -73.811   D Ph(U)     deg
                                         3.9031E-14 E Hdot      W
                                         3.9031E-14 F Edot      W
```

A topologically similar model of a complicated thermoacoustic system using `TBRANCH` and `UNION` is given in the traveling-wave-engine example in Section 6.3. Some more complicated examples of heirarchically nested structured branches and loops are shown in Figs. 6.9 and 11.1.

2.7.6. Copying a segment or group of segments

An entire segment or a group of segments can be copied and pasted, within a model or from one model to another, much like copying and pasting a paragraph or block of paragraphs in a text editor. Dragging the cursor over the segment or segments indicates their selection via a background color change, and right clicking brings up the cut/copy/paste menu. DeltaEC retains `sameas` and master–slave links inside the copied block, while replacing any pointers

to addresses outside the block with appropriate numerical values.

This feature is also useful for killing a large number of segments at once.

2.7.7. Thermophysical properties

The Tools / thermophysical properties pulldown-menu selection gives the user keyboard access to DeltaEC's library of gas and solid properties for a given mean pressure and temperature. This feature is so convenient that we often start DeltaEC simply to look up the transport properties of air, helium, and other common gases.

`RPN` segments, introduced in Chapter 4 and described fully in Section 11.4.1, can also be used to access thermophysical properties. These segments can be inserted anywhere in a model where the user wants to know the gas or solid properties at the local temperature and pressure. By using the incremental plot feature described in Section 3.2, tables and graphs of properties can be generated over ranges of temperature and pressure, by using T_m and p_m as independent plot variables in a `BEGIN` segment upstream of such `RPN` segments.

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3. More about plotting

DeltaEC's built-in plotter, which is accessible in DeltaEC's Display pulldown menu, provides convenient graphical displays of DeltaEC results.

In *state plots*, the state of the wave and other thermoacoustic variables such as temperature and energy flux are displayed vs location x in the model whenever a run has converged properly. Data are tabulated in a file with the extension `<.sp>`.

In *incremental plots*, the relationships among a number of DeltaEC variables are displayed for a set of many converged runs, as one or two independent variables are automatically incremented between runs. Data are tabulated in a file with the extension `<.ip>`. Incremental plotting is also used to let DeltaEC automatically accomplish a large change in some variable via a series of many smaller changes, to avoid target-convergence problems, as described in Section 5.4.2.

DeltaEC's plotter has many built-in, rudimentary formatting features. Curves can be identified by color, line width, or line type. Default axis labels can be overwritten, and the units on axes can be changed by factors of 10. The view can be changed by zooming, and logarithmic scales can be used. Curves can be examined closely by moving the mouse cursor onto them and reading the horizontal and vertical coordinates. Segment numbers can be displayed on graphs, showing where each segment ends. The graphical results can be exported as `<.jpg>`, `<.bmp>`, or other graphics files.

DeltaEC saves the user's most recent plotting and formatting preferences for each DeltaEC output file in auxiliary files with the extensions `<.ssv>` and `<.isv>`, for the `<.sp>` and `<.ip>` files, respectively. (If the plotter is not working, close DeltaEC, delete the `<.ssv>` and `<.isv>` files associated with your model, and try again.)

3.1. State plots

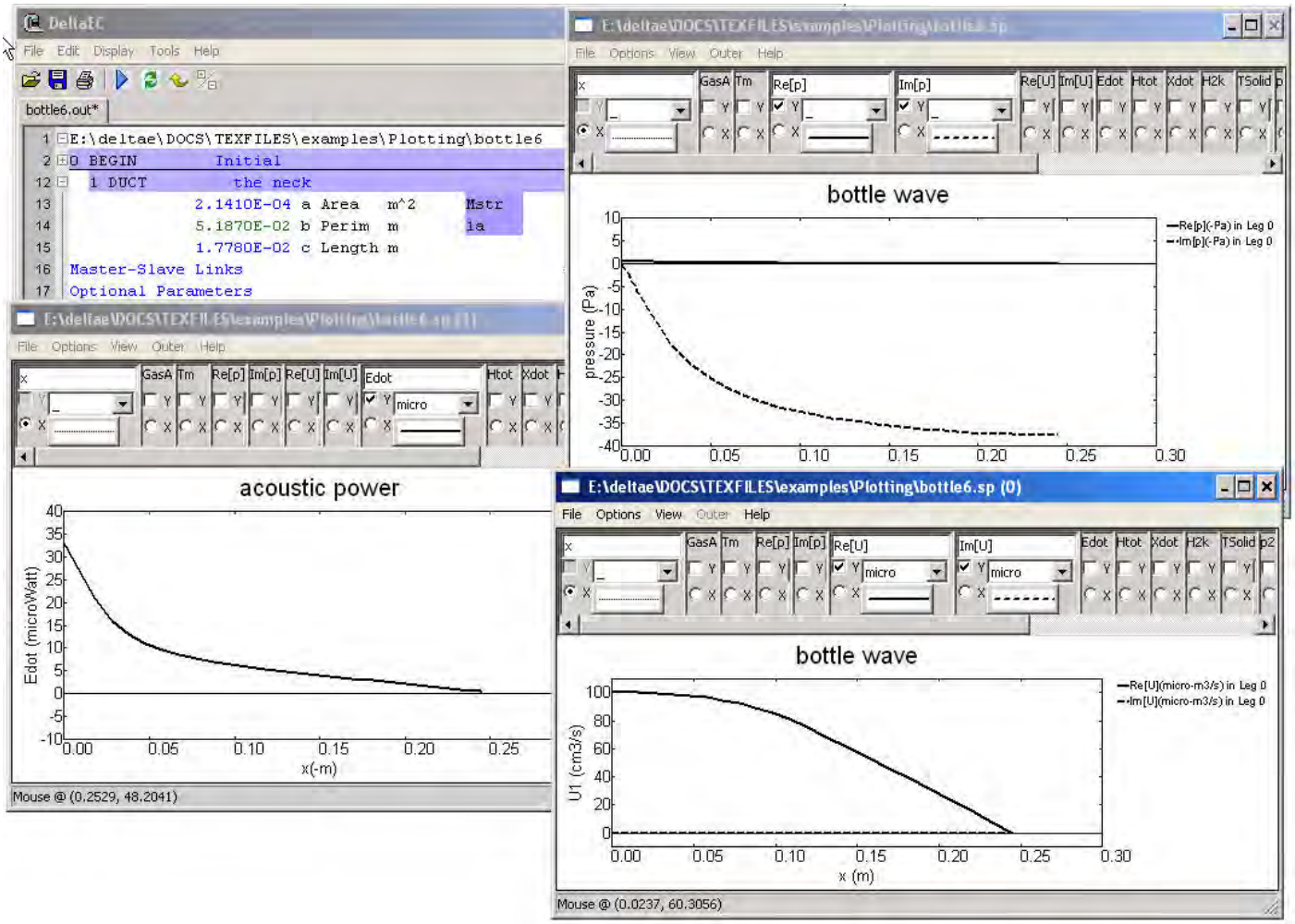
State plots show the current state of the DeltaEC model. DeltaEC's plotter makes state plots from the `<.sp>` file that is created by DeltaEC after every successful run. If the user modifies a parameter in the model, and re-runs it, DeltaEC quickly updates its plot displays to reflect the new reality, just like it updates its numerical displays. This feature was introduced in Section 2.4, where the `<bottle>` model awoke with a state plot showing p_1 and U_1 vs x, and we changed it to display $\dot{E}$ vs x.

DeltaEC subdivides integrable segments like `DUCT`s and `STK**`s into `Nint` pieces, writing `Nint` +1 lines of numbers to the `<.sp>` file for each segment. On the `<.sp>` plot display, each segment is then represented by `Nint` short, straight lines connecting those `Nint` +1 points. The accuracy of the underlying calculation is generally better than what is indicated by the kinkiness of the plot, because the fourth-order Runge-Kutta integration technique

[15] essentially simulates the physics on a finer length scale than the `<.sp>` file actually tabulates.

3.1.1. Cloning

The experienced user will soon want to display more than one plot simultaneously. "Cloning" the state plot lets this occur. Return to `<bottle6>` of Chapter 2, which awakens displaying p_1 vs x. Format this curve for visual appeal, and then "clone" it twice from the plotter's File pulldown menu, editing each clone to arrive at something that looks like this layout:



All five curves displayed here (on three graphs) are updated every time the model runs successfully, and all our effort of formatting and cloning is saved in the `<.ssv>` file if we save when closing the model, so these formatted graphs will reappear if we return to the `<bottle6.out>` model weeks later.

Sometimes it is useful to include A_{gas} vs x in one or more of the state plots, to have a sort of "drawing" of the apparatus lined up with the data graphs. The SegEndMarks option in the plotter's Options menu puts segment numbers on the graphs, at the end of each segment, as another indication of apparatus locations on the data graphs. The schematic view, available on DeltaEC's Display pulldown menu, also gives a picture of the geometry

(see Fig. 2.11 and the Users Guide's cover for examples).

3.1.2. Legs

Models with one or more `TBRANCH`es and/or two or more `BEGIN`s are topologically complicated, so DeltaEC's `<.sp>` file breaks them down into "legs," with each leg starting over at $x = 0$ for purposes of displaying graphs of data. For example, Fig. 3.2 shows a branched resonator, which we modeled with `<legs.out>` (available in the `<Examples\Plotting>` folder). Figure 3.3 shows the standing-wave part of the pressure in that resonator. The side-branch parts of the data form one curve, and the trunk parts of the data form another curve.

Using clone plots and the Legs pulldown menu, the user can show one variable per graph, with all legs plotted in each graph; or can show one leg per graph with all variables of interest plotted in each graph.

3.1.3. Phasor plots

In standing-wave systems, the state plots that display variables such as p_1 and U_1 as functions of x are usually the most useful. However, for traveling-wave systems, phasor diagrams often offer vital insight, so they are included among DeltaEC's standard state-plot options. To enable a phasor plot, check $\mathrm{Re}[p_1]$ or $\mathrm{Re}[U_1]$ instead of x as the "x axis" of the plot, and then use $\mathrm{Im}[p_1]$ or $\mathrm{Im}[U_1]$ as the "y axis." Alternatively, check "Phasor Mode" in the plotter's Options pulldown menu, to ensure equal scaling on the real and imaginary axes and to allow both p_1 and U_1 phasors on the same graph.

Phasor diagrams are only used for complex variables, here p_1 and U_1. (Other examples include transducer current and voltage, but these are not available in DeltaEC's `<.sp>` files.) In a phasor diagram, these complex variables are plotted on the complex plane, with the real part plotted along the horizontal axis and the imaginary part plotted along the vertical axis. Conventionally, arrows from the origin point to each such complex value. One can imagine all of the phasors on a phasor diagram rotating counterclockwise around the origin at radian frequency ω. Then the actual variables represented by the phasors are the time-dependent projections of the phasors on the horizontal axis. Thus a phasor diagram shows the relative magnitudes and time-phase relationships of a number of phasors. The difference between phasors at adjacent places in an apparatus also offers insight into the impedance of what lies between. The difference between pressure phasors is related to the

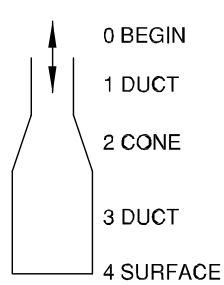


Figure 3.1: The bottle, used as an example in both Chapter 2 and the present chapter.

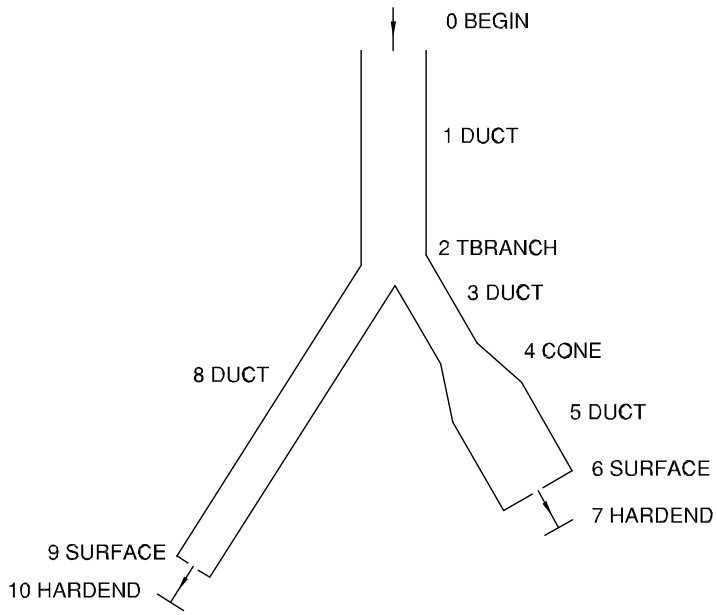


Figure 3.2: A branched resonator. In the model of this resonator, p_1 and U_1 have the same phase at `BEGIN`, and the frequency is a guess. The magnitude of p_1 in `BEGIN` is fixed and the magnitude of U_1 in `BEGIN` is a guess. Both `TBRANCH` impedances are guesses, and the four targets in the two `HARDEND`s are used.

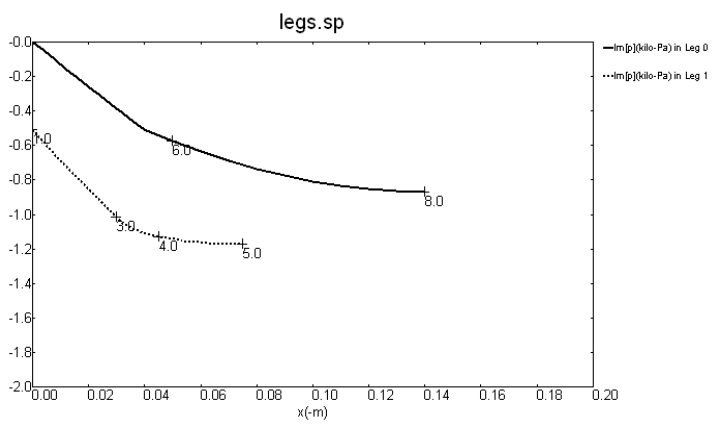


Figure 3.3: The standing-wave part of the pressure oscillation in the branched resonator of Fig. 3.2. There are two curves because the resonator has two legs and both legs have been selected in the Legs pulldown menu. The kink in the upper curve occurs at the same pressure as the start of the lower curve, because these are the same location: the `TBRANCH`.

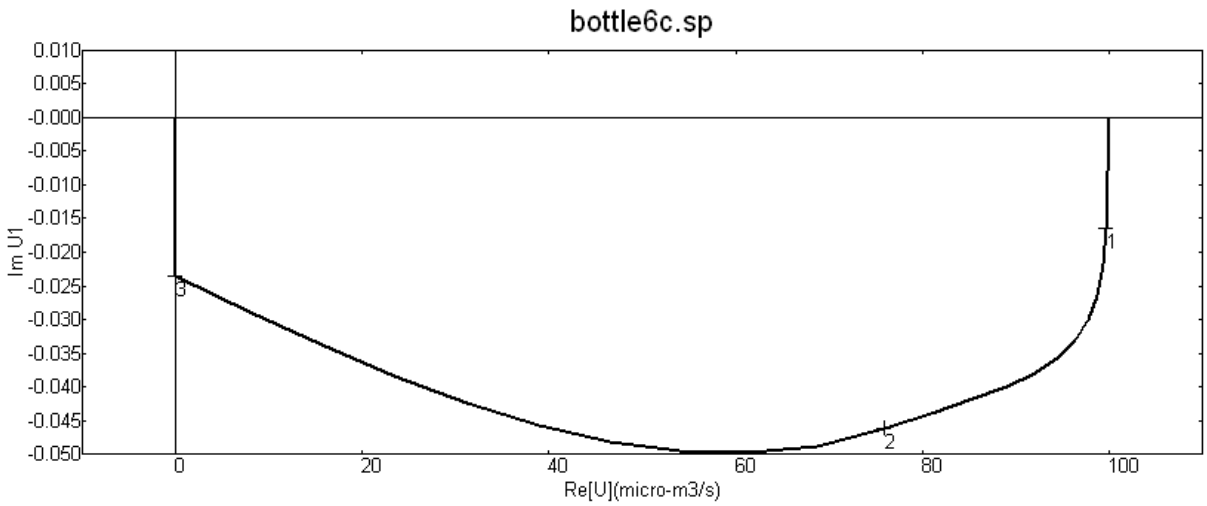


Figure 3.4: The phasor plot displays complex p_1 and/or U_1 on the real–imaginary plane. With SegEndMarks selected in the Options pulldown menu, segment numbers are displayed at the end of each segment.

volume flow rate phasor via the momentum equation, which can include resistance and inertance; the difference between volume-flow-rate phasors is related to the pressure phasor via the continuity equation and the compressibility of the intervening gas [12].

Figure 3.4 shows the U_1 phasors for the bottle.

Another way to look at phasors, based on mouse cursor position in the schematic view, is described in Section 2.7.3.

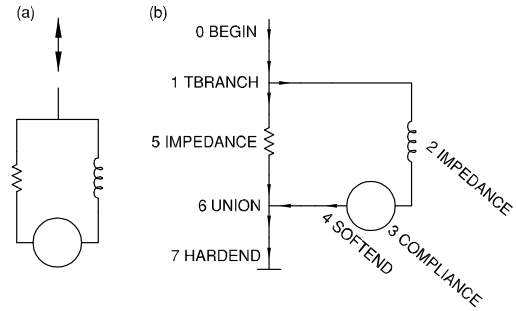
3.2. Incremental plotting

DeltaEC allows for incremental "plotting" by automatically incrementing (or decrementing) an independent variable (or two independent variables) and running repeatedly. This can be used either to let DeltaEC automatically stay converged while changing a single independent variable by many small, "safe" steps when a single leap to a desired new value would fail to converge (see Section 5.4.2), or to tabulate the changing independent variable(s) together with one or more changing output variables in a file named `<.ip>`. Users can then manipulate and/or plot the `<.ip>` file with DeltaEC's built-in plotter (or with their favorite graphics or spreadsheet software). We illustrate these features by continuing two previous examples.

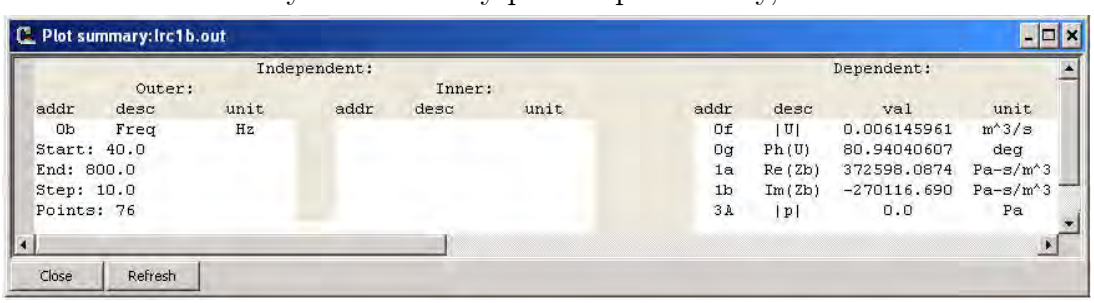
3.2.1. One-dimensional incremental plots

A driven, parallel LRC network was used as an example of structured branches and unions in Section 2.7.5. Here, we use the same example to illustrate a simple incremental plot in DeltaEC. The system is illustrated in the figure below, and the file we start with is `<lrc1.out>` from the `<Examples\Acoustics>` folder. Open that file, and save it as

<lrc1b.out> so subsequent changes don't affect the original. Review the model, to understand that this LRC network is driven at 60 Hz and 2 kPa. Under these circumstances, the pressure amplitude in the COMPLIANCE is 2.28 kPa—higher than at the driver.



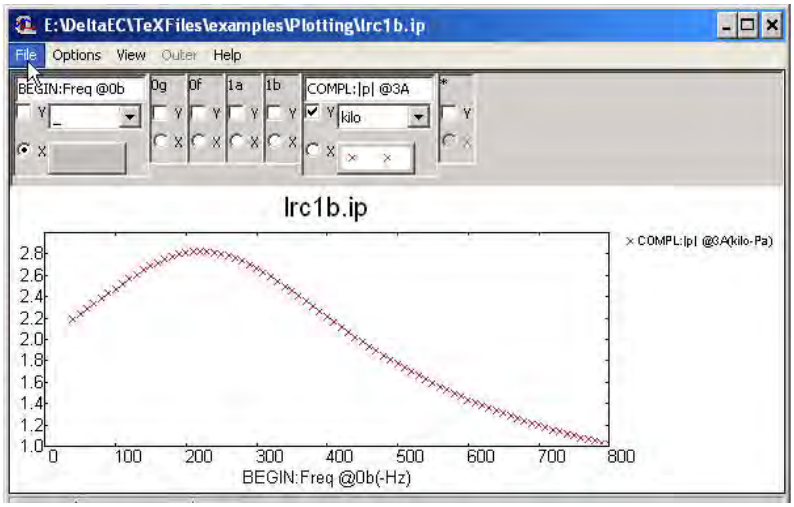
Let's see how the pressure amplitude in the COMPLIANCE depends on frequency, using an incremental plot. Unroll the BEGIN segment, double click on the frequency, and check the Plot box. Let the frequency incrementing start from 40 Hz, end at 800 Hz, and do so in 77 steps so the increments are 10 Hz. Now unroll the COMPLIANCE segment, double click on output A—the pressure amplitude—and select it as a dependent plot variable. Note that 0b and 3A are flagged in the main display as plot variables now. Open the Incremental Plot Summary from the Tools pulldown menu, and see what information is presented. (Note that, in addition to our selection of 3A, all four guesses are listed as dependent plot variables—to let the user return to any incrementally plotted point easily, as described in Section 3.2.3.)



Now start the repetitive run. DeltaEC begins at the initial value of the frequency, 40 Hz, and performs an ordinary run, adjusting the guesses until the targets are met. It writes a line to the incremental-plot file <lrc1b.ip>, recording on one line the independent variable (f), the guesses, and the chosen dependent variables ($|p_1|$ in the COMPLIANCE in the present case). Then DeltaEC increments the independent variable by its step (10 Hz in this case), performs another ordinary run, and records the results on the next line in the <.ip> file...repeating until the final value of the independent variable is reached. In a text editor, the <.ip> file shows this:

```
BEGIN:Freq  BEGIN:|U|   BEGIN:Ph(U) TBRAN:Re(Zb TBRAN:Im(Zb COMPL:|p|
  Hz          m^3/s       deg         Pa-s/m^3    Pa-s/m^3    Pa
     0b          0A          0B          0C          0D          3A
   40.00      3.9237E-03   84.11      5.5931E+05 -4.5475E+05   2186.
   50.00      5.0127E-03   82.56      4.4726E+05 -3.4400E+05   2234.
   60.00      6.1460E-03   80.94      3.7260E+05 -2.7012E+05   2282.
...
```

and opening <lrc1b.ip>, and choosing to plot 3A vs 0b, displays this:



The pressure amplitude starts at 2 kPa at low frequency, and rises with frequency at first, peaking near 200 Hz. Could it be a resonance? The user could now change a parameter such as the resistance in segment 5, repeat the repetitive run, and examine the effects of such a change on the resonance frequency and quality factor. (Don't forget that a frequency-independent $\mathrm{Im}[Z_s]$ in segment 2 is not the same as a frequency-independent inertance!)

3.2.2. Two-dimensional incremental plots

We use the bottle example from Chapter 2 again, starting from `<bottle6.out>` in the `<Examples\Acoustics>` folder. Make the guess–target summary look like this:

```
Guess-Target Summary:
 GUESS       0f          0g
 name   BEGIN:|U|    BEGIN:Ph(U)
 value     1.00E+03      90.
 units      m^3/s         deg
TARGET       5a          5b
 name   HARDEND   :R(1/z HARDEND   :I(1/z
 value      .00          .00
```

Now display the Incremental Plot Summary:

```
Dependent Variables (outputs):
PLOTS      0f         0g
 name   BEGIN:|U|    BEGIN:Ph(U)
 units     m^3/s       deg
```

Keep these parameters as two of the dependent variables to be plotted (they are copies of the guesses).

We will watch these variables as functions of two independent variables: frequency f and mean temperature T_m. To set up these two independent variables to be incremented, modify each of them, checking the Plot box in the Parameter Edit dialog and then entering initial and final values, and the number of steps. Use only three values of mean temperature in the "outer" loop—300, 400, and 500 K—by letting T_m go from 300 to 500 K in three steps of 100 K, and let f go from 100 Hz to 1200 Hz in 551 steps to get exactly 2-Hz increments in the "inner" loop. Also add the pressure in the bottom of the bottle, `5A`, as another dependent plot variable, by double clicking on it and selecting it for plotting. After these actions, check the Incremental Plot Summary again:

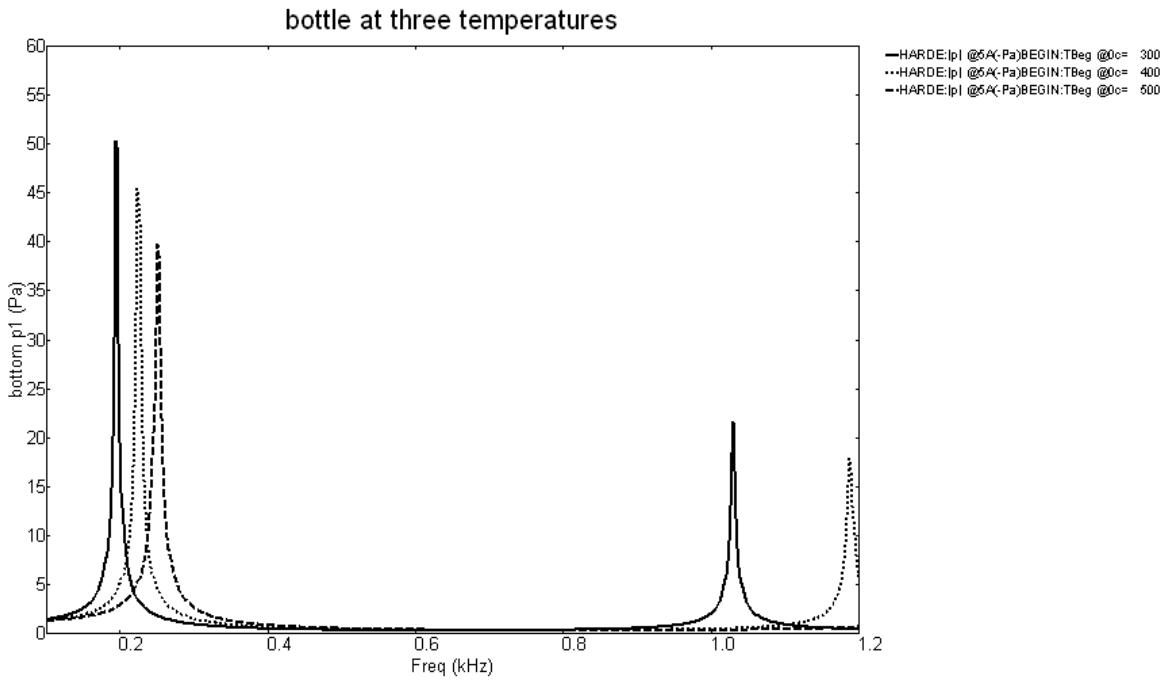


Figure 3.5: Pressure amplitude at the bottom of the bottle as a function of drive frequency, with the pressure amplitude at the mouth held constant at 1 Pa, for three different temperatures.

```
   Outer:                  Inner:                     Dependent:
addr  desc  unit        addr  desc  unit           addr  desc  unit
0c    TBeg  K           0b    Freq  Hz             0f    |U|   m^3/s
Start: 300.0            Start: 100.0               0g    Ph(U) deg
End: 500.0              End: 1200.0                5A    |p|   Pa
Step: 100.0             Step: 2.0
Points: 3               Points: 551
```

Now start a repetitive run. DeltaEC steps through the frequency and mean temperature, taking several seconds. For each value of frequency and mean temperature, it adjusts guesses until targets are met, and then writes one line to the `<.ip>` file, recording the values of the independent and dependent plot variables, including all guesses. When it has finished, use the pc's operating system to find the new file, `<bottle6.ip>`:

```
BEGIN:TBeg  BEGIN:Freq  BEGIN:|U|  BEGIN:Ph(U)  HARDEN:|p|
    K           Hz        m^3/s        deg          Pa
   0c           0b          0f          0g          5A
  300.000     100.000   2.0250E-06 -8.8471E+04      1.38690
  300.000     102.000   2.0953E-06 -8.8471E+04      1.40830
  300.000     104.000   2.1682E-06 -8.8471E+04      1.43090
  .
  .
  .
  500.000    1198.00    1.6259E-07 -8.8481E+04      0.61493
  500.000    1200.00    1.7641E-07 -8.8480E+04      0.62388
```

The results are displayed by DeltaEC's plotter as shown in Fig. 3.5. The plotter's pulldown menu must be used to get all three curves to show on the plot.

3.2.3. Rewinding incremental plots

The rewind feature takes DeltaEC quickly back to a user-selected, previously calculated state that has been saved in the `<.ip>` file. This is useful for:

Selecting a new operating point after using incremental plotting to explore a broad range of behaviors. For example, the user could select a point at the top of one of the peaks in Fig. 3.5.

Selecting a recent well-converged operating point from an incremental plot that contains both converged and unconverged operating points and/or from a long incremental plot that seemed to go awry and was canceled by the user.

The rewind feature is available whenever an `<.ip>` file exists. It is accessible in the Tools pulldown menu or through a button on the toolbar near the top of the main display.

3.2.4. Incremental plots when more than one model is run

Two or more models can be open in DeltaEC at the same time. If these models are run separately, any incremental plots in any of those models will run separately, just as if they were open in separate DeltaEC windows.

Linking two or more models together in a single DeltaEC window and running them together with Run All is described in Section 4.7. The interaction between incremental plots and multi-model running is described near the end of that section.

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4. RPN MATH

4.1. Summary

The RPN segment lets the user create nonstandard guesses, targets, and simple algebraic calculations anywhere in a DeltaEC model. The format of the segment is shown in this example:

```
!-------------------------------- 10 ---------------------------
!  The result(s) of the second line's calculations show up as output(s) in the right column.
 RPN      Dimensionless magnitude of specific acoustic impedance
   16.200     a                                          7.8540E-05 A Znorm
 p1 U1 / mag 9a * rho / a /
```

Input "a" is a real number, which can be a guess, a target, and/or a number used subsequently in calculations via `sameas` links that point to it. The second line (line "b") is a formula expressing a user-defined algebraic procedure in Reverse Polish Notation (RPN), a parenthesis-free algebra encoding technique.

A complete lesson in Reverse Polish Notation can be found in instruction manuals for most HP pocket calculators and on many websites, including Refs. [16] and [17]. DeltaEC users who are unfamiliar with RPN notation are encouraged to consult such a resource. Briefly, as numbers (including pointers to numbers elsewhere in the model, and including variables introduced below) are encountered in the instruction line, these numbers are pushed down onto a stack. Any numbers already present on the stack are pushed deeper into the stack. When a unary operator such as `cos`, `log`, or `sqrt` is encountered in the formula, it takes a single number off the stack ("pops" it), acts on it, and pushes the result back onto the stack. When a binary operator such as + or * is encountered, it pops two numbers off the stack, combines them appropriately, and pushes the result back onto the stack. Other numbers that may be on the stack but are not involved in the current operation simply move up and down during the operation so that no gaps appear in the stack and no uninvolved numbers get lost. (Some of us find it helpful to visualize a spring-loaded stack of dishes at a cafeteria, which adjusts itself so that the top dish is always at the most convenient height no matter how many dishes are hidden underneath it.)

After a simple formula has been processed, there may be only one number remaining on the stack; if so, this number is the "result" of the RPN segment and is displayed as a single output, "A." However, it is often desirable to leave additional results on the stack to be accessed elsewhere or viewed by the user. Whatever is left on the stack after the instruction line is processed is displayed as outputs A, B, C,... For example, if the instruction line is "1 2 3 4 + 45 cos", the segment's output values will be A = 0.7071, B = 7, C = 2, D = 1. To assist in human readability, DeltaEC recognizes the semicolon as a punctuation mark that the user can insert anywhere without affecting the calculation. Thus, the instruction line given above could also be written "1 ; 2 ; 3 4 + ; 45 cos". (Note that blanks must both precede

and follow the semicolon.) Three shades of colored highlighting in the main display of RPN segments also help human readability.

DeltaEC's RPN segment recognizes the common trigonometric, logarithmic, exponential, and algebraic functions present on most pocket calculators, as well as hyperbolic and Bessel functions that are not available on most calculators. These functions accept complex arguments. Other functions convert between complex numbers and real numbers in the usual ways (e.g., real, imaginary, magnitude, phase). For complete lists and short descriptions of DeltaEC's RPN operations, see the tables in Section 11.4.1.

Whenever the command string of an RPN is being modified, clicking the ShowOps button displays a reminder of available operations and variables similar to the following:

```
Valid RPN commands listed below.
   (It is OK to type lower case, except Bessel functions.)

      Basic: +     -     *     /     ^     ~     inv
             log   log10 exp   tenx  sqrt  sqrd  abs
       Trig: cos   acos  sin   asin  tan   atan  atanr pi
             cosr  acosr sinr  asinr tanr  atan2 atan2r
             sinh  cosh  tanh
     Bessel: J0    J1    Y0    Y1
  RPN stack: #     lstx  a<>b  sto   rcl   min   max   inp
    Complex: mag   real  imag  arg   argr  conj  i     cmplx cmplxr

        Gas: gamma a     rho   cp    k     mu    beta  enth  dk    dn    m
    Mixture: nL    kT    D12   mL    mH    NLdot nLzero
      Solid: rhos  cs    ks    ds

      State: Tm    w     f     pm    p1    U1
  Thermodyn: Edot  Htot  H2k   Xdot  Tzero
  Mean flow: Ndot  p20HL p20tot
 Cumulative: F1    vol   x
     Change: =Tm   =p1   =U1   =H2k  =Ndot =NLdot
             =pm   =f    =nL   =F1   =vol  =p20HL
             =Tzero =nLzro
```

Clicking on one of these reminders will save the user a little typing.

Where capital letters appear in the preferred spelling of an RPN operation, DeltaEC will usually accept lower-case typing.

The rest of this chapter introduces many ways that RPN segments can be used.

4.2. Simple non-standard results

The RPN segment is often used to display interesting results for the user.

For example, to calculate and display the gas displacement amplitude

$$|\xi_1| = \frac{|U_1|}{\omega A} \tag{4.1}$$

at the end of, for example, a DUCT at segment 5, an RPN after segment 5 could be written

```
!------------------------- 6 ------------------
RPN   magU1 over omega A (units: meters)
0.00
5C 2 / pi / 0b / 5a /
```

The "0.00" in line a is not used in this example. 5C points to $|U_1|$ in segment 5, 0b points to the frequency in segment 0, and 5a points to the duct area in segment 5. Alternative ways to arrive at the same result in DeltaEC RPN formulas include

```
5C 2 pi * 0b * 5a * /
```

or

```
5C 2 pi 0b 5a * * * /
```

or

```
U1 mag w / 5a /
```

These examples show how DeltaEC recognizes numbers (e.g., "2"), pointers to input and output numbers (e.g., "0b" and "5C"), special constants (here "pi"), and local complex and real variables (here U_1 and ω).

As a second example, the user could display the local values of several thermophysical properties of the gas, namely ρ_m, c_p, a, and μ, by inserting this RPN segment into a model:

```
RPN   A=rho, B=cp, C=sound speed, D=viscosity (MKS units)
0.00
mu a cp rho
```

DeltaEC can perform complex arithmetic in RPNs. For example, it can calculate

$$f_k = \frac{\tanh\left[(1+i)\, y_0/\delta_\kappa\right]}{(1+i)\, y_0/\delta_\kappa} \tag{4.2}$$

for a parallel-plate geometry [e.g., see the first of Eqs. (10.100)]. We let the input parameter, line "a", be equal to the plate half-spacing y_0 so we can change it easily later:

```
RPN  Thermal function for parallel plate geom. "a"=y0 (meters)
 2.0000E-04 a                 (t)     (   0.9244 ,  -0.2392 )  A
inp dk / (1,1) * tanh lstx /
```

Here (1,1) illustrates the entry of a complex constant, $(1+i)$. This complex constant could just as well have been made by entering 1 i + . The input variable of the segment, line a, is referenced via inp in the RPN command string, and the thermal penetration depth δ_κ is accessed via dk.

For complicated algebra that won't fit in one RPN, the user can cascade two or more RPNs, each relying on the output(s) of the previous one(s).

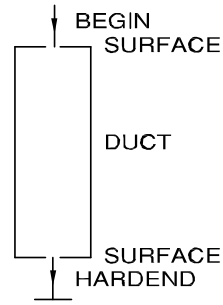
It is dangerous to link the master of a master–slave set to the result of an RPN segment through a sameas link. Such an arrangement works fine until the model is saved to the disk and later re-opened by DeltaEC. While opening such a model, DeltaEC calculates the constant associated with the master–slave link during file parsing, before the model is run and thus before the numerical value of the master is accurately calculated. Thus, the constant might be set inaccurately. If the <.out> file has been modified in a text editor, values of the *output* parameters read from the disk, which are usually irrelevant to subsequent model behavior, can affect such master–slave linkages severely.

4.3. Non-standard inputs

Line a of an RPN segment can be used as an input variable that is unavailable in any other segment. This is convenient for a variable that the user wants to manipulate in non-MKS units, a variable that is frequently changed by the user, or a variable that is changed by DeltaEC as a guess or an independent incremental-plot variable .

For example, suppose that a user is concerned with the volume of a cylindrical resonator, and wants to find out what shape of cylinder requires the lowest drive power for a given

pressure amplitude. The user also prefers to work with the resonator's diameter instead of its area.




```
TITLE     Example 1:  Plane-wave resonator
!Created@13:21:32  8-Jul-06 with DeltaE Vers. 5.5b6 for the IBM/PC-Compatible
!---------------------------------  0 ---------------------------------
 BEGIN      Initialize things
  1.0000E+05 a Mean P    Pa               7903.3     A Freq   G(  0b)     P
  7903.3     b Freq      Hz      G        3.9873E-05 B  |U|   G(  0f)     P
   300.00    c TBeg      K
  1000.0     d  |p|      Pa
     0.0000  e Ph(p)     deg
  3.9873E-05 f  |U|     m^3/s    G
     0.0000  g Ph(U)     deg
 helium     Gas type
!---------------------------------  1 ---------------------------------
 RPN        a=diam (cm), A=cross-sectional area (square meters)
     1.0000  a G or T            (t)      7.8540E-05 A Area
   inp 100 / sqrd pi * 4 /
!---------------------------------  2 ---------------------------------
 RPN        a=volume (cubic cm).  A=length (meters)
    10.000   a G or T            (t)         0.1273  A length
   inp 1E6 / 1A /
!---------------------------------  3 ---------------------------------
 SURFACE     top end cap of resonator (imagine it's a moving driver)
 sameas   1A a Area      m^2              1000.0     A |p|       Pa
                                             0.0000  B Ph(p)     deg
                                          3.9203E-05 C |U|     m^3/s
                                             0.0000  D Ph(U)     deg
                                          1.9602E-02 E Hdot      W
 ideal      Solid type                    1.9602E-02 F Edot      W
!---------------------------------  4 ---------------------------------
 DUCT       the most important piece of this resonator
 sameas   1A a Area      m^2     S= -2     996.65    A |p|       Pa
  3.1372E-02 b Perim      m    Fn(  4a)   5.8368E-04 B Ph(p)     deg
 sameas   2A c Length     m               6.6737E-07 C |U|     m^3/s
                                          5.8368E-04 D Ph(U)     deg
                                          3.3257E-04 E Hdot      W
 ideal      Solid type                    3.3257E-04 F Edot      W
!---------------------------------  5 ---------------------------------
 SURFACE     End of the pipe
 sameas   1A a Area      m^2               996.65    A |p|       Pa
                                          5.8368E-04 B Ph(p)     deg
                                          2.2261E-19 C |U|     m^3/s
                                          -104.32    D Ph(U)     deg
                                         -2.7438E-17 E Hdot      W
 ideal      Solid type                   -2.7438E-17 F Edot      W
!---------------------------------  6 ---------------------------------
 HARDEND    seal the final end of the resonator
     0.0000  a R(1/z)          =  6G?      996.65    A |p|       Pa
     0.0000  b I(1/z)          =  6H?     5.8368E-04 B Ph(p)     deg
                                          2.2261E-19 C |U|     m^3/s
                                          -104.32    D Ph(U)     deg
                                         -2.7438E-17 E Hdot      W
                                         -2.7438E-17 F Edot      W
                                         -1.1503E-16 G R(1/z)
                                           -4.5062E-16 H I(1/z)
```

With this file, the user can modify the diameter (in cm) and the volume (in cm^3) conveniently in `1a` and `2a`, and could use those as independent plot variables or as guesses. RPN instruction `1b` calculates the area (in m^2) and puts it in output `1A` for use in segments 3, 4, and 5. RPN instruction `2b` calculates the length from the given diameter and volume, putting it in output `2A` for use in segment 4.

Note that segments 3, 4, and 5 show how `sameas` can point to DeltaEC outputs (capital letters) as well as inputs (small letters).

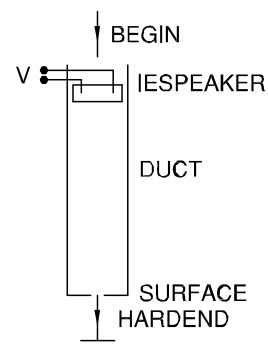
4.4. Non-standard targets

DeltaEC has a few specially assigned input parameters that hold target values in segment types that have outputs very commonly used in targets. Some of these were introduced

in Chapter 2: the complex normalized impedances z_n in `HARDEND` and `SOFTEND`, and the complex pressure p_1 in `UNION`. Other routine, built-in targets are the solid temperatures in heat exchangers, which will be introduced in Chapter 5. DeltaEC knows which output variables in these segments should be associated with their "default" target values when the shooting-method algorithm is using them.

The experienced user will soon hunger for more possibilities. An application may call for specifying acoustic power $\dot{E}$, pressure amplitude $|p_1|$, velocity (magnitude or phase), or piston stroke. The `RPN` segment is used to establish such variables as targets. The first of the `RPN` outputs, "`A`," is recognized by DeltaEC as paired with the `RPN` input "`a`" to comprise a potential target/result.

For example, the stroke of a linear motor is often limited by the flexure characteristics of its suspension. Here is an example using the `RPN` segment to find out how much current must be delivered to a motor to keep the motor's stroke targeted at 2 mm while driving a resonator. The usual `HARDEND` targets are also used, with complex U_1 in `BEGIN` being the corresponding guesses. The `RPN` segment's input, `2a`, which is set by the user at 2.000, is the user's desired target, and the RPN formula yields the motor stroke, which is written in `2A`. DeltaEC has adjusted guess `1h`, the motor current, until the motor stroke met its target.




```
TITLE     Example 1:  Plane-wave resonator
!Created@14:22:59  8-Jul-06 with DeltaE Vers. 5.5b6 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
 BEGIN      Initialize things
  1.0000E+05 a Mean P    Pa
   150.00    b Freq      Hz
   300.00    c TBeg      K
     0.0000  d  |p|      Pa
     0.0000  e Ph(p)     deg
  9.4483E-03 f  |U|     m^3/s    G
   147.39    g Ph(U)     deg     G
 helium     Gas type
!--------------------------------  1 --------------------------------
 IESPEAKER  a linear motor and piston
 sameas   3a a Area      m^2              2443.2     A |p|       Pa
     1.0000  b   R      ohms              -177.67    B Ph(p)     deg
  3.0000E-03 c L         H               9.4248E-03 C |U|     m^3/s
     5.0000  d BLProd   T-m               147.29    D Ph(U)     deg
  1.0000E-02 e   M      kg                  9.4267  E Hdot      W
  1.0000E+04 f   K      N/m                 9.4267  F Edot      W
     0.1000  g   Rm   N-s/m                20.919   G WorkIn    W
     4.7776  h  |I|     A        G         14.031   H Volts     V
     0.0000  i Ph(I)   deg                  4.7776  I Amps      A
                                            51.381   J Ph(Ze)   deg
                                          2443.2     K  |Px|    Pa
 ideal      Solid type                    -177.67    L Ph(Px)   deg
!--------------------------------  2 --------------------------------
 RPN        peak to peak stroke of the motor (mm)
     2.0000  a G or T          =  2A?        2.0000  A stroke
   U1 mag 1a / w / 2 * 1000 *
!--------------------------------  3 --------------------------------
 DUCT       the most important piece of this resonator
  1.0000E-02 a Area      m^2     S= -2    2441.6     A |p|       Pa
     0.3540  b Perim      m    Fn(  3a)   -177.56    B Ph(p)     deg
     6.7000  c Length     m               2.8677E-05 C |U|     m^3/s
                                          -177.56    D Ph(U)     deg
                                          3.5009E-02 E Hdot      W
 ideal      Solid type                    3.5009E-02 F Edot      W
!--------------------------------  4 --------------------------------
 SURFACE     End of the pipe
 sameas   3a a Area      m^2              2441.6     A |p|       Pa
                                          -177.56    B Ph(p)     deg
                                          8.8927E-15 C |U|     m^3/s
                                           -36.711   D Ph(U)     deg
                                         -8.4187E-12 E Hdot      W
 ideal      Solid type                   -8.4187E-12 F Edot      W
!--------------------------------  5 --------------------------------
 HARDEND    seal the final end of the resonator
     0.0000  a R(1/z)          =  5G?     2441.6     A |p|       Pa
     0.0000  b I(1/z)          =  5H?     -177.56    B Ph(p)     deg
                                          8.8927E-15 C |U|     m^3/s
                                           -36.711   D Ph(U)     deg
```

```
                                        -8.4187E-12 E Hdot      W
                                        -8.4187E-12 F Edot      W
                                        -4.6190E-14 G R(1/z)
                                         3.7606E-14 H I(1/z)

! The restart information below was generated by a previous run
! You may wish to delete this information before starting a run
! where you will (interactively) specify a different iteration
! mode.  Edit this table only if you really know your model!
guessz    0f 0g 1h
xprecn  -1.1737E-08  4.2373E-03 -2.9847E-05
targs     2a 5a 5b
SPECIALS    1   3 -2
```

The target parameter of an `RPN`, like any other target, can be used as the independent variable in a plotting loop.

An `RPN` segment should usually be placed downstream of all the results and guesses that it depends on, so that the `RPN` will have current values during each DeltaEC integration sequentially through the segments.

4.5. Changing an integration variable

It is occasionally necessary to abruptly change the value of one or more of DeltaEC's integration variables in the middle of its integration. The `RPN` segment lets the user change these in response to a simple algebraic calculation, by using an RPN operator of the form `=variable`, where `variable` is one of the integration variables. For example, one might have a resonator whose top half is wrapped in a water bath at one temperature and whose bottom half is bathed at another temperature. Splitting the resonator duct into two sequential `DUCT`s and inserting an `RPN` with an appropriate command string between the two `DUCT`s will change the gas temperature T_m halfway along the resonator. To change the temperature from 300 Kelvin (set in the `BEGIN`) to 320 Kelvin, the segment between the `DUCT`s could be

```
RPN
0.00
320 =Tm
```

or

```
RPN
320.0
inp =Tm
```

To increment the temperature by 20 Kelvin, use either

```
RPN
0.00
Tm 20 + =Tm
```

or

```
RPN
20.0
Tm inp + =Tm
```

4.6. Non-standard guesses

Like most other DeltaEC inputs, parameter `a` in an `RPN` segment can be used as a guess. For example, to let DeltaEC guess the temperature increment needed at the present location to hit a target downstream, guess input `a` in an `RPN` segment like this:

```
RPN
20.0     G
Tm inp + =Tm
```

As a contrived example, suppose that the magnetic field B in a speaker depends on temperature T_m according to a strange equation such as

$$B^3 - 5B^2 + 3 = T_m, \tag{4.3}$$

and we want DeltaEC to correctly incorporate a temperature-dependent Bl product in a `SPEAKER` segment. We don't know how to solve Eq. (4.3) for B analytically, but we can make DeltaEC find the root of the cubic equation numerically by guessing B and targeting Eq. (4.3):

```
!--------------------14------------------------
RPN  (a)=magnetic field (tesla), guessed. 1.6 meters of wire.  (A)= BL product.
12.863    a    G or T            A 20.581   B x L
inp 1.6 *
!---------------------15------------------------
RPN   Target this:
0.00    a      G or T             A  0.000 A zero
14a 3 ^ 14a 2 ^ 5 * - 3 + Tm -
!--------------------16-----------------------
 IESPEAKER  a linear motor and piston
 sameas   3a a Area      m^2              2443.2     A |p|       Pa
     1.0000  b   R      ohms              -177.67    B Ph(p)     deg
  3.0000E-03 c L         H               9.4248E-03 C |U|     m^3/s
 sameas 14A  d BLProd   T-m                147.29    D Ph(U)     deg
  1.0000E-02 e   M      kg                  9.4267  E Hdot      W
  1.0000E+04 f   K      N/m                 9.4267  F Edot      W
     0.1000  g   Rm   N-s/m                20.919   G WorkIn    W
     4.7776  h  |I|     A        G         14.031   H Volts     V
     0.0000  i Ph(I)   deg                  4.7776  I Amps      A
                                            51.381   J Ph(Ze)   deg
                                          2443.2     K  |Px|    Pa
 ideal      Solid type                    -177.67    L Ph(Px)   deg
```

4.7. Inter-model linking

Two or more models can be open simultaneously in a single DeltaEC window, and it is sometimes useful to bring a number from one model into another model. This is accomplished with an extended RPN address of the form `filename:addr` in an RPN formula. For example, we have sometimes been interested in a few unequal engines and many unequal refrigerators sharing a common resonator, requiring so many segments and so many guesses and targets that several models are needed to hold them all. Inter-model linking lets such multiple models communicate with one another.

For example, suppose that three models, named <`engine1.out`>, <`engine2.out`>, and <`engine3.out`>, which represent a cascade engine [18], are simultaneously open in DeltaEC. Suppose that hot-heat-exchanger heats are given at address `12e` in all three models. An `RPN` segment near the end of <`engine3.out`> could calculate the total heat required by the set of three engines with the following formula which brings results from <`engine1.out`> and <`engine2.out`> into <`engine3.out`>:

```
engine1:12e engine2:12e + 12e +
```

Extending this example, if the beginning of the <`engine2.out`> model is supposed to be fed by the end of the <`engine1.out`> model, where segment 87 is a `STK**` and 88 is a `HX**`, the key variables can be shared if the beginning of <`engine2.out`> looks like

```
! engine2.out
!-------------------------- 0 ------------------------------
BEGIN This segment is required, even though these variables are re-initialized below!
999.   Pa a Mean P
999.   Hz b Freq
999.    K c TBeg
999.   Pa d |p|
999.  deg e Ph(p)
999. m3/s f |U|
999.  deg g Ph(U)
helium    Gas type
!-------------------------- 1 ------------------------------
RPN  bring pm and f from the other model
engine1:0a =pm  %:0b =f
!-------------------------- 2 ------------------------------
RPN  bring Tm from the other model
engine1:87H =Tm
!-------------------------- 2 ------------------------------
RPN  bring p1 and U1 from the other model
engine1:88B %:88A cmplx =p1 %:88D %:88C cmplx =U1
```

(The notation " % " in an interlinked RPN formula signifies "the previously named model.")

To run all of the models open in DeltaEC together, use Run All, with capital-R as its keyboard shortcut. The order of running will be left to right in the sequence in which the model tabs are laid out. Use the mouse to drag tabs from side to side to change that order. (Drag a tab off the side, top, or bottom of the panel to split the panel into two or more displays so two or more models can be seen simultaneously.) Choose the order of running carefully, so that each model has up-to-date values of any numbers that it gets from other models.

If two or more models are run separately, any incremental plots in any of those models will run separately, just as if they were open in separate DeltaEC windows. However, Run All changes this behavior dramatically, disabling any incremental plots in the separate models and following the choices established by the user in the Master Plot Summary dialog, which is available in the Display pulldown menu. The Master Plot Summary lets the user choose one or two of the independent plot variables that exist in the individual models as the Master independent plot variable (1-dimensional plot) or variables (2-dimensional plot), and choose from among the established dependent plot variables (including guesses) in the individual models too. A Run All will then run all the models, increment an independent plot variable, run all the models again, etc., creating a table of results in a file called <`Master.ip`>. For example, a set of linked models often share the same frequency, linked together via RPN formulas similar to `firstmodelname:0b =f` . If parameter `0b` in model `firstmodelname` is chosen as the independent Master Plot variable, then a Run All will sequentially increment the frequency in all of the linked models.

To view graphs of <`Master.ip`> after such a Run All, click the appropriate button in the Master Plot Summary.

Please do not create more than one set of such inter-linked models with Master Plotting in a single folder, because DeltaEC can have only one <`Master.ip`> file per folder.

4.8. Other math segments

The `CONSTANTS` segment allows the user to insert up to a dozen real constants in one segment. These can then be referred to by other segments (via `sameas`) and can be modified, used as guesses, or used as independent incremental-plot variables. See details in Section 11.4.3.

The `VOLUME` segment automatically adds up the volume in a series of segments. See details in Section 11.4.2. `VOLUME` adds up the volume of gas plus the volume of the solids in porous structures such as heat exchangers, so it gives one estimate of the size of an apparatus. (The RPN variable `vol`, described in Section 11.4.1, is similar to the segment `VOLUME`, but it accesses the cumulative *gas* volume in the model, without involving the solid.)

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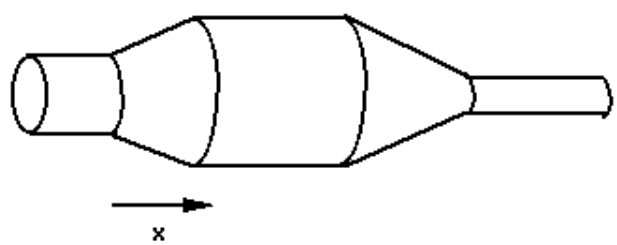
5. Engines & refrigerators

The examples given in the previous Chapters were relatively simple cases of linear acoustics. In this Chapter, we introduce the full thermoacoustic computing power of DeltaEC with some simple engines and refrigerators. After discussing the principles and assumptions that are built into the computation, we present example calculations.

5.1. Principles of calculation



DeltaEC processes one-dimensional strings of acoustic and thermoacoustic elements (or networks of such strings—see Section 2.7.5 and Chapter 11 for `TBRANCH`s and `UNION`s), so the one-dimensional "wave" equation is of the greatest importance. We always assume a time dependence of $e^{i\omega t}$, so the "wave" equation is the second-order Helmholtz differential equation for the complex pressure amplitude $p_1(x)$. As was described at the beginning of Chapter 2, we find it easiest to think of this as two coupled first-order equations in pressure p_1 and volume flow rate U_1. This point of view is taken in Refs. [12] and [19]. In coupled, first-order form, the equations are ready for numerical integration along x.

In Chapter 2, we introduced such equations for `DUCT`s and `CONE`s, in which T_m did not vary with x. Nicholas Rott [20] derived the corresponding equation when $dT_m/dx \neq 0$:

$$\left(1+\frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1+\frac{\rho_m a^2}{\omega^2}\frac{d}{dx}\left(\frac{1-f_\nu}{\rho_m}\frac{dp_1}{dx}\right)-\beta\frac{a^2}{\omega^2}\frac{(f_\kappa-f_\nu)}{(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}\frac{dp_1}{dx}=0, \quad (5.1)$$

which can be rewritten as two coupled first-order equations:

$$\begin{aligned}\frac{dp_1}{dx} &= -\frac{i\omega\rho_m}{A\,(1-f_\nu)}U_1,\\ \frac{dU_1}{dx} &= -\frac{i\omega A}{\rho_m a^2}\left(1+\frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1+\frac{(f_\kappa-f_\nu)}{(1-\sigma)(1+\epsilon_s)}\beta\frac{dT_m}{dx}U_1.\end{aligned} \quad (5.2)$$

These equations are used in most of the `STK**` segments introduced below, which accommodate $dT_m/dx \neq 0$ in engines and refrigerators. The complex variables f_ν and f_κ depend on the size and shape of the pores and on the gas properties. Of course the right sides of these

equations depend implicitly on the local value of T_m, because gas properties such as ρ_m and a depend on T_m.

To complete this picture for many thermoacoustic engines and refrigerators, only two more equations are required, i.e., those for the temperature $T_m(x)$ and the total power $\dot{H}_{\rm tot}(x)$. As for $p_1(x)$ and $U_1(x)$, the equations for $T_m(x)$ and $\dot{H}_{\rm tot}(x)$ depend on the type of segment, and continuity of T_m and $\dot{H}_{\rm tot}$ are imposed at the junctions between segments.

In DELTAEC, the total power is treated as the sum of two terms,

$$\dot{H}_{\rm tot} = \dot{H}_{2,k} + \dot{H}_{\dot{N}}. \tag{5.3}$$

The first term, $\dot{H}_{2,k}$, represents power carried in the x direction by second-order thermoacoustic effects, generally proportional to the lateral spatial average of $\mathrm{Re}[T_1\widetilde{u}_1]$, and by zeroth-order thermal conduction. The second term, $\dot{H}_{\dot{N}}$, represents power carried by nonzero time-averaged flow of gas in the x direction. In the present chapter, we consider only zero time-averaged flow, so $\dot{H}_{\rm tot} = \dot{H}_{2,k}$. (Nonzero $\dot{H}_{\dot{N}}$ is introduced in Chapter 6.)

Most segments, such as `DUCT`s and `CONE`s described in Chapter 2, obey simply $dT_m/dx = 0$ and $d\dot{H}_{\rm tot}/dx = 0$. `STK**` segments have a more complicated differential equation for $T_m(x)$, which is derived from Rott's energy equation [21],

$$\begin{aligned}
\dot{H}_{2,k} &= \frac{1}{2}\mathrm{Re}\left[p_1\tilde{U}_1\left(1-\frac{f_\kappa-\tilde{f}_\nu}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right] \\
&+ \frac{\rho_m c_p|U_1|^2}{2A_{\rm gas}\omega(1-\sigma)|1-f_\nu|^2}\frac{dT_m}{dx}\mathrm{Im}\left[\tilde{f}_\nu+\frac{(f_\kappa-\tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] \\
&- (A_{\rm gas}k + A_{\rm solid}k_s)\frac{dT_m}{dx},
\end{aligned} \tag{5.4}$$

by solving it algebraically for dT_m/dx. The total power flow $\dot{H}_{\rm tot}$ through a `STK**` segment is assumed to be independent of x, because the side walls are assumed to be rigid and thermally insulated. The total power $\dot{H}_{\rm tot}$ changes with x in heat exchanger segments, according to

$$\frac{d\dot{H}_{\rm tot}}{dx} = \dot{q}, \tag{5.5}$$

where $\dot{q}$ is the heat added per unit length in the heat exchanger.

(There is also a "thermally anchored" mode in which $d\dot{H}_{2,k}/dx = d\dot{E}/dx$ in `DUCT`s, `CONE`s, etc. to reflect the assumption that such segments are immersed in heat sinks; this exception to the normal circumstances will be described in Section 11.3.1.)

So, for thermoacoustic calculations, DELTAEC integrates from the `BEGIN`ning through the sequence of subsequent segments, with respect to two complex and two real variables: $p_1(x)$, $U_1(x)$, $T_m(x)$, and $\dot{H}_{\rm tot}(x)$. DELTAEC uses continuity of p_1, U_1, T_m, and $\dot{H}_{\rm tot}$ to pass from the end of one segment to the beginning of the next. Within each segment, it uses equations appropriate to that segment type, governed by local parameters, such as area and perimeter, by global parameters, such as frequency and mean pressure, and by evolving parameters such as p_1, U_1, T_m, and $\dot{H}_{\rm tot}$ themselves and the properties of the gas and solid.

Mathematically, the DeltaEC integration can be regarded as

$$\frac{dp_1}{dx} = \mathcal{F}_{\text{momentum}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega;\ \text{geometry, gas properties, etc.}), \tag{5.6}$$

$$\frac{dU_1}{dx} = \mathcal{F}_{\text{continuity}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \text{geometry, gas properties, etc.}), \tag{5.7}$$

$$\frac{dT_m}{dx} = \mathcal{F}_{x\text{-energy}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \text{geometry, gas properties, etc.}), \tag{5.8}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \mathcal{F}_{\text{lateral energy}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \text{geometry, gas properties, etc.}). \tag{5.9}$$

In reality, as documented here and in Chapter 10, most of the equations for specific segments are simpler than what's suggested by the fully general forms of Eqs. (5.6)–(5.9). For example, the right-hand side of the momentum equation never depends explicitly on p_1 or $\dot{H}_{\text{tot}}$; many segment types have simply $dU_1/dx = 0$ or $dT_m/dx = 0$ or can calculate Δp_1 algebraically instead of with a numerical integration; and dH_{tot}/dx is usually zero or as simple as Eq. (5.5). Nevertheless it is useful to keep in mind the fully general picture of Eqs. (5.6)–(5.9), to help remember what DeltaEC needs to know at each location x as it proceeds with its numerical integration.

5.2. Hofler's refrigerator

Tom Hofler's thermoacoustic refrigerator was described in detail in his Ph.D. thesis [1] and was briefly summarized in Ref. [2]. We use this case to further illustrate capabilities of DeltaEC, generating curves similar to Figs. 16 and 17 in Ref. [1] (Figs. 5 and 6 in Ref. [2]).

The apparatus is shown in Fig. 5.1. We begin with a file (<`hofler1.out`>, in the <`Examples\EnginesAndRefr`> folder) whose geometry is that of Hofler's "long" apparatus. (We originally typed this file in a Notepad text editor while studying Ref. [1].) The list of segments serves as an outline of the model:

```
0 BEGIN:
1 SURFACE: driver end
2 DUCT : ambient temperature duct
3 HX : ambient temperature heat exchanger
4 STKSLAB: Stack
5 HX : Cold heat exchanger
6 DUCT : Cold Duct
7 CONE :
8 COMPLIANCE: end bulb
```

and clicking the toolbar "+" shows details:

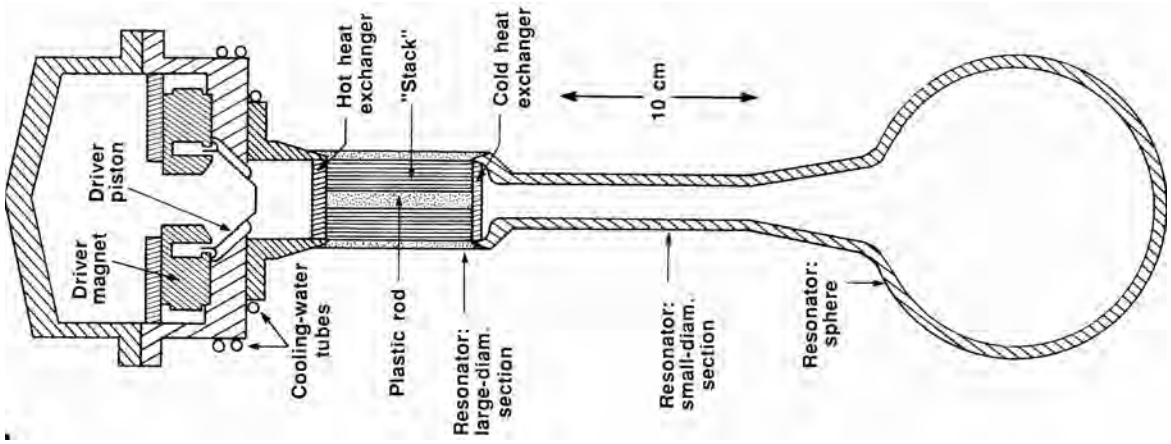
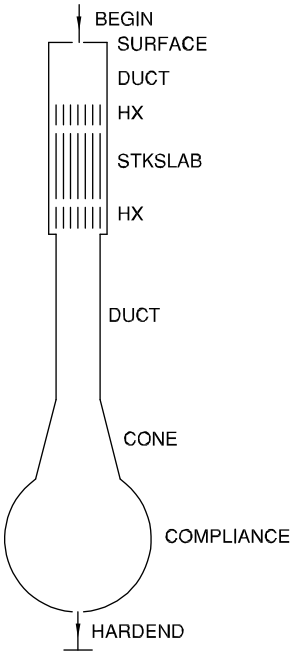


Figure 5.1: Hofler's thermoacoustic refrigerator.

```
TITLE     Hofler's 1986 thermoacoustic refrigerator, hofler1.out
!Created@16:29:08  1-Oct-06 with DeltaE Vers. 6.0g2 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
 BEGIN
  1.0000E+06 a Mean P    Pa
   500.00    b Freq      Hz
   300.00    c TBeg      K
  3.0000E+04 d  |p|      Pa
     0.0000  e Ph(p)     deg
  5.0000E-04 f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 helium     Gas type
!--------------------------------  1 --------------------------------
 SURFACE    driver end
  1.1340E-03 a Area      m^2                0.0000  A |p|       Pa
                                            0.0000  B Ph(p)     deg
                                            0.0000  C |U|     m^3/s
                                            0.0000  D Ph(U)     deg
                                            0.0000  E Htot      W
 ideal      Solid type                      0.0000  F Edot      W
!--------------------------------  2 --------------------------------
 DUCT       ambient temperature duct
 sameas   1a a Area      m^2                0.0000  A |p|       Pa
     0.1190  b Perim      m                 0.0000  B Ph(p)     deg
  4.2600E-02 c Length     m                 0.0000  C |U|     m^3/s
                                            0.0000  D Ph(U)     deg
                                            0.0000  E Htot      W
 ideal      Solid type                      0.0000  F Edot      W
!--------------------------------  3 --------------------------------
 HX         ambient temperature heat exchanger
 sameas   1a a Area      m^2                0.0000  A |p|       Pa
     0.6000  b GasA/A                       0.0000  B Ph(p)     deg
  6.3500E-03 c Length     m                 0.0000  C |U|     m^3/s
  1.9000E-04 d y0         m                 0.0000  D Ph(U)     deg
    -9.6400  e HeatIn     W                 0.0000  E Htot      W
   300.00    f Est-T      K      (t)        0.0000  F Edot      W
                                            0.0000  G GasT     K
 copper     Solid type                      0.0000  H SolidT     K
!--------------------------------  4 --------------------------------
 STKSLAB    Stack
 sameas   1a a Area      m^2                0.0000  A |p|       Pa
     0.7240  b GasA/A                       0.0000  B Ph(p)     deg
  7.8500E-02 c Length     m                 0.0000  C |U|     m^3/s
  1.8000E-04 d y0         m                 0.0000  D Ph(U)     deg
  4.0000E-05 e Lplate     m                 0.0000  E Htot      W
                                            0.0000  F Edot      W
                                            0.0000  G TBeg       K
 kapton     Solid type                      0.0000  H TEnd       K
!--------------------------------  5 --------------------------------
 HX         Cold heat exchanger
 sameas   1a a Area      m^2                0.0000  A |p|       Pa
     0.6700  b GasA/A                       0.0000  B Ph(p)     deg
  2.5400E-03 c Length     m                 0.0000  C |U|     m^3/s
  2.5500E-04 d y0         m                 0.0000  D Ph(U)     deg
     3.0000  e HeatIn     W                 0.0000  E Htot      W
   200.00    f Est-T      K      (t)        0.0000  F Edot      W
                                            0.0000  G GasT     K
 copper     Solid type                      0.0000  H SolidT     K
```



```
!---------------------------------  6 ---------------------------------
 DUCT       Cold Duct
  3.8400E-04 a Area      m^2                 0.0000  A |p|       Pa
  6.9400E-02 b Perim      m                  0.0000  B Ph(p)     deg
     0.1670  c Length     m                  0.0000  C |U|     m^3/s
                                             0.0000  D Ph(U)     deg
                                             0.0000  E Htot      W
 ideal      Solid type                       0.0000  F Edot      W
!---------------------------------  7 ---------------------------------
 CONE
 sameas   6a a AreaI     m^2                 0.0000  A |p|       Pa
 sameas   6b b PerimI     m                  0.0000  B Ph(p)     deg
  6.6800E-02 c Length     m                  0.0000  C |U|     m^3/s
  1.1600E-03 d AreaF     m^2                 0.0000  D Ph(U)     deg
     0.1210  e PerimF     m                  0.0000  E Htot      W
 ideal      Solid type                       0.0000  F Edot      W
!---------------------------------  8 ---------------------------------
 COMPLIANCE end bulb
  4.9000E-02 a SurfAr    m^2                 0.0000  A |p|       Pa
  1.0600E-03 b Volume    m^3                 0.0000  B Ph(p)     deg
                                             0.0000  C |U|     m^3/s
                                             0.0000  D Ph(U)     deg
                                             0.0000  E Htot      W
 ideal      Solid type                       0.0000  F Edot      W
```

The segments that were not introduced in Chapter 2 are `HX` and `STKSLAB`.

`HX`'s have parallel-plate geometry, with plate spacing $2y_o$. Wave propagation through heat exchangers is computed using the momentum and continuity equations, including both viscous and thermal dissipation, much like the calculations in `DUCT`s. There is one additional input: the heat added to the thermoacoustic wave from an external thermal reservoir (or, if negative, the heat rejected to an external reservoir). The heat flow determines the change in total energy flow $\dot{H}_{\text{tot}}$ in the heat exchanger. As an input variable, heat flow can either be specified by the user (and optionally, an independent plot variable) or it can be a guess. There is also a potential target: the temperature of the solid in the heat exchanger, which is assumed to be spatially uniform and which differs from the temperature in the gas in proportion to the heat, as described in Chapter 10. The temperature difference between gas and solid can presently be computed only with an accuracy of about a factor of 2, even in the acoustic approximation; nevertheless, it is included in DeltaEC, to prevent naive users from being led to designs with heat exchangers of negligible surface area that have negligible losses and that would appear to have no disadvantages if this temperature difference were not included. We hope that future revisions of DeltaEC will have a more accurate calculation algorithm for this effect. Meanwhile, however, if you prefer not to use this feature, watch or target the gas mean temperature instead of the solid temperature by using `RPN` (see Chapter 4) or observing the gas temperature in the adjacent stack segment (parameter `G` or `H`).

`STKSLAB` also has parallel-plate geometry. This is the first segment we have encountered in the Tutorial in which T_m can depend on x. DeltaEC obtains the x dependence by integrating the thermoacoustic energy equation, schematically given by Eq. (5.8) and in detail in the equations given in Section 10.5.1, with $\dot{H}_{\text{tot}}$ independent of x.

Using the lessons we learned in Chapter 2, we recognize that we do not know all the initial conditions at `BEGIN`, so we must introduce some necessary guesses and targets. We know that zero U_1 must flow out of the bottom of the refrigerator, so we Insert a `HARDEND` there and use its two impedance targets. Corresponding to those targets, we'll guess two variables in `BEGIN`. These might be complex U_1, but Hofler always ran his refrigerator on resonance, so we'll guess $|U_1|$ and frequency f as guesses to get started, leaving the phases of p_1 and U_1 equal (both zero) as the condition for resonance at the driver. After adding these variables, the guess–target list looks like

```
 GUESS      0b         0f
 name  BEGIN:Freq  BEGIN:|U|
 units     Hz        m^3/s
 value    5.00E+02   5.00E-04

 TARGET      9a          9b
 name  HARDE:R(1/z HARDE:I(1/z
 value     .00         .00
```

Running this case yields $f = 498$ Hz and $|U_1| = 5.50 \times 10^{-4}$ m^3/s. We are gratified that none of the results thus far appear crazy. If you are running this file for yourself, note also that the solid is hotter than the gas in the ambient heat exchanger, where the (positive) heat flows from solid to gas, and that the solid is cooler than the gas in the cold heat exchanger, where the (negative) heat flow is from gas to solid.

Now consider what guesses and targets might be required by the heat and temperature aspects of this model. We know that the bottom of the apparatus was thermally insulated, so no energy can flow out there. We also note from Ref. [1] that Hofler maintained the solid temperature in the ambient heat exchanger at 300 K by vigorous flow of temperature-regulated water through it. These two constraints, not at the `BEGIN`ning, call for the use of two targets: `9c` and `3f`. Hofler applied a known electrical heater power to the cold heat exchanger, so we need not guess that; we leave its value at 3.00 W for now. However, he did not control the heat removed at the ambient heat exchanger, nor the temperature of the *gas* at the ambient end; so DeltaEC must guess those two variables: `0c` and `3e`. After making the necessary changes, the guess–target summary looks like

```
 GUESS      0b          0c          0f          3e
 name  BEGIN:Freq  BEGIN:TBeg  BEGIN:|U|      HX:HeatI
 units     Hz          K          m^3/s         W
 value    498.02      300.0     5.054E-04    -9.640
TARGET      3f          9a          9b          9c
 name     HX:Est-T HARDE:R(1/z HARDE:I(1/z  HARDE: Hdot
 units      K                                    W
 value    300.0        0.00        0.00        0.00
```

before running, and

```
GUESS      0b          0c          0f          3e
 name  BEGIN:Freq  BEGIN:TBeg  BEGIN: |U|     HX:HeatI
 units     Hz          K          m^3/s         W
 value   504.22      300.80     5.4172E-04   -11.126
 TARGET      3f          9a          9b          9c
 name     HX:Est-T HARDE:R(1/z HARDE:I(1/z HARDE:Htot
 units      K                                    W
 value   300.00        0.0000      0.0000      0.0000
result   300.00    -6.0493E-12 -1.0355E-10  1.7764E-15
```

after running. By examining segment 5 in the <`.out`> file, we see that DeltaEC predicts a cold temperature of 223.76 K with this 3.00 W heat load on the cold heat exchanger.

All of these results are reasonable, so now we examine Ref. [1] very closely to consider what else we should do to compared DeltaEC results directly to the published results of Hofler's experiments. Hofler added the *calculated* dissipation in the `DUCT`, `CONE`, and `COMPLIANCE` to his electrically applied heat load, to plot the gross cooling power instead of the net cooling power. So we do the same, with an `RPN` segment at the end of the model. We add another `RPN` to display another of his plotted variables, the temperature ratio. On the basis of the gross cooling power and the measured acoustic power delivered to the resonator by the driver, Hofler also plotted the coefficient of performance, $\mathrm{COP} = \dot{Q}_{C,\mathrm{gross}}/\dot{E}$, relative to Carnot's COP, $\mathrm{COP}_{\mathrm{Carnot}} = T_C/\left(T_0 - T_C\right)$, so we calculate $\mathrm{COP}/\mathrm{COP}_{\mathrm{Carnot}}$ with a third `RPN` at the end. These additions look like

```
+  0 BEGIN
+  1 SURFACE    driver end
+  2 DUCT       ambient temperature duct
+  3 HX         ambient temperature heat exchanger
+  4 STKSLAB    Stack
+  5 HX         Cold heat exchanger
+  6 DUCT       Cold Duct
+  7 CONE
+  8 COMPLIANCE end bulb
+  9 HARDEND    target this to seal the end
!-------------------------------- 10 --------------------------------
 RPN        gross cooling power is net power plus tail dissipation
     0.0000  a G or T            (t)         3.8487  A Qg (W)
   5e 5F +
!-------------------------------- 11 --------------------------------
RPN        temperature ratio, Tc/T0
    0.0000  a G or T            (t)         0.7459  A ratio
  5H 3H /
!-------------------------------- 12 --------------------------------
 RPN        COP / Carnot's COP
     0.0000  a G or T            (t)         0.1614  A COP rel
   10A 1A / 5H 3H 5H - / /
```

We let the net cooling power, i.e., heat `5e` at the cold heat exchanger, be the independent plot variable in DeltaEC, ranging from 1 to 7 W in 0.5 W steps. To obtain the variables we want to display, we add `10A`, `11A`, and `12A` to the list of dependent plot variables. After running this case, we changed $|p_1|$ to 0.015 of p_m, and changed `5e` to 0.5 W, making those changes by hand in a couple of steps to make sure DeltaEC did not get lost. Then we again used `5e` as the independent plot variable, over the range of $\dot{Q}_C$ to 0.5–3.5 W in steps of 0.25 W, and ran it again, appending the results to the previous plot run. We imported the resulting <`.ip`> file into a spreadsheet so we could display it together with Hofler's experimental data. The results are shown in Figs. 5.2. The DeltaEC calculations come reasonably close to the measurements presented in Figs. 16 and 17 of Hofler's thesis [1].

Let's modify this example a little more, using it to illustrate some points about side-branch and series transducers and about DeltaEC's consistency with the first law of thermodynamics. First, let's add a more realistic driver to the system, using `VSPEAKER`. We insert `VSPEAKER` at the beginning, using Ref. [1] for approximate numerical values for the mass, resistance, and force constant for the speaker, and we delete the `SURFACE` segment that was previously near the beginning because `VSPEAKER` accounts for the oscillatory pressurization losses on its surface area. We use the drive voltage as a guess, so we are asking DeltaEC what voltage we must supply to maintain $|p_1| = 30$ kPa in front of the speaker; we estimate it will take about 20 V to drive it, so we use that as our starting value for the guess. We'll also add an `RPN` target after the `VSPEAKER`, to enforce resonance by ensuring that the phases of p_1 and U_1 are equal at the driver. We do this by forcing their difference, computed by subtracting the values appearing as results `1B` and `1L`, to be zero, which the value given in line `2a`.

With this change, `BEGIN` is no longer at a position of nonzero $|U_1|$; `BEGIN` is now at a place where $U_1 = 0$ and p_1 is nonzero, and the subsequent side-branch `VSPEAKER` adds all the volume flow rate that flows into the subsequent segments. We change guesses and initial values accordingly. With these changes, but before running, the guess–target list becomes

```
 GUESS      0b          0c          0e          1h          4e
 name  BEGIN:Freq  BEGIN:TBeg  BEGIN:Ph(p) VSPEA:AplVo    HX:HeatI
 units     Hz          K           deg          V           W
 value   504.22      300.80       100.00      20.000     -11.126
TARGET      2a          4f          10a         10b         10c
 name    RPN:Targe    HX:Est-T HARDE:R(1/z HARDE:I(1/z HARDE:Htot
 units                  K                                  W
 value     0.0000    300.00        0.0000      0.0000      0.0000
result     0.0000    300.00    -7.8635E-15  5.8006E-14 -1.7764E-15
```

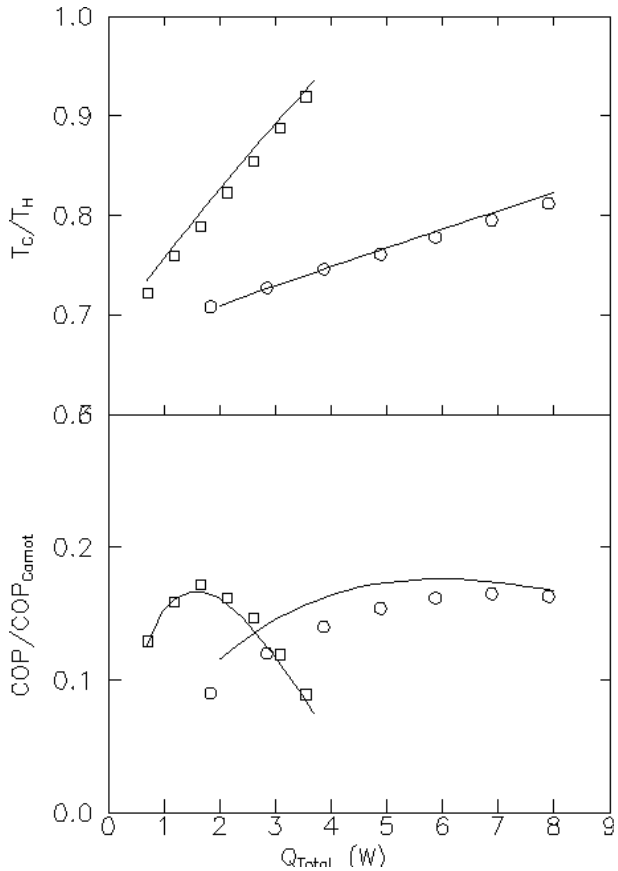
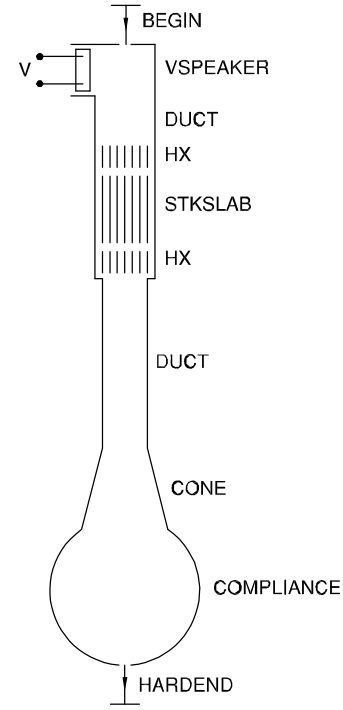


Figure 5.2: Hofler refrigerator results. Lines are DELTAEC results; points are from experimental data presented in Hofler's thesis. Squares, $p_1 = 0.015p_m$. Circles, $p_1 = 0.03p_m$.

and when we run it the guess–target list becomes

```
 GUESS      0b          0c          0e          1h          4e
 name  BEGIN:Freq  BEGIN:TBeg  BEGIN:Ph(p) VSPEA:AplVo    HX:HeatI
 units     Hz          K           deg          V           W
 value   505.59      302.60      152.52       23.385     -36.043
TARGET      2a          4f         10a         10b         10c
 name    RPN:Targe    HX:Est-T HARDE:R(1/z HARDE:I(1/z HARDE:Htot
 units                 K                                  W
 value     0.0000    300.00       0.0000      0.0000      0.0000
result  1.7536E-11   300.00     2.5373E-12 -2.5260E-11 -4.2633E-12
```



and the <`.out`> file becomes

```
TITLE     Hofler's 1986 thermoacoustic refrigerator
!Created@21:47:02  2-Oct-06 with DeltaE Vers. 6.0g2 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
 BEGIN
  1.0000E+06 a Mean P    Pa
   505.59    b Freq      Hz      G
   302.60    c TBeg      K       G
  3.0000E+04 d  |p|      Pa
   152.52    e Ph(p)     deg     G
     0.0000  f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 helium     Gas type
!--------------------------------  1 --------------------------------
 VSPEAKER   electrodynamic driver
  6.0000E-04 a Area      m^2              3.0000E+04 A |p|       Pa
     6.0000  b   R      ohms               152.52    B Ph(p)      deg
     0.0000  c L         H               5.4225E-04 C |U|     m^3/s
     8.0000  d BLProd   T-m                152.52    D Ph(U)      deg
  5.0000E-03 e   M      kg                 33.043    E Htot       W
     0.0000  f   K      N/m                 8.1337   F Edot       W
     0.0000  g   Rm   N-s/m                33.043    G WorkIn     W
    23.385   h AplVol    V       G         23.385    H Volts      V
                                           2.8804   I Amps       A
                                          -11.156    J Ph(Ze)   deg
                                          5.4348E-04 K  |Ux|  m^3/s
 ideal      Solid type                     152.52    L Ph(-Ux   deg
!--------------------------------  2 --------------------------------
 RPN        enforce resonance
     0.0000  a G or T          =  2A?     1.7536E-11 A  null
   1B 1L -
+  3 DUCT       ambient temperature duct
+  4 HX         ambient temperature heat exchanger
+  5 STKSLAB    Stack
+  6 HX         Cold heat exchanger
+  7 DUCT       Cold Duct
+  8 CONE
+  9 COMPLIANCE end bulb
+ 10 HARDEND    target this to seal the end
+ 11 RPN        gross cooling power is net power plus tail dissipation
+ 12 RPN        temperature ratio, Tc/T0
+ 13 RPN        COP / Carnot's COP
```

Comparing the values of the guesses here with those before we inserted the `VSPEAKER` shows two big differences: the phase of the pressure and the heat rejected at the ambient heat exchanger.

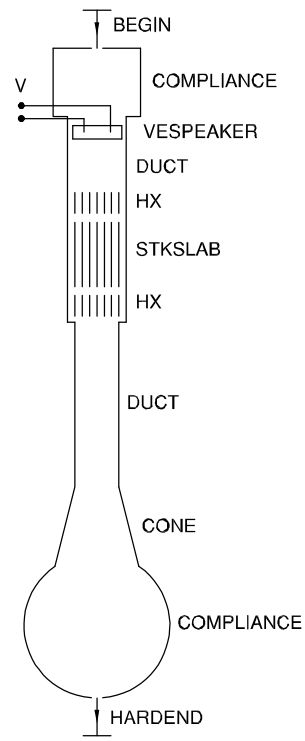
The change in the phase of the pressure is essentially trivial: The `VSPEAKER` segment sets the overall time phase of everything in the model, because the phase of its voltage is fixed at zero. Previously, the phase of U_1 at the top of the model was arbitrarily set to zero by the `BEGIN` segment. Now, the `VSPEAKER` controls the overall time phase of the model, and the nonzero imaginary real part of the speaker's mechanical impedance causes a large phase shift between V_1 and U_1. This simply shifts all p_1 and U_1 phasors by about 30°. A further 180° phase shift enters because of the sign convention for positive voltage built into the segment.

The increase in the heat rejected at the ambient heat exchanger from 11.1 W to 36.0 W is meaningful. The extra heat comes from the `VSPEAKER`, and illustrates how DeltaEC handles energy for `**SPEAKER`s and `**DUCER`s: They are treated as thermally insulated, so all of the electrical energy going into them shows up either as acoustic power or as heat. This

transducer is not remarkably efficient at converting electrical power into acoustic power; most of its electrical power becomes heat, which is rejected at the ambient heat exchanger in this model. The extra load on the ambient heat exchanger increases the temperature difference in that heat exchanger a little, which leads to the small increase in gas temperature seen in `0c`, which in turn leads to the small increase in resonance frequency seen in `0b`.

The speaker can also be included in the model "in series," using a `VESPEAKER` segment, with yet another reconfiguration of the `BEGIN` and associated guesses and targets to reflect new circumstances. As shown below, we include the cavity above the speaker, which appears from Ref. [1] to have a volume of approximately a liter. Above that volume, the `BEGIN` starts with $U_1 = 0$ and with a guessed p_1. We arbitrarily set the time phase of the entire model by setting the phase of p_1 in that volume to zero in the `BEGIN`. We note that Hofler had cooling-water tubes attached to the driver housing, to try to remove as much of the driver's electrical and other dissipation as possible. To crudely model the effect of these tubes, we insert an `RPN` below the driver, which we craft so that it removes 75% of the driver's heat, to illustrate the use of `RPN` for such purposes. With these changes, and after running, the guess–target list becomes

```
 GUESS       0b          0c          0d          2h          2i          7e
 name  BEGIN:Freq  BEGIN:TBeg  BEGIN: |p|  VESPE: |V|  VESPE:Ph(V)    HX:HeatI
 units      Hz          K           Pa          V          deg           W
 value   504.56      301.24      284.59      23.271      116.88      -17.243
TARGET       3a          4a          7f         13a         13b         13c
 name    RPN:Targe   RPN:Targe    HX:Est-T HARDE:R(1/z HARDE:I(1/z HARDE:Htot
 units                             K                                  W
 value     0.0000   3.0000E+04   300.00       0.0000      0.0000      0.0000
result -5.5422E-13  3.0000E+04   300.00     1.7565E-14  1.8398E-13  2.4869E-14
```



and the <`.out`> file becomes

```
TITLE     Hofler's 1986 thermoacoustic refrigerator
!Created@18:52:37  3-Oct-06 with DeltaE Vers. 6.0g2 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
 BEGIN
  1.0000E+06 a Mean P    Pa
   504.56    b Freq      Hz      G
   301.24    c TBeg      K       G
   284.59    d  |p|      Pa      G
     0.0000  e Ph(p)     deg
     0.0000  f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 helium     Gas type
!--------------------------------  1 --------------------------------
 COMPLIANCE the space above the speaker
     0.1000  a SurfAr    m^2               284.59    A |p|       Pa
  1.0000E-03 b Volume    m^3                 0.0000  B Ph(p)     deg
                                          5.4133E-04 C |U|     m^3/s
                                           -90.206   D Ph(U)     deg
                                             0.0000  E Htot      W
 ideal      Solid type                   -2.7680E-04 F Edot      W
!--------------------------------  2 --------------------------------
 VESPEAKER  electrodynamic driver
  6.0000E-04 a Area      m^2              3.0000E+04 A |p|       Pa
     6.0000  b   R      ohms               -90.207   B Ph(p)     deg
     0.0000  c L         H               5.4010E-04 C |U|     m^3/s
     8.0000  d BLProd   T-m                -90.207   D Ph(U)     deg
  5.0000E-03 e   M      kg                 32.668   E Htot      W
     0.0000  f   K      N/m                 8.1015  F Edot      W
     0.0000  g   Rm   N-s/m                32.668   G WorkIn    W
    23.271   h  |V|     V        G         23.271   H Volts     V
   116.88    i Ph(V)   deg       G          2.8605  I Amps      A
                                           -11.041   J Ph(Ze)   deg
                                          3.0002E+04 K  |Px|    Pa
                                           -90.751   L Ph(Px)   deg
!--------------------------------  3 --------------------------------
 RPN        enforce resonance
     0.0000  a G or T          =  3A?    -5.5422E-13 A  null
   2B 2D -
!--------------------------------  4 --------------------------------
 RPN        enforce desired pressure amplitude
  3.0000E+04 a G or T          =  4A?     3.0000E+04 A  |p1|
```

```
   p1 mag
!---------------------------------  5 ---------------------------------
 RPN        (a)= % of driver heat removed by water tubes on driver box
     0.7500  a G or T            (t)        14.243   A ChngeMe
                                            18.425   B ChngeMe
   Htot Edot - inp * sto Htot rcl - =H2k
!---------------------------------  6 ---------------------------------
 DUCT       ambient temperature duct
  1.1340E-03 a Area      m^2              2.9736E+04 A |p|       Pa
     0.1190  b Perim      m                -90.401   B Ph(p)     deg
  4.2600E-02 c Length     m               2.8084E-03 C |U|     m^3/s
                                          -169.43    D Ph(U)     deg
                                            14.243   E Htot      W
 ideal      Solid type                      7.9452  F Edot      W
+  7 HX         ambient temperature heat exchanger
+  8 STKSLAB    Stack
+  9 HX         Cold heat exchanger
+ 10 DUCT       Cold Duct
+ 11 CONE
+ 12 COMPLIANCE end bulb
+ 13 HARDEND    target this to seal the end
+ 14 RPN        gross cooling power is net power plus tail dissipation
+ 15 RPN        temperature ratio, Tc/T0
+ 16 RPN        COP / Carnot's COP
```

Notice how the DeltaEC RPN operator `=H2k` has been used to override DeltaEC's normal continuity of $\dot{H}$ between segments, to simulate the removal of 18.425 W of heat by the water-cooled tubing on Hofler's driver housing.

The standing-wave components of the pressure and volume-flow-rate waves in this refrigerator model are shown in Fig. 5.3—it is essentially a quarter-wave device. In DeltaEC models of thermoacoustic engines and refrigerators, it is often also worthwhile to examine graphs of $\dot{H}$ and $\dot{E}$ and to think about their consistency with the laws of thermodynamics, e.g., to reveal possible fuzzy thinking about how `BEGIN` and `**END` are set up in the model. In the present case, shown in Fig. 5.4, the acoustic power delivered by the driver drops most rapidly in the stack and more slowly in the bottom part of the resonator, and the constancy of $\dot{H}_{\rm tot}$ in each insulated segment (i.e., everywhere but in heat exchangers) is evident. The positive signs of these power flows indicates flows in the positive-x direction, i.e., from the `BEGIN` toward the `HARDEND` in the model.

Another instructive graph displays gas temperature T_m and solid temperature $T_{\rm solid}$ as functions of x, as illustrated for the present example in Fig. 5.5. In heat exchangers, the difference $T_{\rm solid} - T_m$ is due to the time-averaged heat flowing from solid to gas. In other segments, where there is no time-averaged heat flowing from solid to gas, these two temperatures are equal.

5.3. Choosing guesses and targets

The final example in the previous section used six targets and six guesses—the most complicated guess–target summary table we have yet encountered in the Users Guide. In this section, we discuss how we chose its members, and explain some general guidelines for guess–target choices.

Recall the purpose of guesses and targets. DeltaEC is numerically integrating equations

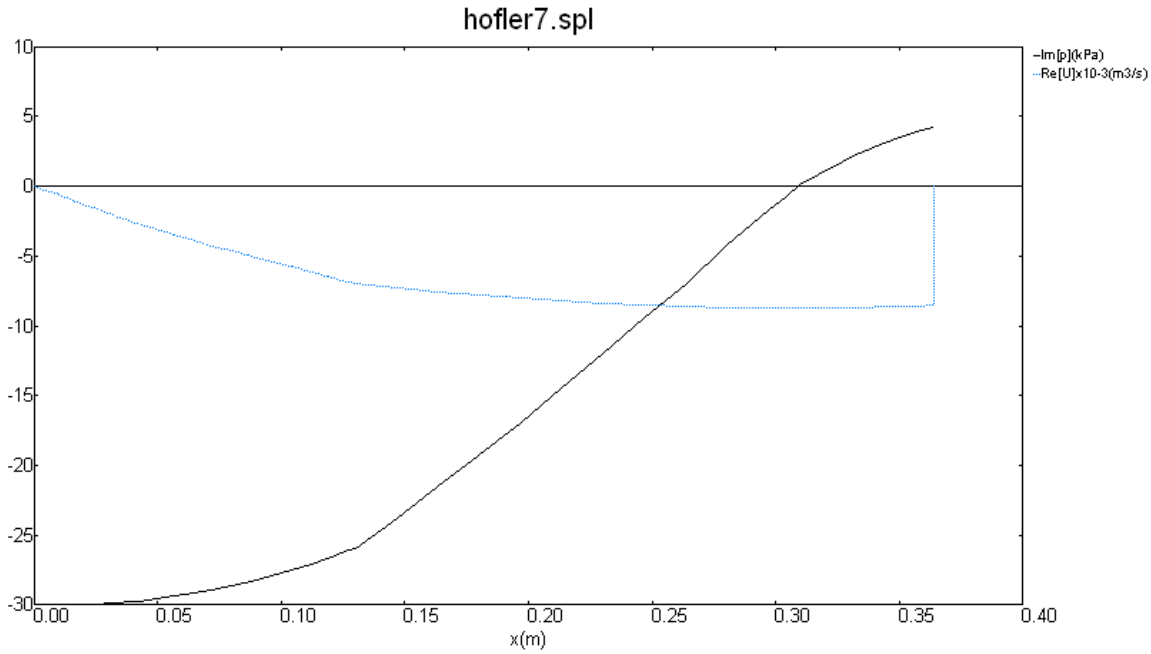


Figure 5.3: Standing-wave parts of the p_1 and U_1 wave in Hofler's refrigerator.

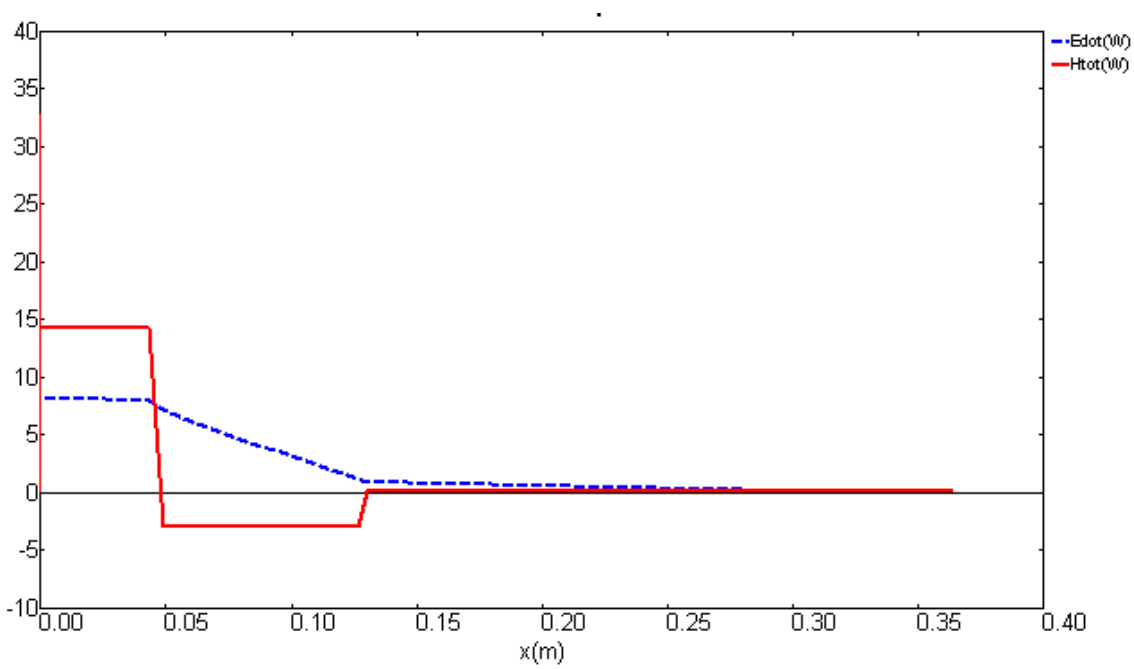


Figure 5.4: Acoustic power $\dot{E}$ (dashed blue) and total power $\dot{H}$ (solid red) as a function of x in the refrigerator model. The `BEGIN`, `COMPLIANCE`, and `VSPEAKER` are all crowded into $x = 0$. The drop in $\dot{H}$ marks the ambient heat exchanger and the smaller rise in $\dot{H}$ marks the cold heat exchanger.

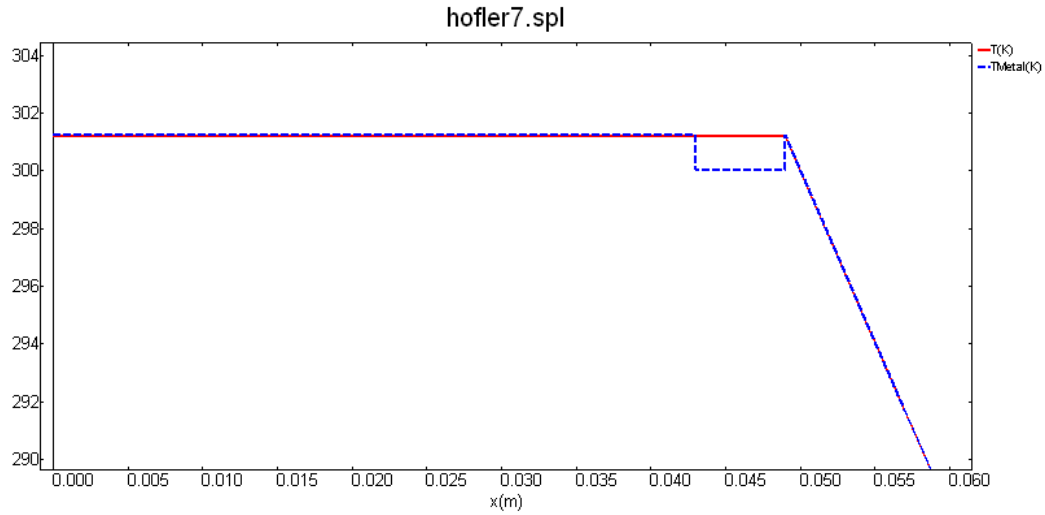


Figure 5.5: DELTAEC results for gas temperature T_m and solid temperature T_{solid} as functions of x in the uppermost part of Hofler's refrigerator. These temperatures differ only in the heat exchanger.

for p_1, U_1, and T_m, which can generally be regarded as

$$\frac{dp_1}{dx} = \mathcal{F}_{\text{momentum}}(\text{local and global parameters and variables}), \tag{5.10}$$

$$\frac{dU_1}{dx} = \mathcal{F}_{\text{continuity}}(\text{local and global parameters and variables}), \tag{5.11}$$

$$\frac{dT_m}{dx} = \mathcal{F}_{x\text{-energy}}(\text{local and global parameters and variables}), \tag{5.12}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \dot{q} \text{ in heat exchangers; } = 0 \text{ elsewhere.} \tag{5.13}$$

from the `BEGIN` segment to the end of the model. Some of the results of that integration are actually known *a priori* by the user, and some of the variables needed by DELTAEC to perform that integration are not known *a priori* by the user. These exceptional variables are specified by the user as targets and guesses, respectively, and DELTAEC uses a shooting-method algorithm, attempting to meet the targets by finding suitable values for the guesses. Initial values for guesses are usually left over from a previous run of DELTAEC, but sometimes they are provided by the user.

For the refrigerator example of the previous section, we chose targets and guesses by considering what integration results were known by us *a priori* and what input variables were not known by us *a priori*. We looked first near the end of the model for possible targets. We definitely needed the three `HARDEND` variables $\text{Re}[1/z_n]$, $\text{Im}[1/z_n]$, and $\dot{H}_{\text{tot}}$ in the target list, all targeted to zero, because there is no hole in the end of the apparatus that would allow nonzero U_1 or $\dot{H}_{\text{tot}}$ to escape—these are integration results that are known to us *a priori*. Experimentally, Hofler maintained the hot heat exchanger at 300 Kelvin; but DELTAEC computes that as a result of each integration pass, so it also had to be a target.

We looked first at the `BEGIN` segment for candidate guesses. The beginning gas temperature was not known *a priori* by us: DELTAEC had to adjust the beginning T_m to arrive at the first `HX` T_{solid} correctly, so `0c` had to be a guess. Next, we did not know the frequency needed to maintain resonance *a priori*, so it too had to be a guess. But how was resonance determined experimentally? By comparing the phases of p_1 and U_1 at the driver. These phases are computed by DELTAEC as results. Hence, we added their difference = 0 as a fifth target. While we were thinking about the driver, we realized that we wanted to imagine that $|p_1|$ just below the driver was fixed, because Hofler adjusted his driver voltage to achieve a desired amplitude. Thus $|p_1|$ at that location—a calculated result in DELTAEC—had to be a target, and the driver voltage had to be a guess. The heat rejected in the first heat exchanger had to be guessed because we didn't know it or control it experimentally, yet it is required by DELTAEC for each numerical integration. By now we had six targets and four guesses, so we needed two more guesses. One guess must be $|p_1|$ at the beginning, which is in the cavity above the loudspeaker, where we had no *a priori* knowledge of pressure. One more guess was needed. The overall time phase of a DELTAEC model is meaningless; somewhere there is always one time-phase variable that can be chosen arbitrarily. However, here we still had two phases set arbitrarily: the phase of p_1 at the `BEGIN` and the phase of V_1 in the driver. One of these had to be used as a guess. We chose to use the phase of V_1 as that guess.

We chose a point of view where we wanted to learn the cold solid temperature `9H` for a given cooling power `9e`. Had we wanted to have DELTAEC tell us the cooling power at a known cold temperature, we would have used an additional guess–target pair, targeting T_C and guessing $\dot{Q}_C$.

Although some situations that we model in DELTAEC are not this confusing, choosing the guesses and targets is never easy for a complicated thermoacoustic system. To choose them wisely, two things are essential: careful thought about the particular thermoacoustic system itself, and an appreciation of what variables are results of a single DELTAEC integration from `BEGIN` to `**END` and what variables are needed by DELTAEC for each such integration. We introduce a few general guidelines for this thought process here.

To understand which variables are candidate targets, you must know which variables are *computed* as results by DELTAEC during each pass. To understand which variables are candidate guesses, you must know which variables are *needed* by DELTAEC for each pass of its integration. To aid this understanding, look at Eqs. (5.10)–(5.13), where computed results are the variables on the left-hand sides, and variables needed are on the right-hand sides. Print these equations (or the more complete versions at the beginning of Chapter 10) and post them near your workstation, so you can look at them for inspiration when you are trying to figure out guesses and targets for a new or modified model. Also look at the columnar structure of the <`.out`> file, where variables needed by DELTAEC for each pass of its integration are in the left column (as are potential targets) and variables that DELTAEC computes as results during each pass are in the right column. For a perfect understanding, there is no good substitute for studying the summaries of the computation algorithms for each segment, as discussed briefly in the tutorial chapters of the Users Guide, and more fully in Chapter 10.

To understand what variables should be regarded as known *a priori* by the user, it is

Table 5.1: Choosing guesses and targets.

	Variables *we* think of as known	Variables *we* think of as results
Variables that DeltaEC needs as inputs for *each pass* of integration. Includes T_m-begin, p_1-begin, U_1-begin, p_m, f, all dimensions, 'ducer coefficients, heat at heat exchangers, gas-mixture mole fraction.	simply fixed	guess
Variables that DeltaEC calculates as results of *each pass* of integration. Includes all T_m, p_1, U_1, $\dot{H}_{\text{tot}}$ except in `BEGIN`; combinations of above such as $\dot{E}$, `RPN` results, and z_n at ends.	target	simply results in <`.out`> and <`.sp`> files

sometimes helpful to think about what variables are (or could be, in principle) experimentally controlled, in contrast to what variables are experimentally observed. These must be compared with the variables that DeltaEC computes as results during each integration pass and those that DeltaEC needs as inputs during each integration pass through the system. Note that our definition of an experimental result is more general than usual. In the Hofler refrigerator case, we considered the drive voltage an experimental result because it is determined experimentally by the condition that the pressure amplitude have the desired value. The viewpoint expressed in Table 5.2 is appropriate for comparison of DeltaEC and experimental data. In this case, geometrical parameters are simply fixed. Targets are experimentally fixed or controlled variables that are results of a single pass of numerical integration, chosen from among T_m, p_1, and U_1 (everywhere but in `BEGIN`); current magnitudes and phases in `VDUCER`s and voltage magnitudes and phases in `IDUCER`s; etc. Guesses may be unknown parameters chosen from among f, the magnitude and phase of U_1-`BEGIN` and p_1-`BEGIN`, T_m-`BEGIN`, heats at heat exchangers, the magnitude and phase of voltage at `VEDUCER`s, etc.

When designing hardware instead of analyzing it, a different viewpoint may be adopted. In this case, many geometrical parameters are not yet fixed, but desired operating temperatures, powers, frequency, etc. have been chosen. Often, several geometrical parameters are included as guesses, and more temperatures and other numerical results are included as targets. Hence, another useful way to think about guesses and targets is represented by Table 5.3.

As we did in our description of the choices of guesses and targets for the refrigerator example above, it is useful to look for targets at the end or middle of the model, then

Table 5.2: Interpreting input and result variables: experimental viewpoint.

	Experimentally controlled variable	Experimental result
Variable needed as input for *each pass* of DeltaEC's integration	simply fixed	guess
Variable computed as a result of *each pass* of DeltaEC's integration	target	simply a result in <`.out`> and <`.sp`> files

Table 5.3: Interpreting input and result variables: design viewpoint.

	Variable *we* want to think of as fixed	Variable *we* want to think of as a result
Variable needed as input for *each pass* of DeltaEC's integration	simply fixed	guess
Variable computed as a result of *each pass* of DeltaEC's integration	target	simply a result in <`.out`> and <`.sp`> files

look for guesses in the beginning or middle of the model. Sometimes it is easy to mentally associate each target with a corresponding guess, as we did for some of them in the discussion above. A few targets, such as the `HARDEND` targets in the example above, may be utterly obvious and necessary, depending only on the model but not on the user's point of view. However, many targets depend on the user's point of view. Does the user want to consider an electrically driven refrigerator to be driven at fixed frequency, or to be kept on resonance by active control? Is the user interested in cooling power at a given cold temperature, or in the cold temperature that is reached for a given cooling load? Thus, it is essential to think carefully about what we want to think of as "known" and what we want to think of as "unknown" for every model and for every use of that model.

It is also important to remember that the overall time phase of a DeltaEC model is arbitrary, to find the one and only one location in the model where that time phase is set (usually in the `BEGIN`, but sometimes in a side-branch transducer), and to make sure there are no other conflicting time-phase input variables (e.g., multiple side-branch transducers can be problematic).

5.4. Meeting targets in an elaborate model

5.4.1. Developing a substantially new model

Occasionally the user can create a completely new model, establish several targets and guesses, hit the blue run button, and enjoy instant success. However, it is much more common to find that this approach yields no more than a message from DeltaEC explaining that it has failed to meet targets. Several strategies can be used to overcome this situation:

Understand approximately what to expect from your model before you even begin working with DeltaEC, so you can provide reasonable initial estimates for guesses. For example, use your knowledge of acoustics to make reasonable initial estimates of values for `TBRANCH` impedances to give DeltaEC a plausible starting point.

Proofread your model carefully. One of the most common problems in DeltaEC models is an order-of-magnitude typographical error in an input variable. Use the schematic view for a quick check of dimensions.

clear all guesses and targets, then run the model and examine the results for approximate sanity. (Better yet, do this sanity check before you establish any guesses and targets in the first place.) If you see a result that is wildly unrealistic, such as a temperature of 10,000 Kelvin, modify something upstream to fix it, so DeltaEC's shooting-method algorithm begins in the correct ballpark.

Even though you may have chosen guesses and targets by thinking about the end of the model first, introduce the guesses and targets starting near the beginning, one target and one guess at a time. To be sure that your first guess will be a suitable partner for your first target, think about the physics of cause and effect. Modify the candidate guess's value by hand before you establish it as a guess, running to confirm that changing the candidate guess has a significant effect on the intended target. Then add

that guess and target, run, and confirm that the target is met. If successful, repeat this process for the next candidate guess and target.

Save your model frequently, so you can backtrack if things go awry. You can save a file with any filename extension, and we frequently find ourselves naming saved files to indicate the order they are created, e.g., <`bottle.001`>, <`bottle.002`>, <`bottle.003`>, or to indicate what they represent to us, e.g., <`bottle.1targ`>, <`bottle.2targs`>, etc. We keep notes to show what each saved file represents.

As the number of guesses and targets increases, DeltaEC's shooting method has more and more work to do. Be kind. Before adding the Nth target and guess, modify the candidate guess by hand to make the intended target variable *very* close to the desired value before asking DeltaEC to take over responsibility for that target and guess.

If an overly bold change has gotten DeltaEC lost, use Restore all guessez, described in Section 5.7, to recover the previous set of guesses.

If DeltaEC seems to be missing targets by only a little, reduce the convergence tolerance or adjust one of the other shooting-method tuneup parameters, which are accessible through the Edit pulldown menu as Options, described in Section 8.3.2.

Consider whether the number of guesses and targets could be reduced by beginning at the `**END` and ending at the `BEGIN`ning. If so, Flip the model as described in Section 5.7.

Consider other ways to reduce the number of guesses and targets. For example use `VSPEAKER` instead of `VESPEAKER` if possible. Or avoid a transducer by using a nonzero U_1 in `BEGIN`.

Avoid a `sameas` or `RPN` that points to a downstream result, if it affects an integration result that is compared with a target. This would confuse DeltaEC by making the targeted result depend on a result left over from the *previous* integration instead of only on the upstream results of the *present* integration. (When several `sameas`'s and `RPN`s point to one another, it can be difficult to see whether they are all independent of downstream results.)

If convergence seems strangely problematical, consider changing the relative importance of different targets. If some targets (such as heat-exchanger temperatures) are aimed at a nonzero number while others (such as `**END` impedance) are aimed at zero, DeltaEC sometimes has trouble deciding how close is close enough. The relative weight can be changed via the Mode parameter described in Section 8.3.2, but it is sometimes more productive to change the weight of an individual target by incorporating it into an `RPN` segment that is aimed at zero and multiplying (or dividing) the calculated target result by a large number to give it more (or less) importance; or adding 1 to what would otherwise be a zero target, to de-emphasize it.

If convergence seems strangely problematical, also consider whether some guesses might essentially have large fixed offsets. The shooting method begins by calculating the

sensitivities of the results to changes in the guesses by changing each guess by a fixed percentage (described in Section 8.3.2). If a part-per-million change in one guess causes part-per-million changes in targeted results, but a part-per-million change in a second guess causes an order of magnitude change in one or more targeted results, the shooting method will fail. In such cases, it helps to use an `RPN` input parameter as small guess, add a large offset to it in the `RPN`'s line `b` to make a number that's the size of the original guess, and `sameas` the original guess to the `RPN` result. This will ensure that ppm changes in all guesses cause ppm changes in all targeted results.

5.4.2. Modifying an existing model

Purely acoustic models with only a few guesses and targets, like the `bottle` in Chapter 2, are usually robust: From a state with all targets met, the user can often change any variable by a factor of two or so, and DeltaEC's shooting method will once again hit the targets. However, DeltaEC's shooting method gets lost more easily for thermoacoustic models with more than about five targets and five guesses, especially with `STKDUCT` and `STKCONE` segments. Some strategies for successful routine modifications of complex models, starting from a state in which targets are met, include:

If an overly bold change has gotten DeltaEC lost, use Restore all guessez, described in Section 5.7, to recover the previous set of guesses.

Use the incremental plot feature to change a variable by small steps. In practice, this usually means: Observe that a big change led to failure to converge; Restore guesses and un-do the big change manually to return to the secure starting condition; Try a smaller change (such as 1/10 of the original change); If the smaller change still led to failure to converge, then restore and undo again, and try an even smaller change; When a small-enough change has been found, use that change as the step in the incremental plot feature to let DeltaEC accomplish the desired big change in 10, 100, or more small steps. (If DeltaEC gets part way through these small steps and then loses its ability to hit targets, use the plot rewind feature, described in Section 3.2.3, to recover a meaningful operating point that is at least part way toward the desired change.)

Insert a new segment gradually, by first inserting it with near-zero volume and near-zero impedance, then gradually increasing these with the incremental plot feature. Often this entails Inserting a new segment with near-zero length and gradually increasing the length. Similarly, the heat of a new heat exchanger can be increased gradually from zero; the impedance of a new side-branch impedance can be reduced gradually from a very large value; the impedance of a series impedance can be increased gradually from zero.

Kill an undesired segment gradually by reducing its length with the incremental plot feature before Killing it.

Save your model frequently, so you can recover from the unexpected. Not all floating-point errors are successfully trapped on all operating systems; some can cause the program to crash and lose unsaved work.

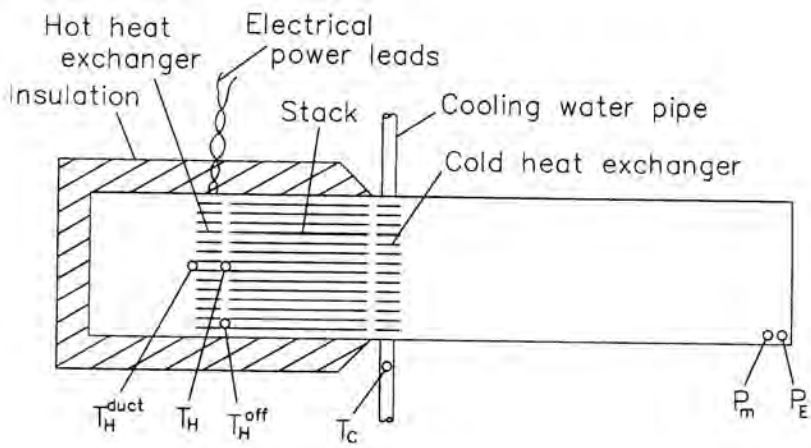


Figure 5.6: Five-inch-diameter standing-wave engine.

The shooting method within DeltaEC can sometimes get stuck around an inappropriate local minimum, particularly when making incremental changes from a model that has already converged. Try manually changing one of the guesses slightly and see if DeltaEC will loose its fondness for this particular sticky point. Or, change the guess-list membership, if you can think of an appropriate alternative to one or more existing guesses.

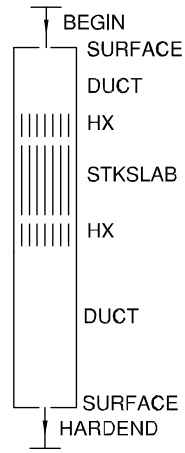
Always check results carefully for reasonableness, particularly when modeling complicated systems or using any of DeltaEC's more elaborate features. While DeltaEC is a useful tool, it is far from foolproof—for example, the shooting method can easily end up generating devices that are several wavelengths long or wander into the completely unrealistic realm of negative values for guessed frequency or length.

5.5. Standing-wave engine

This standing-wave-engine example is described in detail in Ref. [22]. The device described there is used here to illustrate the thermoacoustic capabilities of DeltaEC in the context of an engine. We will reproduce some of the figures in that paper here.

The apparatus is shown in Fig. 5.6. After running the file <`5inch.out`>, the result is:

```
 TITLE     Five-Inch Standing-wave Thermoacoustic Engine
!Created@16:56:43  5-Oct-06 with DeltaE Vers. 6.0g2 for the IBM/PC-Compatible
!---------------------------------  0 ---------------------------------
 BEGIN      Initial
  1.3800E+06 a Mean P    Pa
   121.16    b Freq      Hz      G
   557.92    c TBeg      K       G
  7.3549E+04 d  |p|      Pa      G
     0.0000  e Ph(p)     deg
     0.0000  f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 helium     Gas type
!---------------------------------  1 ---------------------------------
 SURFACE    Hot End
  1.2920E-02 a Area      m^2              7.3549E+04 A |p|       Pa
                                             0.0000  B Ph(p)     deg
                                          3.3316E-05 C |U|     m^3/s
                                           180.00    D Ph(U)     deg
                                             0.0000  E Htot      W
 ideal      Solid type                     -1.2252  F Edot      W
!---------------------------------  2 ---------------------------------
 DUCT       Hot Duct
 sameas   1a a Area      m^2              7.2686E+04 A |p|       Pa
     0.4030  b Perim      m               5.5059E-03 B Ph(p)     deg
     0.2790  c Length     m               8.7694E-02 C |U|     m^3/s
                                           -90.209   D Ph(U)     deg
                                             0.0000  E Htot      W
 ideal      Solid type                     -11.907   F Edot      W
!---------------------------------  3 ---------------------------------
 HX         Hot HX
 sameas   1a a Area      m^2              7.1519E+04 A |p|       Pa
     0.3930  b GasA/A                        0.3876  B Ph(p)     deg
  6.0000E-02 c Length     m               9.6763E-02 C |U|     m^3/s
  4.8300E-04 d y0         m                -91.200   D Ph(U)     deg
  2210.0     e HeatIn     W               2210.0     E Htot      W
   999.00    f Est-T      K      (t)       -95.846   F Edot      W
                                           557.92    G GasT     K
 ideal      Solid type                     563.81    H SolidT     K
!---------------------------------  4 ---------------------------------
 STKCIRC    Honey Stack
 sameas   1a a Area      m^2              6.5700E+04 A |p|       Pa
     0.8100  b GasA/A                        2.7530  B Ph(p)     deg
     0.2790  c Length     m                  0.1599  C |U|     m^3/s
  5.0000E-04 d radius     m                -85.386   D Ph(U)     deg
  5.0000E-05 e Lplate     m               2210.0     E Htot      W
                                           170.63    F Edot      W
                                           557.92    G TBeg       K
 stainless  Solid type                     306.39    H TEnd       K
!---------------------------------  5 ---------------------------------
 HX         Ambient HX
  1.2670E-02 a Area      m^2              6.3081E+04 A |p|       Pa
     0.4860  b GasA/A                        3.2510  B Ph(p)     deg
  5.0800E-02 c Length     m                  0.1676  C |U|     m^3/s
  4.0600E-04 d y0         m                -85.829   D Ph(U)     deg
 -2125.1     e HeatIn     W      G          84.898   E Htot      W
   303.00    f Est-T      K    =  5H?       84.898   F Edot      W
                                           306.39    G GasT     K
 ideal      Solid type                     303.00    H SolidT     K
!---------------------------------  6 ---------------------------------
 ANCHOR    subsequent duct is water jacketed. dHdot/dx = dEdot/dx
!---------------------------------  7 ---------------------------------
 DUCT       Ambient Duct
 sameas   5a a Area      m^2              6.9652E+04 A |p|       Pa
     0.3990  b Perim      m               -176.59    B Ph(p)     deg
     3.6500  c Length     m               1.8500E-05 C |U|     m^3/s
                                          -176.59    D Ph(U)     deg
                                             0.6443  E Htot      W
 ideal      Solid type                       0.6443  F Edot      W
!---------------------------------  8 ---------------------------------
 SURFACE    Ambient End
 sameas   5a a Area      m^2              6.9652E+04 A |p|       Pa
                                          -176.59    B Ph(p)     deg
                                          1.7385E-16 C |U|     m^3/s
                                            95.222   D Ph(U)     deg
                                          1.9100E-13 E Htot      W
 ideal      Solid type                    1.9100E-13 F Edot      W
!---------------------------------  9 ---------------------------------
 HARDEND    target this to seal the end
     0.0000  a R(1/z)          =  9G?     6.9652E+04 A |p|       Pa
     0.0000  b I(1/z)          =  9H?     -176.59    B Ph(p)     deg
                                          1.7385E-16 C |U|     m^3/s
                                            95.222   D Ph(U)     deg
                                          1.9100E-13 E Htot      W
                                          1.9100E-13 F Edot      W
                                          1.3878E-17 G R(1/z)
```



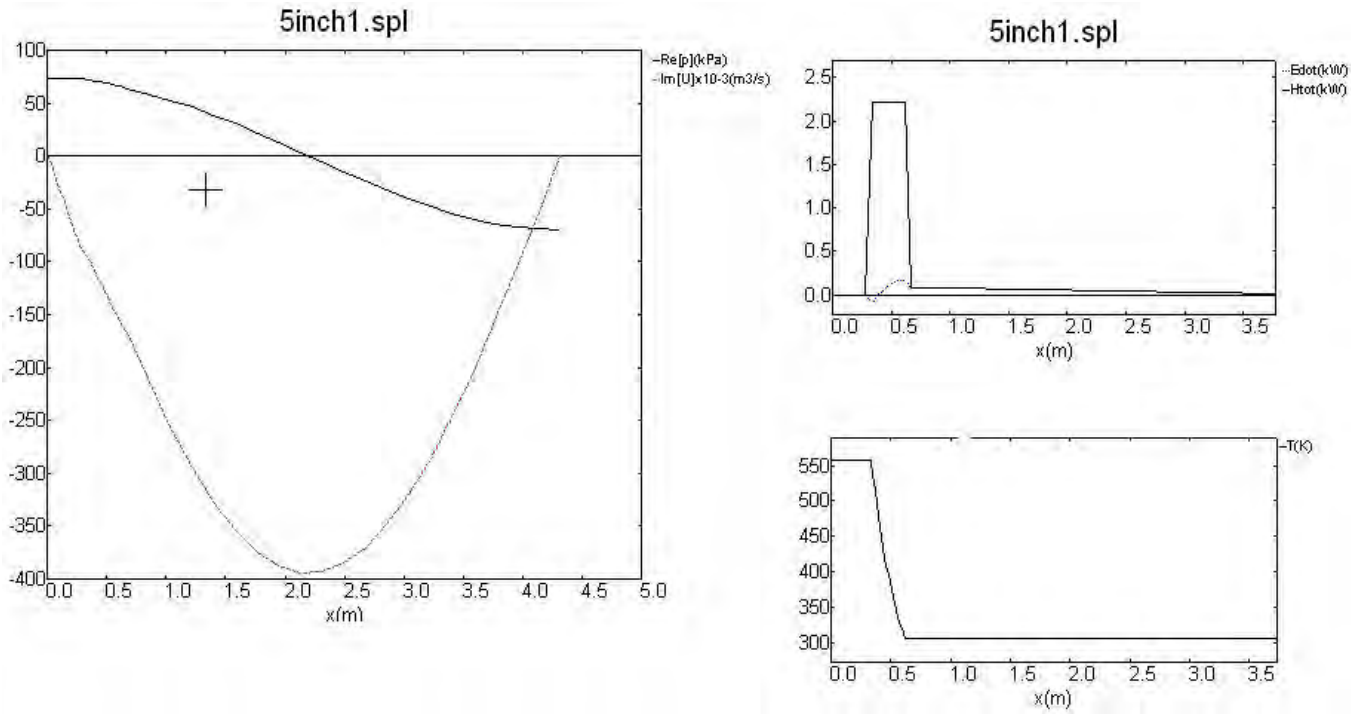


Figure 5.7: Graphs of the wave (left), total power and acoustic power (top right), and temperature (bottom right) as functions of x in <`5inch.out`>. Changing temperature and constant $\dot{H}_{\rm tot}$ show the location of the stack. The heat exchanger locations are indicated by the steep changes in $\dot{H}_{\rm tot}$.

```
                                        -4.3970E-16 H I(1/z)
!--------------------------------- 10 ---------------------------------
 RPN        ensure all ambient-duct dissipation goes to water jacket
     0.0000  a G or T          = 10A?    -5.6843E-14 A  zero
   5E 5F -
```

Examining the guess–target summary, we find

```
 GUESS      0b          0c          0d          5e
 name  BEGIN:Freq  BEGIN:TBeg  BEGIN: |p|     HX:HeatI
 units     Hz          K           Pa          W
 value   121.16      557.92     7.3549E+04 -2125.1
TARGET      5f          9a          9b         10a
 name     HX:Est-T HARDE:R(1/z HARDE:I(1/z   RPN:Targe
 units      K
 value   303.00        0.0000      0.0000      0.0000
result   303.00     1.3878E-17 -4.3970E-16 -5.6843E-14
```

and some <`.sp`> graphs show the wave, power, and temperature profiles in Fig. 5.7.

Targets `9a` and `9b`, the real and imaginary parts of $1/z_n$, are of course necessary to impose $U_1 = 0$ at the closed ambient end of the resonator. Our choices of other targets and guesses were made to reflect the perspective of Ref. [22], in which the as-built hardware was driven by a fixed heat input (ordinary input `3e`) and the ambient heat exchanger was held at fixed temperature (target `5f`). Frequency `0b` is a guess, because we want DELTAEC to tell us the frequency at which it thinks the system should spontaneously oscillate. Initial temperature `0c` and initial pressure amplitude `0d` are also guesses, because we want DELTAEC to tell us the hot temperature and the pressure amplitude at which it thinks the system should oscillate for the given hot heat input. The final target and guess, `10a` and `5e`, allow DELTAEC to tell

us the heat rejected at the ambient temperature, under the constraint that $\dot{E} = \dot{H}_{\rm tot}$ at the interface between the ambient heat exchanger and the long ambient resonator duct. We judge this to be a reasonable energy-closure condition for the model because the resonator duct was water-cooled in this apparatus, so the heat arising from $\dot{E}$ dissipated in it might be expected to be carried away locally by water. (This is of course not strictly true; boundary-layer thermoacoustic processes redistribute energy along x, as was dramatically demonstrated in Ref. [23] long ago.) Consistent with this energy-closure condition, the `ANCHOR` segment just before the final `DUCT` overrides the default behavior of the subsequent segments (thermally insulated, except in heat exchangers), ensuring that $\dot{H}_{2,\rm k} = \dot{E}$ thereafter.

Other choices could be made for the guesses and targets. For instance, the cold-duct length could be substituted for the frequency as one of the guesses, if we were designing an apparatus to operate at a desired frequency. The hot-heat-exchanger temperature `3f` could be added to the target list and, simultaneously, the hot-heat-exchanger heat `3e` could be added to the guess list, if we were designing an apparatus to operate at a desired hot temperature. For now, however, the guesses and targets will be left as they are, since they reflect the point of view adopted in Ref. [22].

To plot some results for this 5-inch engine case, we executed DeltaEC with this file again and prepared parameters so that the plot summary became

```
Dependent Variables (outputs):
PLOTS       0A          0B          0C          0D          3H          8A
 name  BEGIN:Freq  BEGIN:TBeg  BEGIN:|p|  BEGIN:HeatI    HX:Solid HARDE:|p|
 units      Hz          K           Pa           W         K          Pa
Independent Variables (inputs):
 Outer loop:   3e   HX:HeatI Beg=  9.50E+02 End=   50.     Step=  -33.
```

by adding `3H` and `8A` to the dependent-plot-variable list and establishing `3e` as an independent plot variable, setting its initial, final, and step values. (Some other variables such as `TBeg` are of minor interest now, but cannot be deleted from the list of plot variables because guesses appear here by default.)

Next, we modified mean pressure to be 19.2 bar, and ran the code. When completed, we modified mean pressure to 13.8 bar, and ran it again, appending the new results to the <`.ip`> file. Three more runs with mean pressures of 9.6, 6.9, and 5.2 bar completed the data set. We exited from DeltaEC, and read the <`.ip`> file into a spreadsheet/graphics program for a little massaging: converting pressure amplitude at the cold end from Pascals to bar, and then squaring that number; subtracting 303 Kelvin from T_H, and adding the measured heat leak to the room to $\dot{Q}_H$. Plotting these results then yields the curves shown in Fig. 5.8, resembling Figs. 5, 6, and 7 in Ref. [22]. These curves differ slightly from those in the reference, because of DeltaEC's inclusion of the small gas-to-solid temperature differences in the heat exchangers.

More detailed comparison between DeltaEC computations and measurements with this apparatus can be found in Ref. [24]. The agreement is excellent at low amplitude, but disagreement grows large at high amplitude—typical of our experience with standing-wave engines up to $|p_1|/p_m \simeq 0.1$.

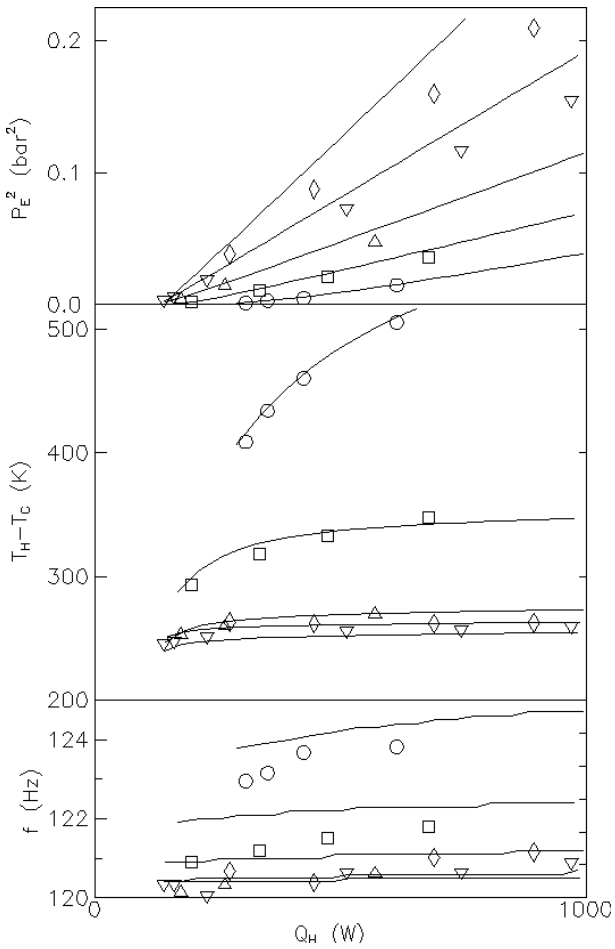


Figure 5.8: Standing-wave engine results. Lines are DeltaEC results; points are from experimental data.

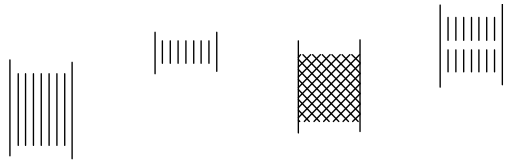


Figure 5.9: More thermoacoustic segments: stacks, regenerators, and heat exchangers.

5.6. More thermoacoustic segments

In this section we list commonly used segments for engines and refrigerators: the several types of heat exchangers, stacks, and regenerators. More details on each can be found in Chapter 10 and Chapter 11.

`STKCIRC` A stack or regenerator with circular pores. We also use this to model hexagonal honeycomb stacks.

`STKSLAB` A stack or regenerator with parallel-plate geometry.

`STKRECT` A stack or regenerator with rectangular (or square) pore geometry.

`STKPIN` A stack or regenerator comprised of an array of pins aligned parallel to x.

`STKDUCT` A "stack" with lateral dimensions much larger than δ_κ, computed in boundary-layer approximation. (Used for pulse tubes and thermal buffer tubes.)

`STKCONE` A tapered "stack" with lateral dimensions much larger than δ_κ, computed in boundary-layer approximation. (Used for pulse tubes and thermal buffer tubes.)

`STKSCREEN` A screen regenerator for Stirling systems.

`STKPOWERLW` A porous medium with friction factor and heat-transfer coefficient specified as power-law functions of Reynolds number.

`HX` A parallel-plate heat exchanger.

`TX` Tube-array heat exchanger, with the thermoacoustic working gas inside the tubes.

`SX` Stacked-screen heat exchangers, valid only for δ_κ greater than hydraulic radius.

`PX` Power-law heat exchangers, with friction factor and heat-transfer coefficient characterized by power laws in Reynolds number.

`VXT1` **and** `VXT2` Heat exchangers with fixed solid temperature, in which gas temperature can vary with x.

`VXQ1` **and** `VXQ2` Heat exchangers with fixed heat per unit length, in which gas temperature can vary with x.

`ANCHOR` This segment overrides what would otherwise be default thermal-insulation mode in subsequent `DUCT`s, `CONE`s, `COMPLIANCE`s, `IMPEDANCE`s, `SURFACE`s, etc., so that DeltaEC treats those segments as if they are immersed in a thermal bath at $T = T_m$.

`INSULATE` This segment is used to re-establish the default thermal-insulation condition if it was previously overridden by `ANCHOR`.

5.7. More user-interface features

These options offer substantial power and convenience for experienced users.

Restore all guessez. Before beginning iterations during a run, DeltaEC saves copies of the guess values. Whenever an unsuccessful run overwrites the guesses (leaving you and DeltaEC lost), you can use this option to restore all the parameters that were changed to their starting point. Simply Restore guessez, modify some variables (e.g., returning anything you recently changed to earlier, more successful values), and try again.

Comment lines. If the heading on the first line of a segment does not provide enough space for the user's notes, additional comment lines can be added below it. Examples are shown at the beginning of the next section, Section 5.8.1.

Split segment. This option, available by right clicking on the top line of a segment or in the Edit pulldown menu, automates the process of splitting any segment with length (e.g., a `DUCT`, `CONE`, or `STK**`) into two segments, each having half of the original length, correcting the `sameas` and `RPN` references everywhere in the model, and correcting the guess, target, and plot lists. (To partition the lengths unequally while keeping the correct total length, you can use master–slave linking (Section 2.7.2) after splitting a segment in equal halves: slave the first length to the second, then modify the first length, then clear the length link to allow the two pieces to be modified independently.)

AutoLayout. Sometimes auxiliary windows such as plots and the highlight display become lost off the edge of the computer's screen. (For example, undocking a laptop from a larger screen can cause this.) The AutoLayout option, available in the Display pulldown menu, brings all of DeltaEC's auxiliary displays close to DeltaEC itself.

Flip model. (Not yet implemented, October 2008.) The number of guesses and targets can sometimes be reduced by beginning the integration of a model from what you had previously considered the "end." Orifice pulse tube refrigerators (described in Section 5.8) are a particularly good example because we naturally think of them as beginning at the driver, but they "end" with an accurately known impedance in the `COMPLIANCE` tank, while the driver impedance is usually less well known; `BEGIN`ning in the tank can often reduce the number of guesses and targets. The "Flip model" operation automates switching back and forth between these two directions of integration, sparing the user from an otherwise very tedious and error-prone process. Flip reverses the order of every segment between the `BEGIN` and the last `HARDEND` or `SOFTEND`. Segments within

TBRANCHes are left in their original order, however. Flip takes care of sameas pointers, RPN segment pointers, CONEs and STKCONEs, and plot references, and an attempt is made to reform the guess and target lists. Always proofread very carefully after a Flip; in complicated models there is usually something that DELTAEC handles incorrectly.

5.8. Stirling and pulse-tube refrigerators

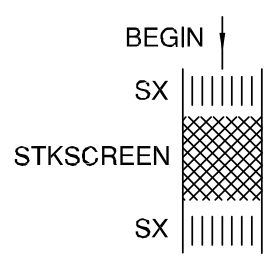
Rott's equations implemented in DELTAEC are valid for any phase difference between p_1 and U_1, and any degree of thermal contact in the "stack." Hence, DELTAEC can be used to model Stirling thermodynamic systems, in which p_1 and U_1 are substantially in phase, as well as standing-wave thermoacoustic devices, in which the phases p_1 and U_1 differ by nearly 90° as described in Sections 5.2 and 5.5. The principal additional DELTAEC segment that is often needed for Stirling systems is one for stacked screen beds, STKSCREEN, because stacked-screen regenerators are more common than parallel-plate, circular-pore, or rectangular-pore regenerators in Stirling systems. In our opinion, the principal shortcomings of DELTAEC for Stirling applications are DELTAEC's acoustic approximation (which leads to reduced accuracy at high pressure amplitudes) and its inability to predict two-dimensional effects and streaming-driven convective heat transport in pulse tubes (shortcomings shared by many other design programs). DELTAEC's main virtues are speed and easy inclusion of some auxiliary components such as ducts, dead volumes, and linear motors in the model.

Harmonic analysis of Stirling systems is introduced by I. Urieli and D. M. Berchowitz in Ref. [25] and by A. J. Organ in Ref. [26].

5.8.1. Alpha Stirling cryocooler

The sample file <Stirling.out> represents a simple 55-Hz, 2-MPa helium Stirling cryocooler with stacked-screen regenerator and heat exchangers. This apparatus is illustrated in Fig. 5.10. (It does not resemble any real cryocooler known to us; we created it only to illustrate DELTAEC's capabilities.) First, we examine <Stirling.out>:

```
TITLE     Bare bones Stirling cryocooler
! The user can type a few lines of notes here, describing the model.
!--------------------------------  0 --------------------------------
 BEGIN      Initialize things.
! Use values that seem reasonable, but are not drawn from
! anything specific in our experience.
  2.0000E+06 a Mean P    Pa
    55.000   b Freq      Hz
   300.14    c TBeg      K       G
  2.8459E+05 d  |p|      Pa      G
   -43.119   e Ph(p)     deg     G
  3.6500E-04 f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 helium     Gas type
!--------------------------------  1 --------------------------------
 SX         aftercooler
! Use an SX because this is low enough power that copper screens
!    will be work ok.
 sameas   2a a Area      m^2              2.8085E+05 A |p|       Pa
     0.6000  b VolPor                      -43.835   B Ph(p)     deg
  1.0000E-03 c Length     m               3.6265E-04 C |U|     m^3/s
 sameas   2d d   rh       m                 -0.3900  D Ph(U)     deg
   -35.833   e HeatIn     W      G           2.0781  E Htot      W
   300.00    f Est-T      K    =  1H?       36.973   F Edot      W
                                           300.14    G GasT     K
 copper     Solid type                     300.00    H SolidT     K
!--------------------------------  2 --------------------------------
 STKSCREEN      regenerator
```



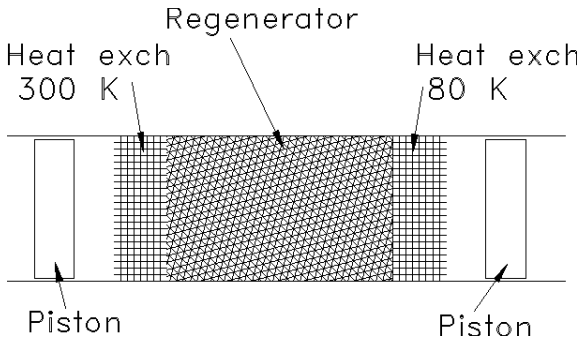


Figure 5.10: A Stirling cryocooler.

```
  1.1670E-04 a Area      m^2              2.2845E+05 A |p|       Pa
     0.6860  b VolPor                     -53.179   B Ph(p)     deg
  5.0000E-02 c Length     m               6.2225E-05 C |U|     m^3/s
  1.3900E-05 d   rh       m                -49.487   D Ph(U)     deg
     0.3000  e ksFrac                        2.0781  E Htot      W
                                             7.0929  F Edot      W
                                            300.14    G TBeg       K
 stainless  Solid type                       79.960   H TEnd       K
!---------------------------------  3 ---------------------------------
 SX         cold heat exch
!  See the file Stirling.xls in this folder for an estimate of
!  the temperature difference from the edge to the center of this
!  heat exchanger, due to finite conductivity of copper.
 sameas   2a a Area      m^2              2.2804E+05 A |p|       Pa
     0.6000  b VolPor                     -53.184   B Ph(p)     deg
  1.0000E-03 c Length     m               6.2000E-05 C |U|     m^3/s
 sameas   2d d   rh       m                -52.000   D Ph(U)     deg
     4.9896  e HeatIn     W      G           7.0677  E Htot      W
    80.000   f Est-T      K    =  3H?        7.0677  F Edot      W
                                            79.960   G GasT     K
 copper     Solid type                      80.000   H SolidT     K
!---------------------------------  4 ---------------------------------
 RPN        enforce Htot = Edot here
     0.0000  a G or T          =  4A?    -4.5359E-12 A
   Htot Edot -
!---------------------------------  5 ---------------------------------
 RPN        U sub 1 at cold end
  6.2000E-05 a G or T          =  5A?     6.2000E-05 A  |U1|
   3C
!---------------------------------  6 ---------------------------------
 RPN        phase(U) at cold end
   -52.000   a G or T          =  6A?      -52.000   A ph(U1)
   3D
```

The physical segments consist of a first heat exchanger, at 300 K, the regenerator, and a second heat exchanger at 80 K. All three are stacked screens. The other segments—`BEGIN` and the `RPN`s—define the end conditions.

The hydraulic radius r_h and volumetric porosity ϕ for the screens can be calculated from the screen manufacturer's specs, using expressions from Ref. [26]:

$$\phi = 1 - \frac{\pi m d}{4}\sqrt{1 + (md)^2}, \tag{5.14}$$

$$r_h = \frac{d}{4}\frac{\phi}{1-\phi}, \tag{5.15}$$

where d is wire diameter and m is mesh number (i.e., number of wires per unit length). The second equation results from the conventional definition of hydraulic radius: Gas volume divided by gas–solid interface area. This regenerator is a little over 1 cm in diameter and is 5 cm long. The heat exchangers are the same diameter but only 1 mm long.

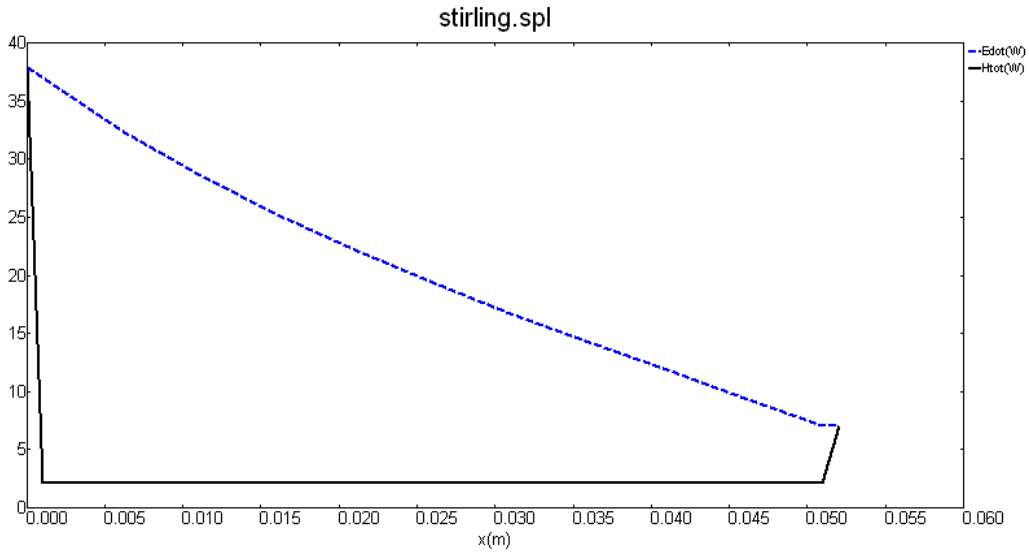


Figure 5.11: Total power (solid black) and acoustic power (dashed blue) vs position in the Stirling cryocooler example. The total power $\dot{H}_{\rm tot}$ is constant and small in the regenerator, and changes in the heat exchangers.

DeltaEC estimates the temperature difference between the helium gas and the copper screen wires in the heat exchangers, but it has no provision for estimating the temperature difference between the screen wires and the "housing" in which they are mounted (a temperature difference caused by the finite thermal conductance of the screen wires themselves). This is not a serious concern for small machines, but should be checked by hand on a case-by-case basis.

Line `e` in the regenerator segment, "`ksFrac`," is the fudge factor by which longitudinal conduction through the regenerator is adjusted due to the spatially intermittent thermal contact between adjacent screens and due to the conduction of the pressure-vessel wall. Following Ref. [27], we often set `ksFrac` somewhere between 0.1 and 0.3, the lower end of the range being appropriate if the conduction of the pressure-vessel wall is negligible.

Our point of view with respect to boundary conditions in this example is most easily displayed by running DeltaEC on this file and examining the guess–target summary,

```
 GUESS       0c          0d          0e          1e          3e
 name  BEGIN:TBeg  BEGIN: |p|  BEGIN:Ph(p)    SX:HeatI    SX:HeatI
 units      K           Pa          deg          W           W
 value   300.14     2.8459E+05   -43.119     -35.833       4.9896
TARGET      1f          3f          4a          5a          6a
 name     SX:Est-T    SX:Est-T   RPN:Targe   RPN:Targe   RPN:Targe
 units      K           K
 value   300.00       80.000       0.0000   6.2000E-05   -52.000
result   300.00       80.000   -2.2675E-12  6.2000E-05   -52.000
```

and the `BEGIN` segment. We are considering the complex volume flow rates at the two ends to be given, as if we have in mind an "alpha" Stirling machine, with the motions of the two pistons determining the volumes of the compression and expansion spaces. The volume flow rate at the hot end is set by lines `0f` and `0g` in the `BEGIN` segment. The 0° phase of line `0g` essentially determines the zero of phase for the entire system. The volume flow rate 3.65×10^{-4} m^3/s of line `0f` (together with the frequency set in line `0b`) implies a volumetric

stroke of 2.1 cm^3 peak-to-peak at the hot end. The RPNs at the cold end ensure that DeltaEC's shooting method arrives there with the desired cold piston stroke and phase. To arrive at these two targets, DeltaEC adjusts two guesses: the pressure amplitude and phase in the BEGIN segment (and hence throughout the cooler). We also insist that the solid temperatures in the two heat exchangers be 300 K and 80 K; DeltaEC achieves these two targets by adjusting two more guesses: the heat extracted at the hot heat exchanger, and the temperature in the BEGIN segment. We also insist that the acoustic power and total power leaving the cold heat exchanger are equal, implying that the cold piston is thermally insulated. (Acoustic power and total power are also equal at the ambient piston, because the BEGIN segment assigns $\dot{H}_{2,k} = \dot{E}$ by default.)

DeltaEC predicts that, under these circumstances, the cooler will reject 36 W at the hot heat exchanger and will have a cooling power of 5 W. This cooling power accounts for heat conduction and enthalpy flow through the regenerator, but does not account for any heat load imposed by frictional irreversibilities in the cold piston, nor any heat load imposed by the regenerator *case* conduction except to the extent that it is included in ksFrac.

We now make or suggest a few simple modifications to this file to illustrate additional features of DeltaEC.

To discover what temperature the cooler would maintain with a heat load of 10 W instead of 5 W, we remove 3f—the cold heat exchanger temperature—from the target list, and we remove 3e—the cooling power—from the guess list, and modify it to 10 W. Running DeltaEC shows that the shooting method is lost with such a dramatic change, so we Restore guessez, modify 3e back to 5 W, and use an incremental plot of 3e from 5 W to 10 W, step 0.2 W, to reach the desired heat load. Under these circumstances the cold temperature will be 232 K.

Using 3e as an independent plot variable running from 10 W to 2 W with steps of, say, 0.2 W, and using 3H (cold solid temperature) as a dependent plot variable will generate a table of cold temperature (and other defaults) vs heat load.

Insertion of two *ESPEAKER segments, before the aftercooler and after the cold heat exchanger, would model use of linear motors and pistons there.

In the next subsection, we will use TBRANCH and UNION to change this model from an "alpha" Stirling machine to a "beta" or "gamma" machine, with one power piston on the hot end and a displacer piston in parallel with the heat exchange elements.

5.8.2. Beta or gamma cryocooler

The use of TBRANCH and UNION for hardware with branches and loops was introduced in Section 2.7.5. Recall the method of computation: At a TBRANCH, the branch impedance determines how the (complex) volume flow rate splits up; volume flow rates are added at the UNION. In purely acoustic models, the complex branch impedance is most often used as a pair of guesses that DeltaEC adjusts in its usual way to get the complex pressure at the SOFTEND of the TBRANCH to come out to be the same as it is at the UNION. In models with thermoacoustic components, a third guess–target pair is also employed: The fraction of the energy flow $\dot{H}_{\mathrm{tot}}$ going down the TBRANCH is guessed, and the temperature at the SOFTEND of the TBRANCH is targeted to equal the temperature at the UNION. Models with

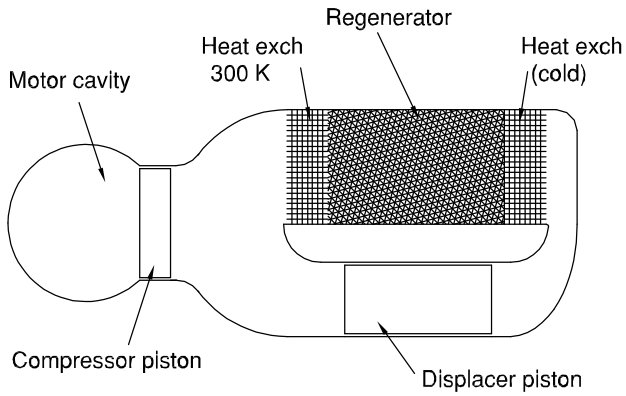


Figure 5.12: "Gamma"-style Stirling machine.

TBRANCHs and UNIONs tend to have many guesses and targets, since every SOFTEND and UNION contributes two or three targets (and a few more targets are almost always needed for temperatures, heats, etc.). Keep in mind that UNION targets differ from the usual RPN, **END and heat-exchanger-temperature targets because their actual values are not specified by the user—they vary from one integration to the next, depending on what is happening at the associated SOFTEND segment, as the shooting method varies the guesses.

As an example of use of TBRANCH and UNION in a thermoacoustic setting, we return to the Stirling cryocooler example above, and convert it to a "displacer"-style [25] Stirling machine, with a compressor piston at the hot end and a driven displacer piston connecting the hot and cold ends. In the previous example, PU power (i.e., $\dot{E}$ in DeltaEC vocabulary) flowed in at the BEGIN and out at the end; no explicit segments indicated the pistons there. Here, with a displacer piston, the cold-end PU power is returned automatically to the hot end, reducing the hot-end PU power requirement.

The apparatus layout is illustrated in Fig. 5.12. The corresponding display of segment headings is

```
  0 BEGIN:G  P Initialize things
  1 COMPLIANCE:     space around power-piston motor
  2 IESPEAKER:     the power piston
  3 COMPLIANCE:     a little space in front of the power piston
  4    TBRANCH:G    branch to displacer
  5    IESPEAKER:     a spring-mounted, driven moving mass
  6    COMPLIANCE:     a little volume for the connection
  7    SOFTEND:     reconnect at UNION
  8 SX   :GT   aftercooler
  9 STKSCREEN:     regenerator
 10 SX   :GT   cold heat exch
 11 UNION: T   displacer cold end
 12 HARDE: T   close the end
 13 RPN  : T   simulate insulation in the displacer piston
```

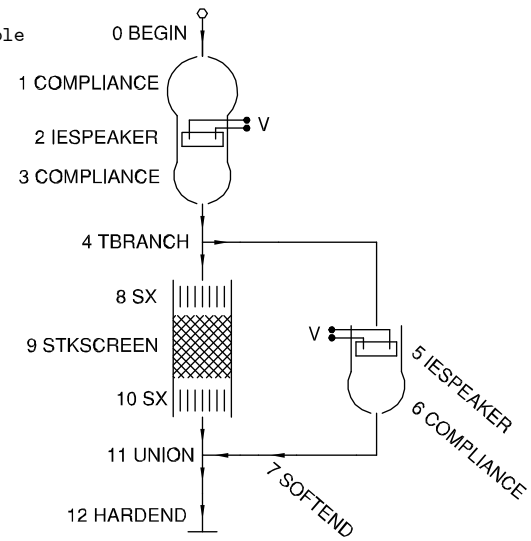
and the guess–target summary is the largest we have yet encountered in the Users Guide:

```
 GUESS       0c          0d          0e          4a          4b          4c          8e         10e
 name  BEGIN:TBeg  BEGIN: |p|  BEGIN:Ph(p) TBRAN:Re(Zb TBRAN:Im(Zb TBRAN:HtotB   SX:HeatI    SX:HeatI
 units      K           Pa          deg      Pa-s/m^3    Pa-s/m^3       W            W           W
 value   300.22     6373.3       114.43    -3.0131E+09 -3.2836E+09    -6.7651    -45.267       4.5794
TARGET      8f         10f         11b         11c         12a         12b         12c         13a
 name     SX:Est-T    SX:Est-T UNION:|p|SO UNION:Ph(p) HARDE:R(1/z HARDE:I(1/z HARDE:Htot    RPN:Targe
 units      K           K          Pa          deg                                  W
 value   300.00       80.000    2.5375E+05   -44.127       0.0000      0.0000      0.0000      0.0000
result   300.00       80.000    2.5375E+05   -44.127    8.9507E-14  2.8896E-13 -2.4998E-11 -4.8383E-11
```

One can think of these guesses and targets as paired up in the following way. The `TBeg` guess lets DeltaEC hit the T-ambient target. (These two are so nearly equal, and so trivially related, that they could easily be omitted.) The two branch-impedance guesses let DeltaEC reach the two p_1 targets at the `UNION`. The branch-Htot guess lets DeltaEC hit the `RPN` target at segment 13, which enforces thermal insulation on the cold end of the displacer piston. The heat removed at the hot heat exchanger lets DeltaEC reach the target temperature at the cold heat exchanger. Finally, the two p_1 guesses in the `BEGIN` segment and the heat in the cold heat exchanger allow DeltaEC to achieve $U_1 = 0$ and $\dot{H}_{\text{tot}} = 0$ at the `HARDEND` at the end of the apparatus.

The corresponding <`.out`> file is

```
TITLE     Stirling cooler w displacer piston, illustrating TBRANCH--UNION
!-gamma.out
!Created@14:22:52  7-Oct-06 with DeltaE Vers. 6.0g2 for the IBM/PC-Compatible
!---------------------------------  0 ---------------------------------
 BEGIN      Initialize things
  2.0000E+06 a Mean P    Pa
    55.000   b Freq      Hz
   300.22    c TBeg      K       G
  6373.3     d  |p|      Pa      G
   114.43    e Ph(p)     deg     G
     0.0000  f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 helium     Gas type
!---------------------------------  1 ---------------------------------
 COMPLIANCE space around power-piston motor
     0.1000  a SurfAr    m^2              6373.3     A |p|       Pa
  5.0000E-04 b Volume    m^3               114.43    B Ph(p)     deg
                                         3.3041E-04 C |U|     m^3/s
                                           23.550   D Ph(U)     deg
                                            0.0000  E Htot      W
 ideal      Solid type                  -1.6157E-02 F Edot      W
!---------------------------------  2 ---------------------------------
 IESPEAKER  the power piston
  2.0000E-04 a Area      m^2              2.9239E+05 A |p|       Pa
     1.0000  b   R      ohms               -39.277   B Ph(p)     deg
     0.0000  c L         H               3.3020E-04 C |U|     m^3/s
    10.000   d BLProd   T-m                 23.620   D Ph(U)     deg
  8.3000E-02 e   M      kg                  40.076   E Htot      W
  1.0000E+04 f   K      N/m                 21.992   F Edot      W
     0.0000  g   Rm   N-s/m                40.076   G WorkIn    W
     6.0000  h  |I|     A                   19.931   H Volts     V
   140.00    i Ph(I)   deg                   6.0000  I Amps      A
                                          -312.09    J Ph(Ze)   deg
                                          2.9812E+05 K  |Px|    Pa
                                           -39.820   L Ph(Px)   deg
!---------------------------------  3 ---------------------------------
 COMPLIANCE a little space in front of the power piston
  1.0000E-03 a SurfAr    m^2              2.9239E+05 A |p|       Pa
  2.0000E-06 b Volume    m^3               -39.277   B Ph(p)     deg
                                         2.7635E-04 C |U|     m^3/s
                                           18.316   D Ph(U)     deg
                                           40.076   E Htot      W
 ideal      Solid type                     21.652   F Edot      W
!---------------------------------  4 ---------------------------------
 TBRANCH    branch to displacer
 -3.0131E+09 a Re(Zb) Pa-s/m^3   G        2.9239E+05 A |p|       Pa
 -3.2836E+09 b Im(Zb) Pa-s/m^3   G         -39.277   B Ph(p)     deg
    -6.7651  c HtotBr   W        G        6.5609E-05 C |U|     m^3/s
                                           93.262   D Ph(U)     deg
                                           -6.7651  E HtotBr    W
                                           -6.4850  F EdotBr    W
                                           28.137   G EdotTr    W
!---------------------------------  5 ---------------------------------
 IESPEAKER  a spring-mounted, driven moving mass
  5.0000E-05 a Area      m^2              2.5375E+05 A |p|       Pa
     0.5000  b   R      ohms               -44.127   B Ph(p)     deg
     0.0000  c L         H               6.5762E-05 C |U|     m^3/s
     1.0000  d BLProd   T-m                 93.396   D Ph(U)     deg
  7.0000E-03 e   M      kg                  -6.1541  E Htot      W
  1190.0     f   K      N/m                 -6.1538  F Edot      W
     0.0000  g   Rm   N-s/m                 0.6110  G WorkIn    W
     1.0000  h  |I|     A                    1.6425  H Volts     V
   -30.000   i Ph(I)   deg                   1.0000  I Amps      A
                                           -41.930   J Ph(Ze)   deg
                                          4.4991E+04 K  |Px|    Pa
```



```
                                          169.20    L Ph(Px)   deg
!--------------------------------  6 --------------------------------
 COMPLIANCE a little volume for the connection
  1.0000E-06 a SurfAr    m^2              2.5375E+05 A |p|       Pa
  5.0000E-07 b Volume    m^3               -44.127   B Ph(p)     deg
                                          5.7703E-05 C |U|     m^3/s
                                           103.08    D Ph(U)     deg
                                            -6.1541  E Htot      W
 ideal      Solid type                     -6.1541  F Edot      W
!--------------------------------  7 --------------------------------
 SOFTEND    reconnect at UNION
     0.0000  a Re(z)             (t)      2.5375E+05 A |p|       Pa
     0.0000  b Im(z)             (t)       -44.127   B Ph(p)     deg
     0.0000  c Htot     W        (t)      5.7703E-05 C |U|     m^3/s
                                           103.08    D Ph(U)     deg
                                            -6.1541  E Htot      W
                                            -6.1541  F Edot      W
                                           -56.531   G Re(z)
                                           -36.427   H Im(z)
                                           300.22    I   T     K
!--------------------------------  8 --------------------------------
 SX         aftercooler
 sameas   9a a Area      m^2              2.8988E+05 A |p|       Pa
     0.6000  b VolPor                      -39.755   B Ph(p)     deg
  1.0000E-03 c Length     m               2.6449E-04 C |U|     m^3/s
 sameas   9d d   rh       m                  4.0408  D Ph(U)     deg
   -45.267   e HeatIn     W      G           1.5747  E Htot      W
   300.00    f Est-T      K    =  8H?       27.670   F Edot      W
                                           300.22    G GasT     K
 copper     Solid type                     300.00    H SolidT     K
!--------------------------------  9 --------------------------------
 STKSCREEN      regenerator
  1.1670E-04 a Area      m^2              2.5407E+05 A |p|       Pa
     0.6860  b VolPor                      -44.169   B Ph(p)     deg
  5.0000E-02 c Length     m               5.6226E-05 C |U|     m^3/s
  1.3900E-05 d   rh       m                -74.284   D Ph(U)     deg
     0.3000  e ksFrac                        1.5747  E Htot      W
                                             6.1785  F Edot      W
                                           300.22    G TBeg       K
 stainless  Solid type                      79.962   H TEnd       K
!-------------------------------- 10 --------------------------------
 SX         cold heat exch
 sameas   9a a Area      m^2              2.5375E+05 A |p|       Pa
     0.6000  b VolPor                      -44.127   B Ph(p)     deg
  1.0000E-03 c Length     m               5.7703E-05 C |U|     m^3/s
 sameas   9d d   rh       m                -76.923   D Ph(U)     deg
     4.5794  e HeatIn     W      G           6.1541  E Htot      W
    80.000   f Est-T      K    = 10H?        6.1541  F Edot      W
                                            79.962   G GasT     K
 copper     Solid type                      80.000   H SolidT     K
!-------------------------------- 11 --------------------------------
 UNION      displacer cold end
     7.0000  a SegNum                     2.5375E+05 A |p|       Pa
  2.5375E+05 b |p|SOF    Pa    = 11A?      -44.127   B Ph(p)     deg
   -44.127   c Ph(p)S    deg   = 11B?     1.4140E-15 C |U|     m^3/s
   300.22    d T_SOFT    K       (t)        28.663   D Ph(U)     deg
                                         -2.4998E-11 E Htot      W
                                          5.3083E-11 F Edot      W
                                            79.962   G T         K
!-------------------------------- 12 --------------------------------
 HARDEND    close the end
     0.0000  a R(1/z)          = 12G?     2.5375E+05 A |p|       Pa
     0.0000  b I(1/z)          = 12H?      -44.127   B Ph(p)     deg
     0.0000  c Htot     W      = 12E?     1.4140E-15 C |U|     m^3/s
                                            28.663   D Ph(U)     deg
                                         -2.4998E-11 E Htot      W
                                          5.3083E-11 F Edot      W
                                          8.9507E-14 G R(1/z)
                                          2.8896E-13 H I(1/z)
!-------------------------------- 13 --------------------------------
 RPN        simulate insulation in the displacer piston
     0.0000  a G or T          = 13A?    -4.8383E-11 A  null
   6E 6F -

! The restart information below was generated by a previous run
! You may wish to delete this information before starting a run
! where you will (interactively) specify a different iteration
! mode.  Edit this table only if you really know your model!
guessz    0c 0d 0e 4a 4b 4c 8e 10e
xprecn  -3.8009E-03  2.3931E-03 -4.4166E-04  1.2042E+04 -4.9579E+04  3.1353E-05  1.1774E-04  9.4330E-06
hilite    10e
targs     8f 10f 11b 11c 12a 12b 12c 13a
```

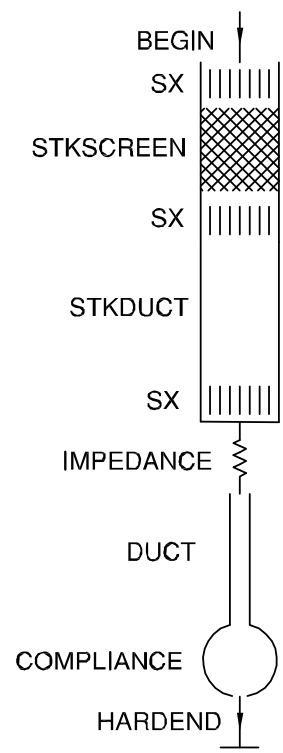
We are using `IESPEAKER` segments for both the displacer and the driver, because linear motors and loudspeakers share the same physical transduction mechanism.

The user will soon discover that this is a surprisingly robust model, considering the large number of guesses and targets: The model tolerates steps in many independent variables of several percent without getting lost. The user might next generate cooling-power curves by using the cold-temperature target as an independent plot variable and the cooling power as dependent plot variable; or the user might explore the frequency dependence of the cooler, by using frequency as an independent plot variable.

5.8.3. Pulse tube refrigerator

Changing a Stirling cryocooler into an orifice pulse tube refrigerator (OPTR) is a simple matter of replacing the cold piston with a pulse tube, heat exchanger, orifice, inertance (optional), and compliance volume in series. Figure 5.13 represents such a cooler. The sample file <`optr.out`> represents a 300 Hz, 3 MPa helium orifice pulse tube refrigerator. (It is not representative of any real hardware known to us. In fact, it is deliberately inefficient, to serve as a lesson in the next section.) The <`.out`> file is:

```
TITLE     a crude cooler design, not optimal
!---------------------------------  0 ---------------------------------
 BEGIN      Start with 8% p osc
  3.0000E+06 a Mean P    Pa
   300.00    b Freq      Hz
   300.10    c TBeg      K       G
  2.4000E+05 d  |p|      Pa
     0.0000  e Ph(p)     deg
  6.9787E-03 f  |U|     m^3/s    G
    52.898   g Ph(U)     deg     G
 helium     Gas type
!---------------------------------  1 ---------------------------------
 SX         Aftercooler
  1.0290E-03 a Area      m^2              2.3018E+05 A |p|       Pa
     0.6900  b VolPor                       -3.5084  B Ph(p)     deg
  1.2500E-02 c Length     m               5.8509E-03 C |U|     m^3/s
  6.4500E-05 d   rh       m                 46.900   D Ph(U)     deg
  -454.47    e HeatIn     W      G          50.700   E Htot      W
   300.00    f Est-T      K    =  1H?      429.15    F Edot      W
                                           300.10    G GasT     K
 copper     Solid type                     300.00    H SolidT     K
!---------------------------------  2 ---------------------------------
 STKSCREEN    Regenerator
 sameas   1a a Area      m^2              1.7241E+05 A |p|       Pa
     0.7300  b VolPor                      -19.695   B Ph(p)     deg
  5.5000E-02 c Length     m               1.2191E-03 C |U|     m^3/s
  2.4000E-05 d   rh       m                -29.387   D Ph(U)     deg
     0.3000  e ksFrac                       50.700   E Htot      W
                                           103.59    F Edot      W
                                           300.10    G TBeg       K
 stainless  Solid type                     149.90    H TEnd       K
!---------------------------------  3 ---------------------------------
 SX         Cold heat exchanger
 sameas   4a a Area      m^2              1.1527E+05 A |p|       Pa
     0.6900  b VolPor                      -15.367   B Ph(p)     deg
  2.0000E-03 c Length     m               1.2192E-03 C |U|     m^3/s
  6.4500E-05 d   rh       m                -29.702   D Ph(U)     deg
    14.510   e HeatIn     W      G          65.210   E Htot      W
   150.00    f Est-T      K    =  3H?       68.082   F Edot      W
                                           149.90    G GasT     K
 copper     Solid type                     150.00    H SolidT     K
!---------------------------------  4 ---------------------------------
 STKDUCT      Pulse tube
  5.6870E-05 a Area      m^2     S= -2    1.0482E+05 A |p|       Pa
  2.6740E-02 b Perim      m    Fn(  4a)    -44.000   B Ph(p)     deg
     0.2000  c Length     m               1.3018E-03 C |U|     m^3/s
  1.0000E-05 d WallA     m^2               -51.089   D Ph(U)     deg
                                            65.210   E Htot      W
                                            67.704   F Edot      W
                                           149.90    G TBeg       K
 stainless  Solid type                     300.20    H TEnd       K
!---------------------------------  5 ---------------------------------
 SX         Hot heat exchanger
 sameas   4a a Area      m^2              2.2288E+04 A |p|       Pa
     0.6900  b VolPor                      -19.900   B Ph(p)     deg
```



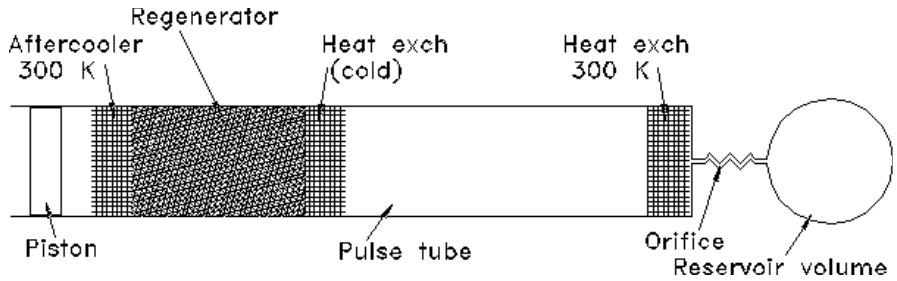


Figure 5.13: An orifice pulse tube refrigerator (OPTR). The orifice can be purely resistive, as shown here, but adding inertance in series with the resistive orifice improves performance.

```
  5.0000E-03 c Length     m                1.3025E-03 C |U|     m^3/s
  6.4500E-05 d   rh       m                 -51.420   D Ph(U)     deg
   -65.210   e HeatIn     W      G        -2.8422E-14 E Htot      W
   300.00    f Est-T      K    =  5H?        12.373   F Edot      W
                                            300.20    G GasT     K
 copper     Solid type                     300.00    H SolidT     K
!---------------------------------  6 ---------------------------------
 IMPEDANCE  The orifice
  1.0000E+07 a Re(Zs) Pa-s/m^3             1.3094E+04 A |p|       Pa
     0.0000  b Im(Zs) Pa-s/m^3               11.433   B Ph(p)     deg
                                            1.3025E-03 C |U|     m^3/s
                                             -51.420   D Ph(U)     deg
                                           -2.8422E-14 E Htot      W
                                               3.8908  F Edot      W
!---------------------------------  7 ---------------------------------
 DUCT       inertance
  1.0000E-05 a Area      m^2     S= -2    2.3020E+04 A |p|       Pa
  1.1210E-02 b Perim      m    Fn(  7a)   -141.30    B Ph(p)     deg
  3.0000E-02 c Length     m                1.3017E-03 C |U|     m^3/s
  5.0000E-04 d Srough                       -51.434   D Ph(U)     deg
                                           -2.8422E-14 E Htot      W
 ideal      Solid type                     3.3976E-02 F Edot      W
!---------------------------------  8 ---------------------------------
 COMPLIANCE Reservoir volume
  1.2680E-02 a SurfAr    m^2   Fn(  8b)   2.3020E+04 A |p|       Pa
  1.5000E-04 b Volume    m^3     S= -5    -141.30    B Ph(p)     deg
                                            2.1371E-13 C |U|     m^3/s
                                            -178.84    D Ph(U)     deg
                                           -2.8422E-14 E Htot      W
 ideal      Solid type                     1.9505E-09 F Edot      W
!---------------------------------  9 ---------------------------------
 HARDEND    The end
     0.0000  a R(1/z)          =  9G?     2.3020E+04 A |p|       Pa
     0.0000  b I(1/z)          =  9H?     -141.30    B Ph(p)     deg
     0.0000  c Htot     W      =  9E?     2.1371E-13 C |U|     m^3/s
                                            -178.84    D Ph(U)     deg
                                           -2.8422E-14 E Htot      W
                                            1.9505E-09 F Edot      W
                                            3.6105E-09 G R(1/z)
                                           -2.7741E-09 H I(1/z)

! The restart information below was generated by a previous run
! You may wish to delete this information before starting a run
! where you will (interactively) specify a different iteration
! mode.  Edit this table only if you really know your model!
guessz    0c 0f 0g 1e 3e 5e
xprecn  -3.3810E-03 -1.7732E-08  2.2426E-04 -2.7531E-03  3.4751E-04  7.8445E-05
hilite    3e
targs     1f 3f 5f 9a 9b 9c
SPECIALS    3   4 -2   7 -2   8 -5
```

The Stirling part of the system is modeled as a stacked-screen regenerator `STKSCREEN` and two stacked-screen heat exchangers `SX`. We model the pulse tube itself as a `STKDUCT`, using Rott's wave equation and enthalpy flux equation in boundary-layer approximation, because the pulse-tube diameter is $\gg \delta_\kappa$. The heat exchanger at the hot end of the pulse tube is the `HX`. The orifice, inertance, and compliance are easily modeled as DeltaEC `IMPEDANCE`, `DUCT`, and `COMPLIANCE`, respectively. Our use of zero for the imaginary part of the `IMPEDANCE` reflects our intention that this orifice should truly be resistive, with its pressure drop in phase

with mass flux.

For purposes of illustration here, we will regard the geometry of the apparatus as given, and will explore its performance for a given, fixed input $|p_1|$. The guess–target summary indicates our point of view:

```
 GUESS      0c          0f          0g          1e          3e          5e
 name  BEGIN:TBeg  BEGIN: |U|  BEGIN:Ph(U)    SX:HeatI    SX:HeatI    SX:HeatI
 units      K          m^3/s        deg          W           W           W
 value   300.10     6.9787E-03    52.898    -454.47       14.510     -65.210
TARGET      1f          3f          5f          9a          9b          9c
 name     SX:Est-T    SX:Est-T    SX:Est-T HARDE:R(1/z HARDE:I(1/z HARDE:Htot
 units      K           K           K                                W
 value   300.00      150.00      300.00        0.0000      0.0000      0.0000
result   300.00      150.00      300.00     3.6105E-09 -2.7741E-09 -2.8422E-14
```

Three of the six targets fix the hot and cold temperatures at 300 K and 150 K. We leave the amplitude of the oscillatory pressure at the `BEGIN`ning at 8% of mean pressure, and leave the frequency fixed at 300 Hz. Hence, we are asking: What is the cooling power at 150 K, and how much input power, volume flow rate, etc. are required, for fixed frequency and pressure amplitude, and for this geometry? The result, given in the file listings above: 14.5 W of cooling power, requiring over 500 W of input power from the compressor.

As with most OPTR models we have worked with in DeltaEC, this one is not particularly "robust." An abrupt change of a typical variable by 20% or 30% will likely cause DeltaEC to get hopelessly lost, unable to adjust its guesses to hit the targets. Part of the "fragility" of OPTR models (and other models with a `STKDUCT` or `STKCONE`) in DeltaEC is due to the fact that small changes in variables near the `BEGIN`ning, such as p_1, U_1, and the heat removed at a heat exchanger, have a large effect on temperatures at the end of the pulse tube. Some of the fragility is due to the fact that OPTR models typically have a large number of guesses and targets. When you encounter a fragile DeltaEC model, try to reduce the number of guesses and targets as much as possible (particularly in initial design explorations when you are changing things dramatically) and, once you have a convergent model, make only small changes in variables. Try reducing DeltaEC's convergence tolerance (Section 8.3.2) if you have to use more than 5 guesses and 5 targets. Remember to subdivide a large change in a variable into a series of small changes, using an incremental plot. A fast computer and frequent hard-disk saving of satisfactory converged models will minimize frustration.

Examination of the pulse tube segment `STKDUCT` above shows a possible problem: The pulse tube figure of merit $\dot{H}_{\rm tot}/\dot{E}$, which can theoretically be as high as 1.000 but is usually more like 0.8, may be unrealistically high: $\dot{H}_{\rm tot}/\dot{E} \simeq 65$ W/68 W$\simeq 0.96$. DeltaEC knows nothing about jet- or streaming-driven convection, and pulse-tube experimentalists are only beginning to learn how to reliably avoid such convection. (For a discussion of streaming-driven convection, see Refs. [28] or [29], and [30] and [12].) To force DeltaEC to accommodate a reduced pulse-tube figure of merit, the user can introduce an `RPN` and an additional guess–target pair. The `RPN` instruction line can compute the ratio of $\dot{H}_{\rm tot}$ to $\dot{E}$ in the pulse tube, and the target value in line `a` can be set to something like 0.8. The user can simulate the thermal loading of streaming-driven convection, etc. by letting DeltaEC guess an unphysically large value for the cross section of the pulse tube solid wall (line `4d`), which then conducts significant heat from hot to cold, allowing DeltaEC to meet its target of 0.8. We will not do so here.

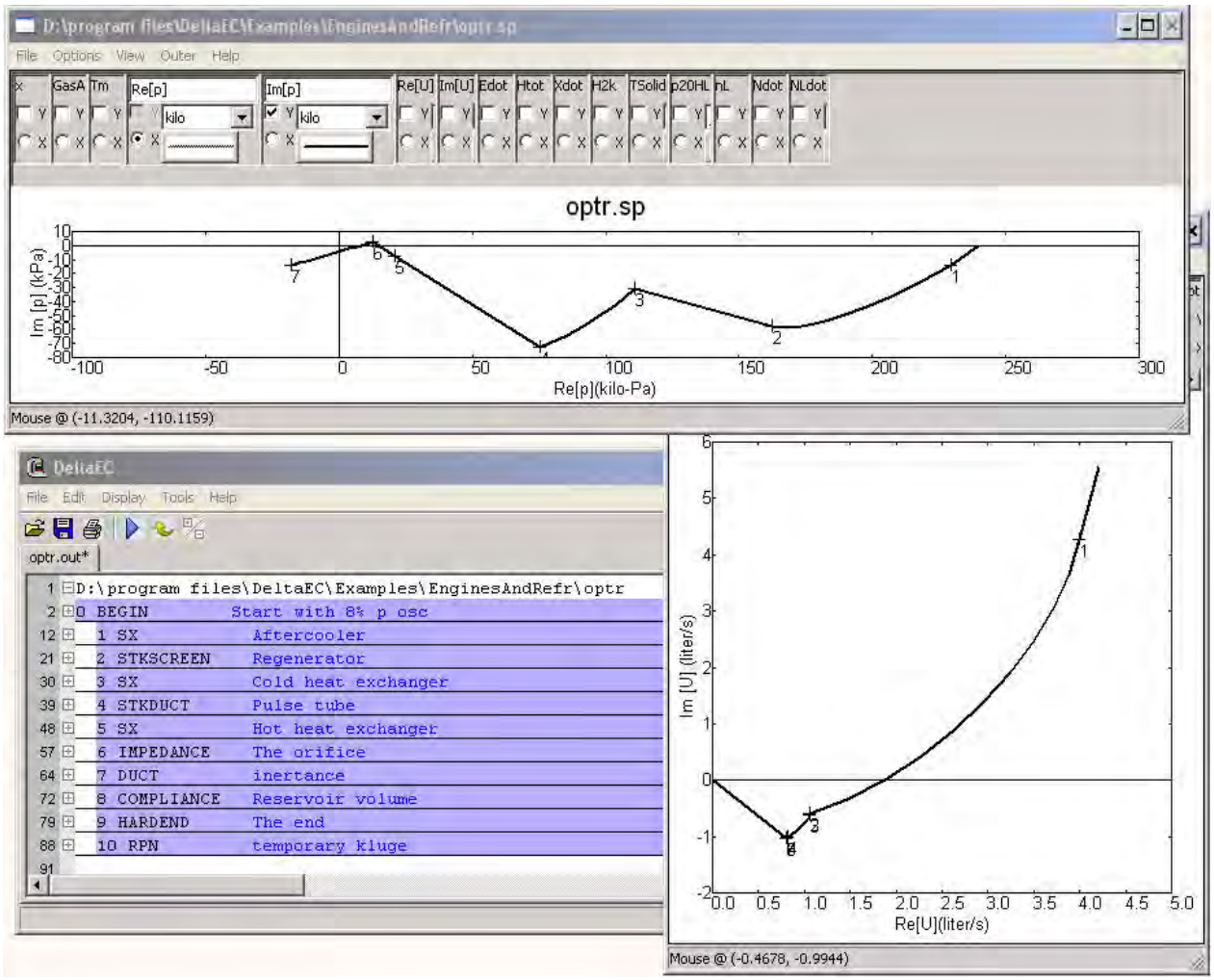


Figure 5.14: Plotting <`optr.sp`> using $\mathrm{Re}[p_1]$ as the horizontal axis and $\mathrm{Im}[p_1]$ as the vertical axis yields a pressure-phasor plot. The SegEndMarks option tags the curve with numbers indicating the ends of the segments. Cloning the plot and switching p_1 to U_1 yields a volume-flow-rate phasor plot. These can also be created with Phasor Mode in the plotter's Options pulldown menu.

The `JOIN` segment should be used at the ends of a pulse tube to model the temperature overshoots and adiabatic–isothermal interface losses there, adding more reality to DELTAEC modeling of the pulse tube itself. See Section 10.8.

Our choices for nearly all input variables in this model were random, not guided by good design sense or any real cryocooler. In particular, our choice of $\mathrm{Re}[Z_s] = 1 \times 10^7$ for the orifice impedance above was random. To find a better orifice setting, use $\mathrm{Re}[Z_s]$ as an independent plot variable, letting it range from 1×10^7 to 1×10^8. The cooling power peaks at 14.75 W for $\mathrm{Re}[Z_s] = 2.2 \times 10^7$. This is not much improvement. For guidance on what to try changing next to improve the performance, we consider *exergy* in the next section.

5.9. Exergy flow

Among the many important thermodynamic variables having units of power (e.g., Watts) is *exergy* flow. In thermodynamics, exergy represents the ability to do useful work, in principle, assuming unhindered access to an infinitely large thermal reservoir at an "ambient" temperature T_0 at which heat has no value [31].

Novice thermoacousticians sometimes suppose that the acoustic power $\dot{E}$ represents the ability to do useful work, because the treatment of intensity and power in adiabatic waves in introductory acoustics courses implies that the face of a moving piston is a perfect interface between mechanical power (Fu) and acoustic power (pU). However, thermoacoustic waves are generally not adiabatic. The "heat" carried near solid surfaces by the $s_1\widetilde{u}_1$ term in the second-order enthalpy flow represents an ability to do useful work that is separate from $\dot{E}$, because this term in the enthalpy flow can be usefully coupled to the assumed thermal reservoir at T_0 through a heat engine if $s_1\widetilde{u}_1$ is flowing at a temperature different from T_0. Taking this into account [12], the exergy flow associated with a thermoacoustic wave is

$$\dot{X} = \frac{T_0}{T_m}\dot{E} + \left(1 - \frac{T_0}{T_m}\right)\dot{H}_{2,k}. \tag{5.16}$$

(For an adiabatic sound wave, $\dot{H}_{2,k} = \dot{E}$, so this reduces to $\dot{X} = \dot{E}$, independent of T_m, as expected.) The ambient temperature T_0 is set to 300 K by default in the `BEGIN` segment; this default can be overridden by using `=Tzero` in a subsequent `RPN` segment.

The usefulness of the concept of exergy arises from its relationship to irreversible entropy generation, expressed by the Guoy-Stodola theorem [31],

$$\Delta\dot{X} = T_0\dot{S}_{\mathrm{gen}}. \tag{5.17}$$

In this simplified expression of the theorem, $\Delta\dot{X}$ represents the difference in exergy flow between two locations in an apparatus between which exergy cannot flow in or out laterally, and the theorem says that this difference is the product of T_0 and the rate of irreversible entropy generation between those two locations. In a perfectly efficient apparatus, there is no irreversible entropy generation, so $\Delta\dot{X} = 0$ (except where $\dot{X}$ flows in or out, e.g., at a heat exchanger or a `**DUCER` or `**SPEAKER`). Thus, a graph of $\dot{X}$ vs x can help show the user what parts of an apparatus are responsible for its inefficiency.

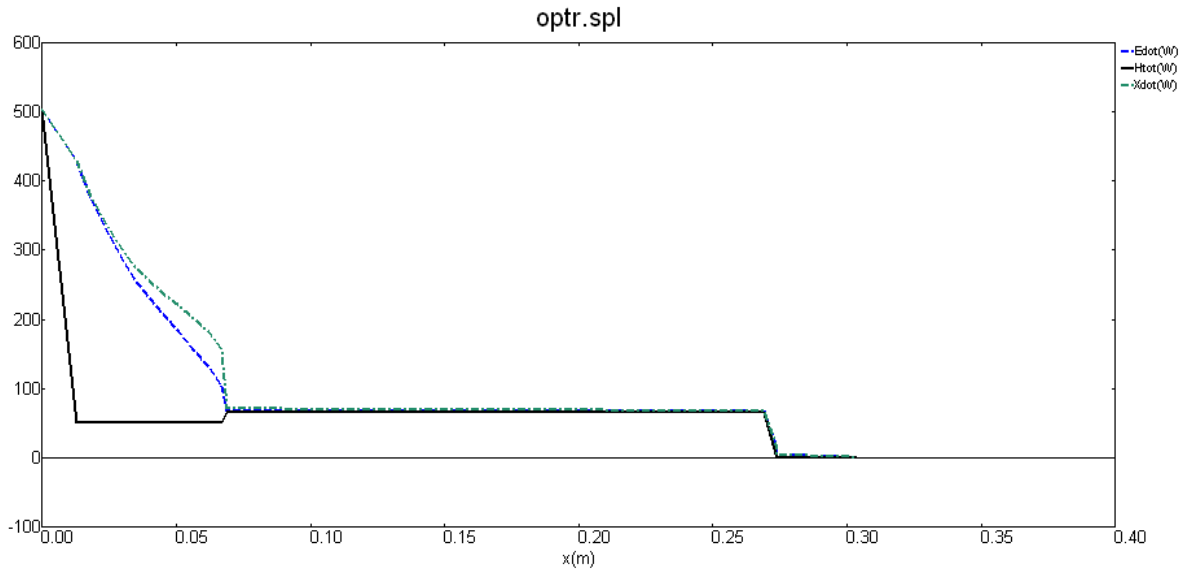


Figure 5.15: Graph of total power (black solid line), acoustic power (blue dashed line), and exergy flow (green dot-dashed line) in the OPTR example. The locations where $\dot{H}_{\mathrm{tot}}$ changes are the three heat exchangers.

For example, consider the OPTR described in Section 5.8.3. The <`.sp`> plot of $\dot{E}$, $\dot{H}_{\mathrm{tot}}$, and $\dot{X}$ is shown in Fig. 5.15. Most of the exergy is lost in the regenerator, and the rate of exergy loss is steepest near the ambient end of the regenerator. This suggests that viscous losses in the regenerator may be severe. Indeed, the <`.out`> file shows a large pressure drop in the regenerator. The user might try changing the dimensions of the regenerator to improve the performance. Figure 5.15 also shows significant drops in $\dot{X}$ in the ambient and cold heat exchangers. There, the analysis of the situation is a little more complicated, because the heats flowing into a heat exchanger contribute exergy according to [12]

$$\Delta\dot{X} = \dot{Q}\left(1 - T_0/T_m\right), \tag{5.18}$$

where T_0 is ambient temperature and T_m is the local temperature. At the ambient heat exchanger, $T_m = T_0$, so the drop in exergy there is entirely lossy. At the cold heat exchanger, $T_m \neq T_0$, but the loss in thermoacoustic exergy due to $\dot{Q}$ is only 15 W, so most of the drop in $\dot{X}$ in Fig. 5.15 at the cold heat exchanger is lossy. Thus, the heat exchangers are also contributing significantly to the poor performance in this OPTR. At this point, the user might suspect that the poor performance of this OPTR arises from trying to force too much flow through pores that are too tight or an area that is too small.

Subsequent chapters describe additional steady-flow and mixture-separation features in DeltaEC. These phenomena also contribute to $\dot{X}$, and the terms that they add to Eq. (5.16) are given in Eq. (8.9).

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6. Steady flow, too

The superposition of steady flow and oscillating flow creates many interesting and important effects that can be modeled with DeltaEC. A nonzero steady flow profoundly affects $T_m(x)$ in stacks and regenerators, creates Doppler shifts [32] and increases the turbulent dissipation of acoustic power in `DUCT`s, and causes a small x dependence in the time-averaged pressure. In systems such as Reid's refrigerator (shown below in Fig. 6.1), externally forced steady flow provides the ability to deliver cold gas from a thermoacoustic refrigerator without a cold heat exchanger. In more conceptually subtle situations, the acoustic oscillations themselves cause the steady flow, which is then called "streaming" by acousticians [33]. In some circumstances, such as the thermoacoustic-Stirling hybrid engine shown in Fig. 1.7 or 6.4, we try to understand the streaming well enough to eliminate it. In other circumstances, such as the self-circulating heat exchanger shown in Fig. 1.9, we try to make streaming as large as possible without undue dissipation of acoustic power.

6.1. Principles of calculation

DeltaEC accounts for steady flow as $\dot{N}$, the moles per second[1] flowing through the segments in the positive-x direction. It can be specified or guessed in `BEGIN`, and it splits up at `TBRANCH`s, adds up at `UNION`s, and can be abruptly changed in `RPN`s. Otherwise, $\dot{N}$ flows unchanged through segments. (But note that it seldom makes sense to imagine that nonzero $\dot{N}$ could flow *through* one of the *enclosed* `*EDUCER` or `*ESPEAKER` segments.)

Steady flow carries power

$$\dot{H}_{\dot{N}} = \dot{N} m w_m, \tag{6.1}$$

where m is the molar mass and w_m is the mean enthalpy per unit mass. The total power

$$\dot{H}_{\text{tot}} = \dot{H}_{2,k} + \dot{H}_{\dot{N}} \tag{6.2}$$

is the power that obeys the first law of thermodynamics for open control volumes. It is the sum of the second-order time-averaged thermoacoustic enthalpy flux, the ordinary thermal conduction in the x direction, and $\dot{H}_{\dot{N}} = \dot{N} m w_m$.

In DeltaEC, mean enthalpy is referenced to zero at temperature T_0, which is set to 300 Kelvin by default in the first `BEGIN` segment of a model. This default can be overridden

[1] DeltaEC's moles are gram-moles: 6.022×10^{23} particles (e.g., helium's molar mass is 0.004002 kg/mol). We might have chosen to use mass flow or volume flow instead of molar flow as DeltaEC's steady-flow parameter. We rejected volume flow because it changes with x when T_m depends on x, which seemed unnecessarily confusing. We chose molar flow instead of mass flow for convenience in the context of mixture separation, which is introduced in Chapter 7.

by using the `=Tzero` operation in an `RPN` segment. For an ideal gas, Eq. (6.1) can then be regarded as

$$\dot{H}_{\dot{N}} = \dot{N} m c_p \left(T_m - T_0\right). \tag{6.3}$$

Nonzero $\dot{N}$ also contributes a term

$$\dot{N}\left(m w_m - T_0 m s_m\right) \tag{6.4}$$

to exergy flow $\dot{X}$. Like the enthalpy per unit mass w_m, the entropy per unit mass s_m is referenced[2] to zero at T_0.

DeltaEC also introduces a new integration variable, the second-order time-averaged head-loss pressure $p_{2,0,HL}$, to keep track of the small but important effects of steady flow on the time-averaged pressure. The second-order head-loss pressure $p_{2,0,HL}$ is initialized at zero in `BEGIN` and evolves with x in most other segments. When $\dot{N}$ and $p_{2,0,HL}$ are involved in a DeltaEC calculation, the numerical-integration picture that is summarized in Eqs. (5.6)–(5.9) for thermoacoustics with $\dot{N} = 0$, namely

$$\begin{aligned}
\frac{dp_1}{dx} &= \mathcal{F}_{\text{momentum}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega;\ \text{geometry, gas properties, etc.}),\\
\frac{dU_1}{dx} &= \mathcal{F}_{\text{continuity}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \text{geometry, gas properties, etc.}),\\
\frac{dT_m}{dx} &= \mathcal{F}_{x\text{-energy}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \text{geometry, gas properties, etc.}),\\
\frac{d\dot{H}_{\text{tot}}}{dx} &= \mathcal{F}_{\text{lateral energy}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \text{geometry, gas properties, etc.}).
\end{aligned}$$

must be replaced by this slightly more complicated picture:

$$\frac{dp_1}{dx} = \mathcal{F}_{\text{momentum}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \dot{N};\ \text{geometry, gas properties, etc.}), \tag{6.5}$$

$$\frac{dU_1}{dx} = \mathcal{F}_{\text{continuity}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \dot{N}, \text{geometry, gas properties, etc.}), \tag{6.6}$$

$$\frac{dT_m}{dx} = \mathcal{F}_{x\text{-energy}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \dot{N}, \text{geometry, gas properties, etc.}), \tag{6.7}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \mathcal{F}_{\text{lateral energy}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \dot{N}, \text{geometry, gas properties, etc.}), \tag{6.8}$$

$$\frac{dp_{2,0,HL}}{dx} = \mathcal{F}_{\text{head loss}}(p_1, U_1, T_m, \dot{H}_{\text{tot}};\ p_m, \omega, \dot{N}, \text{geometry, gas properties, etc.}). \tag{6.9}$$

[2] Entropy per unit mass is referenced to zero at $T = T_0$ and $p = p_m$. Since all parts of a thermoacoustic apparatus share the same p_m, this unconventional reference pressure does not affect any entropy or exergy *differences* [e.g., Eq. (5.17)] within a DeltaEC model.

Our confidence in the accuracy of the DELTAEC implementation of these equations is lower than in the equations described in Chapters 2 and 5. We are very confident of the effect of nonzero $\dot{N}$ on $\mathcal{F}_{x\text{-energy}}$ and $\mathcal{F}_{\text{lateral energy}}$ under all circumstances we have encountered, and confident of the effect of nonzero $\dot{N}$ on all the equations when the steady and oscillating flows are both laminar, but for turbulent flow our understanding of $\mathcal{F}_{\text{head loss}}$ and of the effects of nonzero $\dot{N}$ on $\mathcal{F}_{\text{momentum}}$ and $\mathcal{F}_{\text{continuity}}$ are still evolving, and are mostly based on the quasi-steady approximation, namely that the behavior at any instant of time t is the same as it would be for steady flow with volume flow rate $U = \dot{N}m/\rho_m + \text{Re}[U_1 e^{i\omega t}]$, an approximation with obvious shortcomings in acoustics where inertial effects of gas acceleration are usually as important as anything else. The complicated details of the current steady-flow DELTAEC implementation are given in Chapter 10.

One other variable is sometimes useful in the context of nonzero $\dot{N}$. The (total) second-order, time-averaged pressure, $p_{2,0,\text{tot}}$ corresponds to pressure changes as measured by side-wall pressure transducers [34], because `p20tot` is given by

$$p_{2,0,\text{tot}} = p_{2,0,\text{rev}} + p_{2,0,HL} \tag{6.10}$$

where the reversible part of the second-order time-averaged pressure [35] is given by

$$p_{2,0,\text{rev}}(x) = \frac{|p_1|^2}{4\rho_m a^2} - \frac{\rho_m |U_1|^2}{4A_{\text{gas}}^2}. \tag{6.11}$$

The variable `p20tot` is available in `RPN` segments.

In the rest of this chapter, we illustrate DELTAEC's steady-flow features with three examples. In the first example, nonzero $\dot{N}$ is externally driven and $p_{2,0,HL}$ is of little interest. In the second example, $\dot{N} = 0$ is desired, but must be accomplished through careful balancing of small $p_{2,0,HL}$ effects. In the third example, nonzero $\dot{N}$ is driven by large $p_{2,0,HL}$ effects.

6.2. Reid's refrigerator

Bob Reid's standing-wave refrigerator, shown in Fig. 6.1 and described in Refs. [36, 37, 38], was built for the study of large nonzero $\dot{N}$ through the stacks. The standing wave in the resonator had pressure nodes at the top-center and bottom-center locations, and out-of-phase pressure antinodes near the left and right speaker pairs. Steady flow of ambient-temperature working gas was injected into the resonator at the upper pressure node. This flow was cooled as it passed through the stacks, so cold gas flowed out of the resonator at the lower pressure node. There was no concern that acoustic power might flow out into the steady-flow piping because the steady-flow connections were made at pressure nodes.

To model this apparatus in DELTAEC, we will consider only one half of the apparatus, because the other half is identical (though shifted in time phase by 180°). Focusing only on the thermal aspects, we begin the model just below the speakers in the right half, and end it where the right half joins the left half, at the exit port at the bottom. The `BEGIN` and the guess–target summary reflect the following perspective: The refrigerator is driven at a known frequency and with a known pressure amplitude near the drivers; a known steady flow of gas at a known temperature goes down through the apparatus; the ambient heat

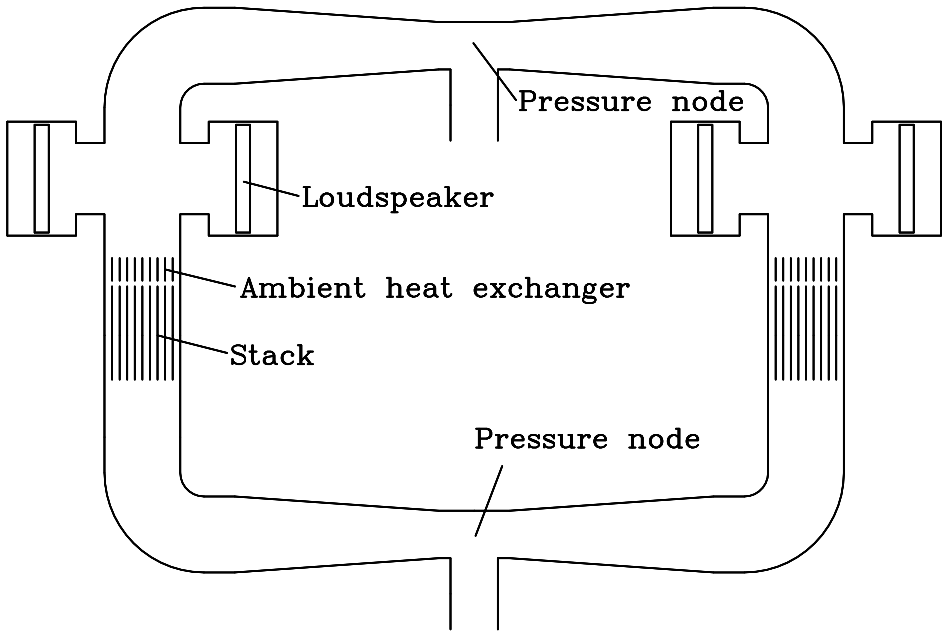
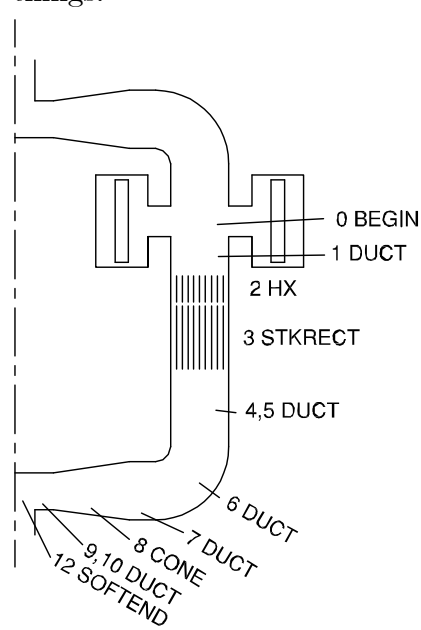


Figure 6.1: Standing-wave refrigerator with nonzero steady flow through the stacks, studied by Reid [36, 37, 38]. Steady flow was injected at the upper pressure node at ambient temperature and removed at the lower pressure node at a cold temperature.

exchanger solid is held at a known temperature by cooling water. We want to learn the temperature of the exiting steady flow of cold gas, among other things.

```
 GUESS      0f          0g          2e          2g
 name  BEGIN:|U|   BEGIN:Ph(U)    HX:HeatI    HX:FracQ
 units  m^3/s       deg          W
 value -5.5064E-02   188.72     -135.67     6.4297E-02
TARGET      2f          12a         12b         12c
 name     HX:Est-T SOFTE:Re(z) SOFTE:Im(z) SOFTE:Htot
 units  K                                   W
 value   300.00        0.0000      0.0000    -66.741
result   300.00    -5.3646E-17  3.9259E-16   -66.741


 TITLE     segments 1-7 < Bob 9/97;  segs 8-14 < Hiller ntbk, pg one, 10/97
!->reid4.out
!---------------------------------  0 ---------------------------------
 BEGIN      Initialize in main duct where drivers are attached
  3.2388E+05 a Mean P Pa
    92.000   b Freq   Hz
   303.00    c T-beg  K
  6500.0     d |p|    Pa
    90.000   e Ph(p)  deg
 -5.5064E-02 f |U|    m^3/s      G
   188.72    g Ph(U)  deg        G
     0.6700  i Ndot   mol/s
     0.9200  j nL
 HeAr       Gas type
!---------------------------------  1 ---------------------------------
 DUCT       Pre-stack Duct
  1.8430E-02 a Area   m^2                 6234.1     A |p|    Pa
     0.4813  b Perim  m                     89.694   B Ph(p)  deg
     0.1494  c Length m                  7.3748E-02 C |U|    m^3/s
                                            6.4137  D Ph(U)  deg
                                            68.925   E Htot   W
 ideal      Solid type                      26.898   F Edot   W
!---------------------------------  2 ---------------------------------
 HX         Ambient Heat exchanger
  1.7211E-02 a Area   m^2                 6169.1     A |p|    Pa
     0.5227  b GasA/A                       89.800   B Ph(p)  deg
  1.2700E-02 c Length m                  7.4625E-02 C |U|    m^3/s
  6.3500E-04 d y0     m                     6.2375  D Ph(U)  deg
```



```
  -135.67    e HeatIn W          G         -66.741   E Htot   W
   300.00    f Est-T  K        =  2H?       25.808   F Edot   W
  6.4297E-02 g FracQN            G         302.37    G GasT   K
                                           300.00    H SolidT K
 copper     Solid type                     -99.799   I H_2k   W
!---------------------------------  3 ---------------------------------
 STKRECT    rectangular-pore stack
  1.8824E-02 a Area   m^2                 5547.7     A |p|    Pa
     0.7050  b GasA/A                       93.316   B Ph(p)  deg
     0.1524  c Length m                  9.1542E-02 C |U|    m^3/s
  4.0640E-04 d a      m                     3.8407  D Ph(U)  deg
  1.1811E-04 e Lplate m                   -66.741   E Htot   W
  6.3500E-03 f b      m                     2.3231  F Edot   W
                                          302.37    G T-beg  K
                                          295.21    H T-end  K
 kapton     Solid type                  -6.8212E-13 I H_2k   W
+  4 DUCT       From cold hx to bolt flange, + a half inch
+  5 DUCT       Duct, beginning 1/2 in below big bolt flange
+  6 DUCT       Elbow, 6 inch diam, 5 inch radius of curvature on centerline
+  7 DUCT       straight part of big black cone
+  8 CONE       The long black plastic cone
+  9 DUCT       little straight section of black cone
+ 10 DUCT       half the white "tee", up to the symmetry midpoint
!--------------------------------- 11 ---------------------------------
 RPN        Expect Htot = Edot + Ndot*m*enth
     0.0000  a Target            (t)       -66.741   A RPNval
   Edot Ndot m * enth * +
!--------------------------------- 12 ---------------------------------
 SOFTEND    pressure node
     0.0000  a Re(z)           = 12G?     4.5144E-12 A |p|    Pa
     0.0000  b Im(z)           = 12H?      101.31    B Ph(p)  deg
 sameas  11A c Htot   W        = 12E?        0.1597  C |U|    m^3/s
                                             3.5311  D Ph(U)  deg
                                           -66.741   E Htot   W
                                          -4.8796E-14 F Edot   W
                                          -5.3646E-17 G Re(z)
                                           3.9259E-16 H Im(z)
                                           295.21    I T      K
                                           -23.968   J p20hl  Pa
!--------------------------------- 13 ---------------------------------
 RPN        U mean here (liter/sec)
   999.00    a Target            (t)         5.0775  A RPNval
   Ndot m * rho / 1000 *
```

Output `3H` tells us that the refrigerator cools the gas to 295 Kelvin. The <`.sp`> plots shown in Figs. 6.2 and 6.3 show details of the temperature profile in the stack and the heat exchanger upstream of it, and how the energy flow is independent of x in the stack even as energy shifts from $\dot{N}mc_pT_m$ to $\dot{H}_{2,k}$. The temperature profile is so severely curved—essentially horizontal near the ambient end—that an increase in flow above the present value of 0.67 mol/s (segment 0), which is about 5 liters per second (segment 13), would probably be undesirable. Note that the solid temperature equals the gas temperature everywhere except in the heat exchanger, where the solid is cooler than the gas so it can remove ambient heat from the gas.

When $\dot{N} \neq 0$, the energy boundary conditions at the `BEGIN`ning and `END` of a model often require careful thought. The most commonly desired boundary condition is

$$\dot{H}_{\text{tot}} = \dot{E} + \dot{N}mw_m. \tag{6.12}$$

On its face, this condition sets total power equal to the sum of acoustic power and steady-flow enthalpy. Comparison of this expression with Eqs. (6.1) and (6.2) here and with Eq. (5.29) in Ref. [12] shows what this condition deliberately omits: axial thermal conduction in the gas and solid, and boundary-layer, bucket-brigade thermoacoustic entropy transport in the gas. Thus, this boundary condition is appropriate whenever a long, large-diameter, unremarkable `DUCT` or other segment starts or ends a model with $\dot{N} \neq 0$. This condition is encountered so frequently that it taken to be the default in `BEGIN`, which can be overridden by accessing the Default Htot / Other Htot choice in the Optional Parameters in `BEGIN`. The Default

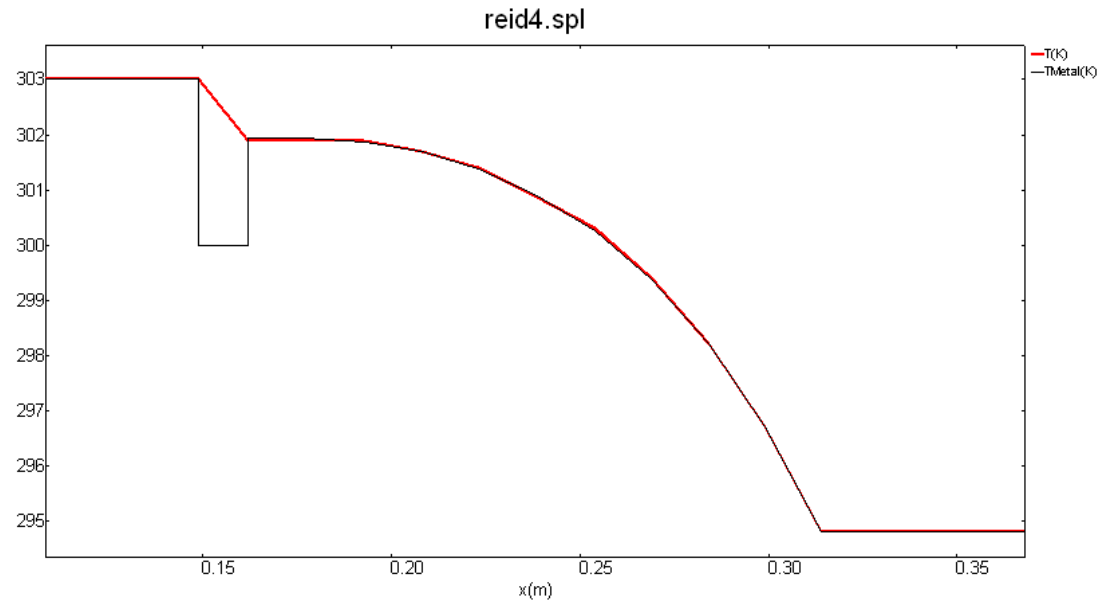


Figure 6.2: Temperatures vs x in Reid's refrigerator. Red line, gas temperature $T_m(x)$. Black line, solid temperature $T_{\text{solid}}(x)$.

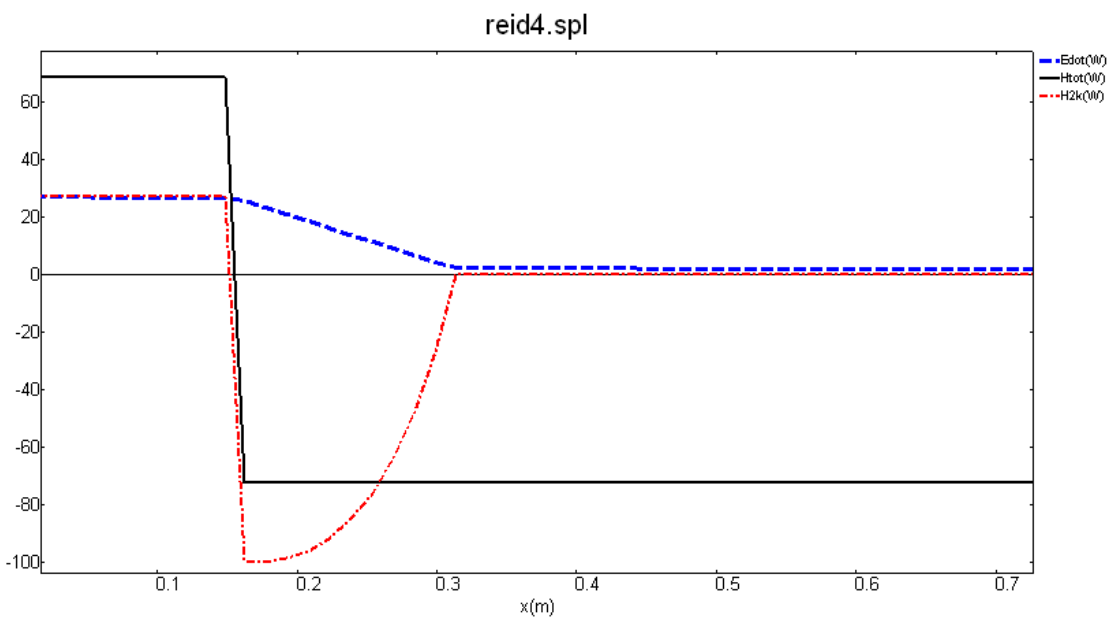


Figure 6.3: Power flows in Reid's refrigerator. Blue dashed line is $\dot{E}$. Solid black line is $\dot{H}_{\text{tot}}$. Dot-dashed red line is $\dot{H}_{2,k}$.

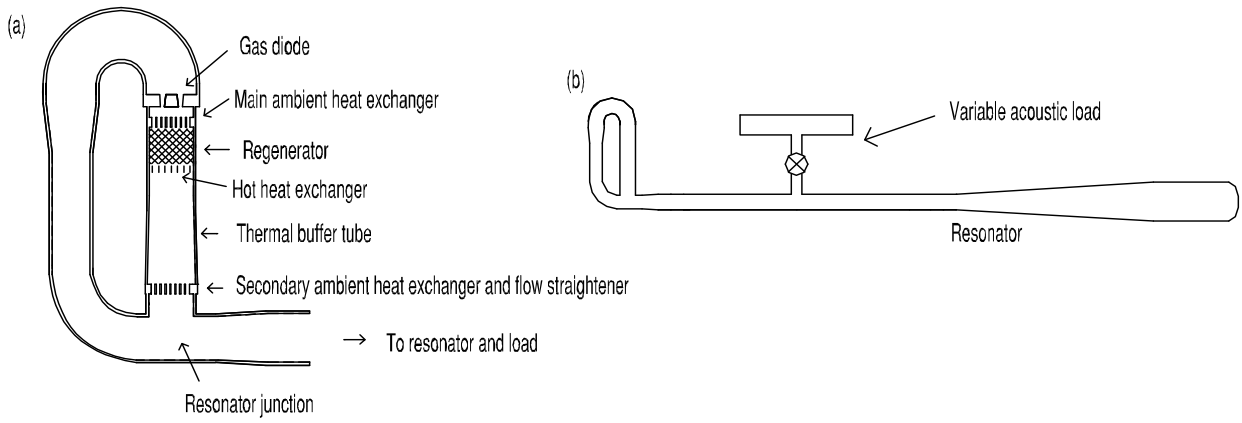


Figure 6.4: Scale drawings of the TASHE. (a) The toroidal part of the TASHE, containing the heat exchangers. (b) Overall view, with the toroidal part at the left end.

Htot choice sets the initial value of $\dot{H}_{\rm tot}$ according to Eq. (6.12). This condition must also sometimes be enforced in subsequent `SOFTEND`s, where an `RPN` that expresses Eq. (6.12) can be used, as illustrated above in segments 11 and 12.

A new input, `FracQN`, has appeared in the `HX` segment now that $\dot{N} \neq 0$. This variable, $F_{\dot{Q}\dot{N}}$, tells DELTAEC what fraction of the heat exchanger's heat $\dot{Q}$ should be allocated to changing energy in the steady flow through the heat exchanger; the remainder is allocated to $\dot{H}_{2,k}$. We often use $F_{\dot{Q}\dot{N}}$ as a guess, to let DELTAEC meet temperature and energy-flow targets. If it is not needed as a guess, it can be set to a simple value like 0 or assigned a varying value like $\left(1 + m|\dot{N}|/\rho_m\,|U_1|\right)^{-1}$ by `sameas`'ing it to the result of an `RPN` calculation.

Output `J` in segment 12 gives $p_{2,0,HL} = -24$ Pa. Since $p_{2,0,HL}$ is initialized to zero in `BEGIN`, this shows that only 24 Pa of steady pressure is needed to push the 5 liters per second through the stack and heat exchanger. To learn how much of that falls across the heat exchanger and how much falls across the stack, we could insert an `RPN` segment between the heat exchanger and the stack, with `p2OHL` in line `b`, or display $p_{2,0,HL}$ in a <`.sp`> plot.

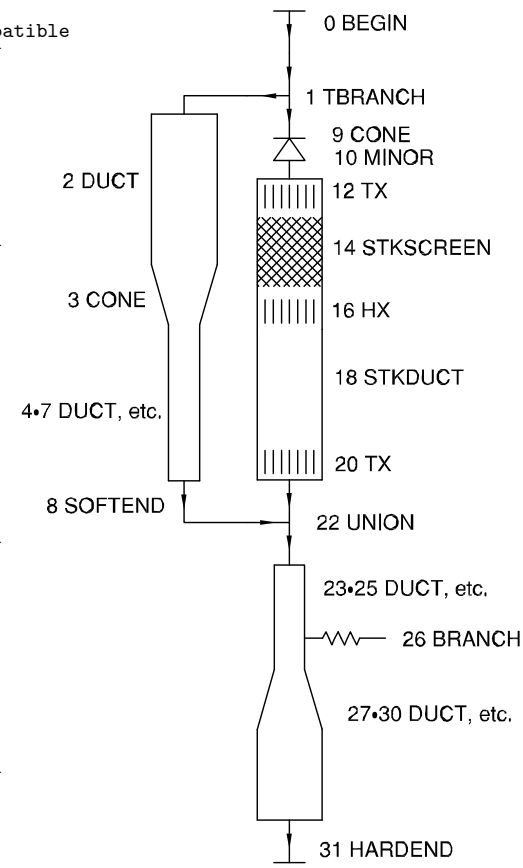
6.3. TASHE

The thermoacoustic-Stirling hybrid engine (TASHE), shown in Fig. 6.4 and described in detail in Refs. [5, 39], will be used to illustrate the use of DELTAEC to design a toroidal thermoacoustic apparatus without Gedeon streaming [40]. Without intelligent design, Gedeon streaming around the loop of the TASHE can convect an enormous amount of heat away from the hot heat exchanger.

We created this DELTAEC model starting from Backhaus's 1998 DELTAE model (Appendix B4 of Ref. [12]; also www.lanl.gov/thermoacoustics/Tashe.out). We removed some inessential segments to reduce the total number, in an effort to save room in the Users Guide. We modified some segment types to bring the model up to date in DELTAEC version 6. For selecting guesses and targets, we have adopted the following perspective: At what

frequency, amplitude, and hot temperature will this engine run, for a given $\dot{Q}_H$ applied at the hot heat exchanger? Ambient temperature and gas mean pressure are given, and hardware dimensions are fixed. In anticipation of adding more targets and guesses later, we make a special effort to minimize their numbers here. Thus, we do not use gas temperature `0c` as a guess and ambient heat-exchanger solid temperature `12f` as a target; we simply set `0c` manually to a value that brings `12H` near 25°C. We also avoid guessing heat-exchanger heat `20e` and targeting `HARDEND` $\dot{H}_{\text{tot}} = 0$ by using an `RPN` calculation at segment 19 to anticipate what the heat-exchanger heat must be based on global first-law considerations. With those simplifications, we still have 31 segments, 5 targets, and 5 guesses. This file is in the <`Examples\SteadyFlow`> folder, as <`tashe1.out`>.

```
!Created@11:41:05  9-Oct-06 with DeltaE Vers. 6.0g2 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
 BEGIN      the setup
  3.1030E+06 a Mean P    Pa
    85.748   b Freq      Hz      G
   325.00    c TBeg      K
  3.1748E+05 d  |p|      Pa      G
     0.0000  e Ph(p)     deg
     0.0000  f  |U|     m^3/s
     0.0000  g Ph(U)     deg
 helium     Gas type
!--------------------------------  1 --------------------------------
 TBRANCH    Split up the flow
 -1.8483E+07 a Re(Zb) Pa-s/m^3   G        3.1748E+05 A |p|       Pa
  1.3735E+07 b Im(Zb) Pa-s/m^3   G           0.0000  B Ph(p)     deg
 -1263.0     c HtotBr   W                 1.3787E-02 C |U|     m^3/s
                                           -143.38    D Ph(U)     deg
                                          -1263.0     E HtotBr    W
                                          -1756.6     F EdotBr    W
                                           1756.6     G EdotTr    W
+  2 DUCT       180 bend plus brass connecting flange (pg 27 book 4)
+  3 CONE       4" to 3" Concentric reducer (pg 36 book 4)
+  4 DUCT       3" FB Duct - Length given in concept.skf
+  5 CONE       3.5" to 3" Long radius reducing elbow (pg 36 book 4)
+  6 MINOR      minor loss, small end of elbow
+  7 DUCT       FB connector/part of tee (Pg 55 book 4 concept.skf)
!--------------------------------  8 --------------------------------
 SOFTEND    End of feedback branch
     0.0000  a Re(z)             (t)      2.6397E+05 A |p|       Pa
     0.0000  b Im(z)             (t)         1.1149  B Ph(p)     deg
     0.0000  c Htot     W        (t)         0.2056  C |U|     m^3/s
                                            -92.896   D Ph(U)     deg
                                          -1263.0     E Htot      W
                                          -1897.7     F Edot      W
                                             -0.1143  G Re(z)
                                              1.6308  H Im(z)
                                            325.00    I   T     K
+  9 CONE       gas diode plate
!-------------------------------- 10 --------------------------------
 MINOR      minor loss here
 sameas   9d a Area      m^2              3.0872E+05 A |p|       Pa
     0.8000  b   K+                         -1.9125  B Ph(p)     deg
  7.0000E-02 c   K-                       1.3576E-02 C |U|     m^3/s
                                            35.646   D Ph(U)     deg
                                          1263.0     E Htot      W
                                          1661.2     F Edot      W
+ 11 DUCT       jetting space
!-------------------------------- 12 --------------------------------
 TX         Main room temp water HX (pg 90 book 3)
  6.6580E-03 a Area      m^2              3.0885E+05 A |p|       Pa
     0.2275  b GasA/A                       -2.0019  B Ph(p)     deg
  2.0400E-02 c Length     m               1.1093E-02 C |U|     m^3/s
  1.2700E-03 d radius     m                 13.880   D Ph(U)     deg
 -1594.9     e HeatIn     W      G        -331.91    E Htot      W
   300.00    f Est-T      K      (t)      1647.6     F Edot      W
                                           325.00    G GasT     K
 stainless  Solid type                     292.83    H SolidT     K
+ 13 DUCT       Regen cold end dead space due to ribs (pg 91 book 3)
!-------------------------------- 14 --------------------------------
 STKSCREEN  Regenerator (pg 92 book 3) (Ks frac est:pg 20 book 4)
  6.2070E-03 a Area      m^2              2.6942E+05 A |p|       Pa
     0.7190  b VolPor                        1.5875  B Ph(p)     deg
  7.3000E-02 c Length     m               3.2517E-02 C |U|     m^3/s
  4.2200E-05 d   rh       m                -38.620   D Ph(U)     deg
     0.3000  e ksFrac                     -331.91    E Htot      W
                                          3345.2     F Edot      W
```



```
                                          325.00    G TBeg       K
 stainless  Solid type                    825.00    H TEnd       K
+ 15 DUCT       All regen hot end dead space (pg 92 book 3)(area is avg)
!--------------------------------- 16 ---------------------------------
 HX         HHX (pg 93 book 4) heat xfer area used/not acoustic area
  5.6970E-03 a Area      m^2              2.6933E+05 A |p|       Pa
     0.9867  b GasA/A                        1.5724  B Ph(p)     deg
  6.3500E-03 c Length     m               3.3918E-02 C |U|     m^3/s
  7.9400E-04 d y0         m                -41.855   D Ph(U)     deg
  3700.0     e HeatIn     W               3368.1     E Htot      W
     0.0000  f Est-T      K      (t)      3317.3     F Edot      W
                                           825.00    G GasT     K
 stainless  Solid type                    917.21    H SolidT     K
+ 17 DUCT       hhx dead space (pg 94 book 3) stainless used for Qdot
+ 18 STKDUCT    Straight section of pulse tube (pg 101 bk 4)
!--------------------------------- 19 ---------------------------------
 RPN        how much heat must be rejected here? Resonator is insulated
     0.0000  a G or T            (t)     -2105.1     A Heat
   16e 12e + ~
!--------------------------------- 20 ---------------------------------
 TX         Small water Xger
  6.6580E-03 a Area      m^2              2.6551E+05 A |p|       Pa
     0.2690  b GasA/A                        1.2238  B Ph(p)     deg
  1.0160E-02 c Length     m               7.5413E-02 C |U|     m^3/s
  2.2860E-03 d radius     m                -69.552   D Ph(U)     deg
 sameas  19A e HeatIn     W               1263.0     E Htot      W
   290.00    f Est-T      K      (t)      3296.6     F Edot      W
                                           325.00    G GasT     K
 ideal      Solid type                    195.21    H SolidT     K
+ 21 DUCT       PT connector (see pg 55 book 4 and concept.skf)
+ 22 UNION      Rejoin
+ 23 DUCT       Section of 3.5" tee
+ 24 CONE       3.5" to 4" cone to adapt to resonator (pg 93 book 4)
+ 25 DUCT       Initial section of resonator
+ 26 BRANCH     RC dissipator
+ 27 DUCT       Continuation of resonator
+ 28 CONE       7 degree cone - 10.02" final diameter
+ 29 DUCT       10" duct                     P8
+ 30 SURFACE    end cap of resonator
!--------------------------------- 31 ---------------------------------
 HARDEND    end of duct
     0.0000  a R(1/z)          = 31G?     9.5273E+04 A |p|       Pa
     0.0000  b I(1/z)          = 31H?     -178.61    B Ph(p)     deg
     0.0000  c Htot     W        (t)      1.0381E-15 C |U|     m^3/s
                                           -82.323   D Ph(U)     deg
                                         -4.5475E-13 E Htot      W
                                         -5.4178E-12 F Edot      W
                                         -1.1441E-16 G R(1/z)
                                          1.0380E-15 H I(1/z)

 GUESS      0b          0d          1a          1b          12e
 name  BEGIN:Freq  BEGIN: |p|  TBRAN:Re(Zb TBRAN:Im(Zb    TX:HeatI
 units     Hz          Pa       Pa-s/m^3    Pa-s/m^3        W
 value    85.748    3.1748E+05 -1.8483E+07  1.3735E+07 -1594.9
TARGET     22b         22c         22d         31a         31b
 name  UNION:|p|SO UNION:Ph(p) UNION:T_SOF HARDE:R(1/z HARDE:I(1/z
 units     Pa          deg         K
 value  2.6397E+05     1.1149    325.00        0.0000      0.0000
result  2.6397E+05     1.1149    325.00    -1.1441E-16  1.0380E-15
```

With 3700 W applied to the hot heat exchanger, the apparatus resonates at 85.7 Hz with an amplitude of 317 kPa at `BEGIN`. The hot gas temperature is 825 K and the hot solid temperature is 917 K.

One strange-looking number appears above: Segment 1, `TBRANCH`, input `c`, -1263 W, `HtotBr`. DeltaEC arrives at the `TBRANCH` with some value for $\dot{H}_{\mathrm{tot}}$. (It is zero in the present example, but not always.) $\dot{H}_{\mathrm{tot,br}}$ tells DeltaEC how much $\dot{H}_{\mathrm{tot}}$ to send into the branch; the balance goes into the trunk. Neither we nor DeltaEC may know the right value for this number in the present model, because the contribution of Rayleigh- and jet-driven streaming to heat convection in the part of the torus that connects main ambient heat exchanger (segment 12) and secondary ambient heat exchanger (segment 20) is unknown. Such streaming helps determine how much of the heat that comes from dissipation of $\dot{E}$ in the load and resonator (assumed insulated here) shows up in each of those two heat exchangers. Fortunately, the performance of the apparatus is insensitive to $\dot{H}_{\mathrm{tot,br}}$: If we change its value

from -1263 W to -1763, the resonance frequency, pressure amplitude, and hot temperature change by less than 0.1%, even though the heat rejected at the main ambient heat exchanger increases by 500 W and that in the secondary ambient heat exchanger decreases by the same amount. In the absence of any evidence to the contrary, it is sometimes reasonable to set

$$\dot{H}_{\mathrm{tot,br}} = \dot{E}_{\mathrm{br}} + \dot{N}_{\mathrm{br}} m w_m, \tag{6.13}$$

and this is available as a check-box option in the `TBRANCH` segment. A third reasonable option here would be to add $\dot{H}_{\mathrm{tot,br}}$ to the guess list, using it to allow DeltaEC to maintain equal solid temperatures in the two ambient-temperature heat exchangers. Since we are trying to minimize the number of guesses and targets right now, we choose the first option.

Now we let DeltaEC begin to consider Gedeon streaming, by checking the "allow steady flow" box in `BEGIN`. This enables nonzero $\dot{N}$ and the calculation and display of $p_{2,0,HL}$ throughout the model. For now, we leave $\dot{N} = 0$ and examine the `UNION` at segment 22:

```
!------------------------------- 22 ----------------------------
UNION      Rejoin
    8.0000  a SegNum                     2.6479E+05 A |p|       Pa
 2.6479E+05 b |p|Sft    Pa    = 22A?        1.1020  B Ph(p)     deg
    1.1020  c Ph(p)S    deg   = 22B?        0.2861  C |U|     m^3/s
  325.00    d TSoft     K     = 22G?      -86.769   D Ph(U)     deg
   -0.6445  e p20HLS    Pa      (t)      2.2737E-13 E Htot      W
                                         1407.3     F Edot      W
                                          325.00    G T         K
                                        -4128.8     H p20HL     Pa
```

"Allowing steady flow" has turned on the calculation of $p_{2,0,HL}$. Parameter `e` here shows that $p_{2,0,HL} = -0.6$ Pa at `SOFTEND` segment 8, which is the end of the `TBRANCH` and is supposedly reconnected to the trunk here at segment 22. However, parameter `H` shows that $p_{2,0,HL} = -4129$ Pa here in the trunk.

There is a mismatch of 4 kPa in second-order time-averaged pressure in this `UNION` segment. We have already targeted p_1 and T_m in the `UNION`, to satisfy acoustic and thermodynamic boundary conditions; now we must add $p_{2,0,HL}$ in the `UNION` to the target list. Our corresponding guess will be $\dot{N}_{\mathrm{br}}$ in segment 1: We ask DeltaEC how much Gedeon streaming will flow around the torus, and what effects that will have on operation of the apparatus.

We make a small increase in $\dot{N}_{\mathrm{br}}$ and watch segment 22, to make sure $p_{2,0,HL}$ changes in the right direction, to make sure we understand what we're doing. We note that DeltaEC gets lost if we change $\dot{N}_{\mathrm{br}}$ by 0.1 mol/s, so we make smaller changes of 0.01 mol/s until $p_{2,0,HL}$ in segments 8 and 22 are nearly matched. Then we add `1d` and `22e` to the guess and target lists, respectively.

Making those changes and running, we notice that the state plot of $T_m(x)$ is not smooth in the thermal buffer tube. We increase the number of steps in the numerical integration of `STKDUCT` (and the other integrated segments), by increasing `Nint`. The irregularities in $T_m(x)$ go away above `Nint` $= 30$. Running with `Nint` $= 40$, the guess–target summary becomes

```
 GUESS      0b          0d          1a          1b          1d          12e
 name  BEGIN:Freq  BEGIN: |p|  TBRAN:Re(Zb TBRAN:Im(Zb TBRAN:Ndotb  TX:HeatI
 units     Hz          Pa       Pa-s/m^3    Pa-s/m^3    mol/s           W
 value    86.111    2.0298E+05 -2.0040E+07  1.3489E+07  0.29709       -3197.8
TARGET     22b         22c         22d         22e         31a         31b
 name  UNION:|p|SO UNION:Ph(p) UNION:T_SOF UNION:p20HL HARDE:R(1/z HARDE:I(1/z
 units     Pa          deg         K           Pa
 value  1.7115E+05     1.0374    325.00       -0.9237      0.0000    0.0000
result  1.7115E+05     1.0374    325.00       -0.9237  -1.2258E-13 -1.4742E-13
```

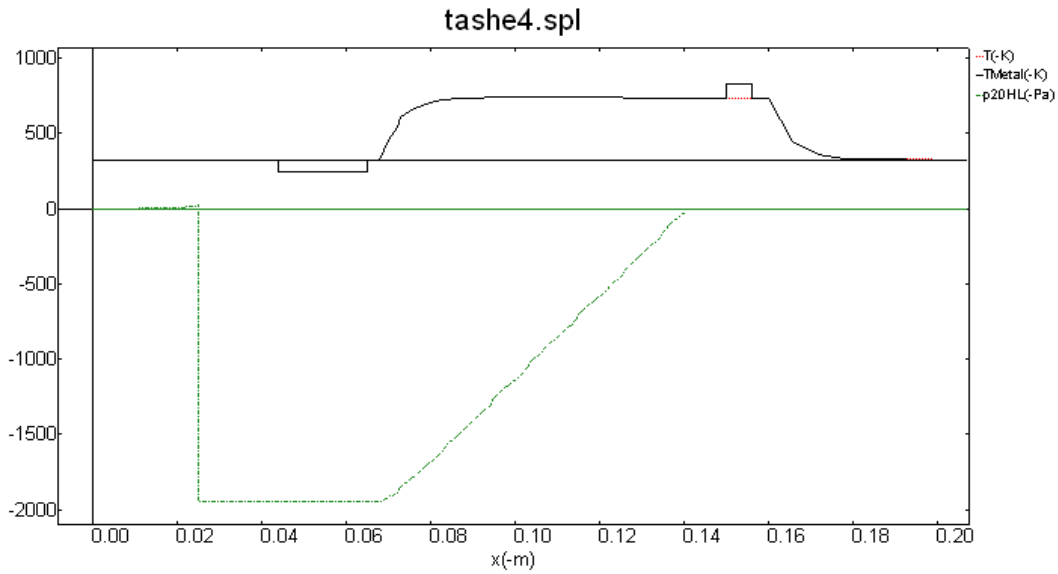


Figure 6.5: State plot of the TASHE example, with steady-flow calculations enabled but Gedeon streaming not properly eliminated. Solid black curve is T_{solid}, dashed red curve is T_m, and dot-dashed green curve is $p_{2,0,HL}$. The locations of the three heat exchangers are evident as short plateaus in T_{solid}.

and the interesting characteristics of the engine become $f = 86$ Hz, $|p_1| = 203$ kPa, $T_{m,H} = 726$ K, and $T_{\text{solid},H} = 816$ K. This estimate of performance is probably much more realistic than the values of 317 kPa, 825 K, and 917 K we found earlier, because now we have included the Gedeon streaming, which is carrying away a lot of heat from the hot heat exchanger without contributing to any thermoacoustic power generation.

To decide what to do about the Gedeon streaming, we can either look at the sign of $\dot{N}_{\text{br}}$ or examine temperatures and $p_{2,0,HL}$ in the state plot shown in Fig. 6.5. The regenerator is running far on the hot side of a linear temperature profile, and $\dot{N}$ is streaming through it from hot to cold. Apparently the gas diode is too tight, overcompensating for the tendency of the regenerator to stream in the direction of $\dot{E}$. No other segments contribute significantly to $p_{2,0,HL}$. Examination of the gas-diode segments,

```
!--------------------------------  9 ------------------------------
CONE       gas diode plate
 5.8880E-04 a AreaI     m^2              2.0362E+05 A |p|       Pa
    0.4210  b PerimI     m                 -0.54813 B Ph(p)     deg
 2.5399E-02 c Length     m               8.2830E-03 C |U|     m^3/s
 1.1400E-04 d AreaF     m^2                 32.908   D Ph(U)     deg
    0.1850  e PerimF     m                 553.06    E Htot      W
 5.0000E-04 f Srough                       703.55    F Edot      W
ideal      Solid type
!-------------------------------- 10 ------------------------------
MINOR      minor loss here
sameas   9d a Area      m^2              2.0012E+05 A |p|       Pa
    0.8000  b   K+                         -1.2126  B Ph(p)     deg
 7.0000E-02 c   K-                       8.2830E-03 C |U|     m^3/s
                                           32.908   D Ph(U)     deg
                                          553.06    E Htot      W
                                          686.12    F Edot      W
```

shows that they are qualitatively in order, with the minor-loss coefficient larger in the $+x$ direction than in the $-x$ direction, as necessary to fight Gedeon streaming. We surmise

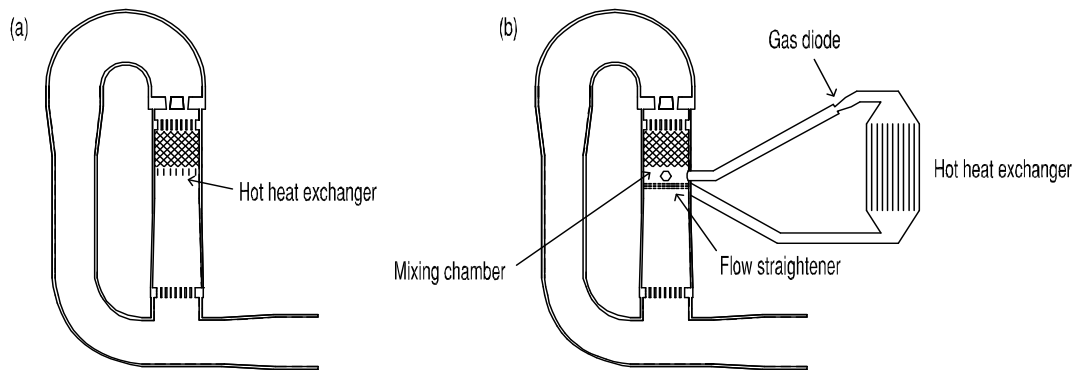


Figure 6.6: (a) The TASHE of the previous section, with a traditional hot heat exchanger between the regenerator and the thermal buffer tube. (b) The same TASHE, with the hot heat exchanger replaced by a mixing chamber, self-circulating loop, and flow straightener.

that the area at the small end of the gas diode, 1.14 cm^2, may be too small. We tentatively increase that area about 1% (leaving its perimeter constant, since this was an adjustable slit in the experimental hardware), and we are pleased to see that DeltaEC converges on a solution with reduced streaming, higher amplitude, and lower hot temperatures. We use the incremental plot to continue increasing that area, watching $\dot{N}_{\rm br}$ and stopping the sequence when it gets close to zero. We then clear $\dot{N}_{\rm br}$ from the guess list and set it equal to zero, and add the gas-diode's small-end area to the guess list. A run then yields <`tashe5.out`> in the <`Examples\SteadyFlow`> folder, which shows that an area of 2.54 cm^2 stops Gedeon streaming, and with this area the apparatus runs with $f = 86$ Hz, $|p_1| = 300$ kPa, $T_{m,H} = 701$ K, and $T_{\rm solid,H} = 791$ K—much better than before. A quick glance at the state plot shows near-linear temperature profiles in the stack and thermal buffer tube, even with `Nint` back down at its default value of 10.

6.4. Self-circulating heat exchanger

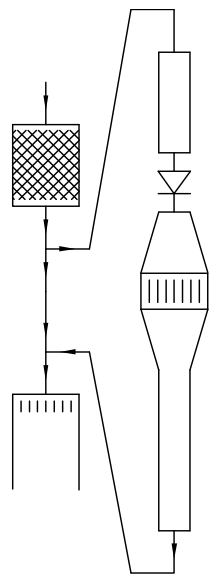
References [8] and [41] describe the use of a gas diode in a resonant loop to induce a large $\dot{N}$ for deliberate heat transfer. This gives us the freedom to design heat exchangers that are not constrained by geometrical proximity to a regenerator or stack. In the present section, we will continue to explore the use of nonzero $\dot{N}$ in DeltaEC by replacing the hot `HX` in the TASHE example of the previous section with a self-circulating hot heat-transfer loop. This replacement is illustrated in Fig. 6.6, which builds on Fig. 6.4. The heat-transfer loop here bears a qualitative resemblance to the system described in Ref. [8], but the quantitative details here are entirely made up and no details have been optimized or even thoughtfully chosen. The purpose of this section is only to illustrate the development of a topologically complicated DeltaEC model.

We began with the realization that this is going to be a complicated model, so initial simplification is important. To begin that simplification, we stripped <`tashe5.out`>, described at the end of the previous section, down to only a few segments in the vicinity of the hot heat exchanger:

```
!--------------------------------  0 --------------------------------
 BEGIN      the setup
  3.1030E+06 a Mean P    Pa
    85.825   b Freq      Hz
   325.00    c TBeg      K
  2.9866E+05 d  |p|      Pa
    -0.9297  e Ph(p)     deg
  1.3553E-02 f  |U|     m^3/s
    13.134   g Ph(U)     deg
  -302.68    h Htot       W
  0.0000     i Ndot    mol/s
 helium     Gas type
+  1 STKSCREEN  Regenerator (pg 92 book 3) (Ks frac est:pg 20 book 4)
+  2 DUCT       All regen hot end dead space (pg 92 book 3)(area is avg)
+  3 HX         HHX (pg 93 book 4) heat xfer area used/not acoustic area
+  4 DUCT       hhx dead space (pg 94 book 3) stainless used for Qdot
+  5 STKDUCT    Straight section of pulse tube (pg 101 bk 4)
```

We set the initial conditions in `BEGIN` to match the conditions of the segment just upstream of the `STKSCREEN` in <`tashe5.out`>, so the integration proceeds through the rest of the model as it did in <`tashe5.out`>, with no need for guesses and targets.

Next, we install a hot self-circulating loop instead of the hot heat exchanger. In place of the `HX`, we insert, as a side-branch loop, in sequence: a quarter wavelength of duct, a gas diode, an adapter cone, a `VXQ1` heat exchanger, another adapter, and a quarter-wavelength return duct. We also insert a flow straightener above the thermal buffer tube. We keep guesses and targets at a minimum: those that are necessary at the `UNION` and `TBRANCH`, and one additional guess–target pair, guessing the input heat and targeting the thermal buffer tube's final temperature.



```
!->E:\DeltaEC\TeXFiles\examples\SteadyFlow\hotloop3.out
!Created@17:18:20  25-Dec-2007 with DeltaEC version 6.1a17 under win32,
using Win 5.1.2600 (Service Pack 2) under Python DeltaEC.
!--------------------------------  0 --------------------------------
BEGIN      the setup
 3.1030E+06 a Mean P Pa
   85.825   b Freq   Hz
  325.00    c TBeg   K
 2.9866E+05 d |p|    Pa
   -0.9297  e Ph(p)  deg
 1.3553E-02 f |U|    m^3/s
   13.134   g Ph(U)  deg
 -302.68    h Htot   W
 1.0000E-20 i Ndot   mol/s
helium           Gas type
!--------------------------------  1 --------------------------------
STKSCREEN  Regenerator (pg 92 book 3) (Ks frac est:pg 20 book 4)
 6.2070E-03 a Area   m^2                 2.5423E+05 A |p|    Pa
    0.7190  b VolPor                        1.9555  B Ph(p)  deg
 7.3000E-02 c Length m                   3.1876E-02 C |U|    m^3/s
 4.2200E-05 d rh     m                    -31.288   D Ph(U)  deg
    0.3000  e ksFrac                    -302.68    E Htot   W
                                        3388.9     F Edot   W
                                         325.00    G TBeg   K
                                         702.66    H TEnd   K
stainless           Solid type          -302.68    I H2k    W
!--------------------------------  2 --------------------------------
DUCT       top half of mixing chamber
 6.2000E-03 a Area   m^2      Mstr       2.5420E+05 A |p|    Pa
    0.2791  b Perim  m        2a            1.9444  B Ph(p)  deg
 1.0000E-02 c Length m                   3.2804E-02 C |U|    m^3/s
                                          -33.708   D Ph(U)  deg
                                        -302.68    E Htot   W
ideal           Solid type              3387.9     F Edot   W
!--------------------------------  3 --------------------------------
TBRANCH    branch into hot loop
 8.8445E+07 a Re(Zb) Pa-s/m^3 G          2.5420E+05 A |p|    Pa
 4.5827E+06 b Im(Zb) Pa-s/m^3 G             1.9444  B Ph(p)  deg
    0.77977 d NdotBr mol/s    G          2.8702E-03 C |U|    m^3/s
                                            -1.0217  D Ph(U)  deg
                                        6890.7     E HtotBr W
                                         364.32    F EdotBr W
                                        3023.6     G EdotTr W
!--------------------------------  4 --------------------------------
DUCT       quarter-wave duct from engine to diode
```

```
 7.0000E-04 a Area   m^2      Mstr       9632.4     A |p|    Pa
 9.3748E-02 b Perim  m        4a         -162.74    B Ph(p)  deg
    4.6000  c Length m                   5.4063E-02 C |U|    m^3/s
 5.0000E-04 d Srough                      -87.786   D Ph(U)  deg
                                         6890.7     E Htot   W
ideal           Solid type                 67.606   F Edot   W
!---------------------------------  5 ---------------------------------
RPN        calculate Borda-Carnot minor loss here
     0.0000 a G or T                    0.18367                   A ChngeMe
1 6a 4a / - sqrd
!---------------------------------  6 ---------------------------------
MINOR      asymmetric flow at gas diode
 4.0000E-04 a Area   m^2                 9324.3     A |p|    Pa
 5.0000E-02 b K+                        -173.83    B Ph(p)  deg
sameas   5A c K-                        5.4063E-02 C |U|    m^3/s
                                         -87.786   D Ph(U)  deg
                                         6890.7     E Htot   W
                                           17.401   F Edot   W
!---------------------------------  7 ---------------------------------
CONE       expansion to plenum
sameas   6a a AreaI  m^2      Mstr       1.9369E+04 A |p|    Pa
 7.0899E-02 b PerimI m        7a         -176.76    B Ph(p)  deg
    0.4000  c Length m                   4.9458E-02 C |U|    m^3/s
sameas   9a d AreaF  m^2      Mstr        -87.861   D Ph(U)  deg
    0.4342  e PerimF m        7d         6890.7     E Htot   W
 5.0000E-04 f Srough                        9.1879  F Edot   W
ideal           Solid type
!---------------------------------  8 ---------------------------------
RPN        watch that angle
    5.0000  a G or T                   16.125                    A ChngeMe
7e 7b - pi / 7c / atan
!---------------------------------  9 ---------------------------------
VXQ1       plenty of room for heat transfer
 1.5000E-02 a Area   m^2                 1.6733E+04 A |p|    Pa
    0.4000  b GasA/A                     -176.06    B Ph(p)  deg
    0.1000  c SolA/A                     5.6533E-02 C |U|    m^3/s
 1.0000E-02 d rh     m                     92.036   D Ph(U)  deg
 1.0000E-02 e LenTS1 m                   9816.2     E Htot   W
    6.0000  f Len P1 m                    -15.696   F Edot   W
 1.0000E-02 g LenTS2 m                    705.49    G TSolBeg P1 K
 2925.4     h HeatP1 W        G           886.08    H TSolEnd P1 K
    1.0000  i FracQN                      364.32    I H2k    W
stainless           Solid type
!--------------------------------- 10 ---------------------------------
CONE       adapt between heat exchanger and quarter-wave tube
sameas   7d a AreaI  m^2                 9914.8     A |p|    Pa
sameas   7e b PerimI m                   -174.13    B Ph(p)  deg
sameas   7c c Length m                   6.0744E-02 C |U|    m^3/s
sameas  11a d AreaF  m^2                   92.157   D Ph(U)  deg
sameas  11b e PerimF m                   9816.2     E Htot   W
 5.0000E-04 f Srough                      -19.515   F Edot   W
ideal           Solid type
!--------------------------------- 11 ---------------------------------
DUCT       quarter wave connection tube
sameas   4a a Area   m^2                 2.5420E+05 A |p|    Pa
sameas   4b b Perim  m                      1.9444  B Ph(p)  deg
    5.0000  c Length m                   4.4245E-03 C |U|    m^3/s
sameas   4d d Srough                      138.07    D Ph(U)  deg
                                         9816.2     E Htot   W
ideal           Solid type               -405.38    F Edot   W
!--------------------------------- 12 ---------------------------------
SOFTEND    end of the branch; this will reconnect to trunk at UNION
     0.0000 a Re(z)                      2.5420E+05 A |p|    Pa
     0.0000 b Im(z)                         1.9444  B Ph(p)  deg
     0.0000 c Htot   W                   4.4245E-03 C |U|    m^3/s
                                          138.07    D Ph(U)  deg
                                         9816.2     E Htot   W
                                         -405.38    F Edot   W
                                           -9.8025  G Re(z)
                                           -9.4244  H Im(z)
                                          883.14    I T      K
                                         1603.6     J p20HL  Pa
!--------------------------------- 13 ---------------------------------
UNION      reconnect the "final" end of the loop here
 12         a SegNum                     2.5420E+05 A |p|    Pa
sameas  12A b |p|Sft Pa       =13A          1.9444  B Ph(p)  deg
sameas  12B c Ph(p)S deg      =13B       2.6026E-02 C |U|    m^3/s
sameas  12I d TSoft  K                    -35.728   D Ph(U)  deg
sameas  12J e p20HLS Pa       =13H       2622.7     E Htot   W
                                         2618.2     F Edot   W
                                          702.66    G T      K
                                         1603.6     H p20HL  Pa
!--------------------------------- 14 ---------------------------------
DUCT       bottom half of mixing chamber
sameas   2a a Area   m^2                 2.5417E+05 A |p|    Pa
```

```
sameas   2b b Perim  m                      1.9358  B Ph(p)  deg
sameas   2c c Length m                   2.7060E-02 C |U|    m^3/s
                                          -38.505   D Ph(U)  deg
                                         2622.7     E Htot   W
ideal           Solid type               2617.3     F Edot   W
!--------------------------------- 15 ---------------------------------
SX         flow straightener (no heat input)
sameas  14a a Area   m^2                 2.5397E+05 A |p|    Pa
    0.6500  b VolPor                        1.9618  B Ph(p)  deg
 5.0000E-03 c Length m                   2.7301E-02 C |U|    m^3/s
 5.0000E-04 d rh     m                    -39.682   D Ph(U)  deg
     0.0000 e HeatIn W                   2622.7     E Htot   W
     0.0000 f SolidT K                   2590.6     F Edot   W
     0.0000 g FracQN                      702.66    G GasT   K
                                          702.66    H SolidT K
ideal           Solid type               2622.7     I H2k    W
!--------------------------------- 16 ---------------------------------
STKDUCT    Straight section of pulse tube (pg 101 bk 4)
 7.0000E-03 a Area   m^2                 2.5136E+05 A |p|    Pa
    0.2963  b Perim  m                      1.6872  B Ph(p)  deg
    0.2400  c Length m                   6.5671E-02 C |U|    m^3/s
 1.1600E-03 d WallA  m^2                  -70.112   D Ph(U)  deg
                                         2622.7     E Htot   W
                                         2578.0     F Edot   W
                                          702.66    G TBeg   K
                                          300.00    H TEnd   K
stainless           Solid type           2622.7     I H2k    W
!--------------------------------- 17 ---------------------------------
RPN        temperature target
  300.00    a G or T          =17A      300.00                    A ChngeMe
Tm


 GUESS      3a          3b          3d          9h
 name  TBRAN:Re(Zb TBRAN:Im(Zb TBRAN:Ndotb  VXQ1:HeatP
 units  Pa-s/m^3    Pa-s/m^3    mol/s          W
 value  8.8445E+07  4.5827E+06      0.77977   2925.4
TARGET     13b         13c         13e       17a
 name  UNION:|p|SO UNION:Ph(p) UNION:p20HL   RPN:Targe
 units     Pa          deg         Pa
 value  2.5420E+05     1.9444   1603.6       300.00
result  2.5420E+05     1.9444   1603.6       300.00
```

Several features that have not been previously encountered or discussed in the Users Guide are displayed in this file. Details about each of these features are given in Chapter 10.

Segment `MINOR` is employed here with unequal minor-loss coefficients, as appropriate for an asymmetrical geometry and as necessary to create a pumping effect to move $\dot{N} \simeq 0.8$ mole/sec around the loop [41]. Figure 6.7 shows the state plot of $p_{2,0,HL}$ around the loop. The rise near $x = 4$ m is the abrupt effect of the gas diode's `MINOR` segment. The gradual decreases elsewhere are the pressure gradients in the `DUCT`s (and, much smaller, in the `VXQ1`), which the gas-diode pump is overcoming. The gradient $dp_{2,0,HL}/dx$ varies along each `DUCT`, despite the constancy of the area and flow rate, because of the nonlinear interaction between U_1 and $\dot{N}$, which increases $dp_{2,0,HL}/dx$ where $|U_1|$ is largest. The nature of that nonlinear interaction is controlled in part by the value of the roughness parameter in `DUCT` segments 4 and 11. As described in detail in Section 10.1.1, the interaction between turbulent steady flow and turbulent oscillating flow is also governed by which of the Optimistic, Average, or Pessimistic boxes are checked in a `DUCT`. The `p2OHL` target in the `UNION` ensures that $p_{2,0,HL}(x = 16.5 \text{ m}) = p_{2,0,HL}(x = 0)$. Recall that the zero of $p_{2,0,HL}$ is set arbitrarily in the `BEGIN` segment, so the vertical offset of this graph is meaningless for the present discussion; it only indicates $\Delta p_{2,0,HL}$ across the regenerator when $\dot{N} = 0$ in the regenerator.

Segment 9, `VXQ1`, is one of the family of "variable" heat exchanger, `VX`, segments. The other types of heat exchanger segments—`HX`, `TX`, `SX`, and `PX`—are assumed to be short, to have x-independent solid temperatures, and to have constant dT_m/dx. The `VX**` segments are numerically integrated with `Nint` steps, so they can be long. `VXT1` and `VXT2` are intended

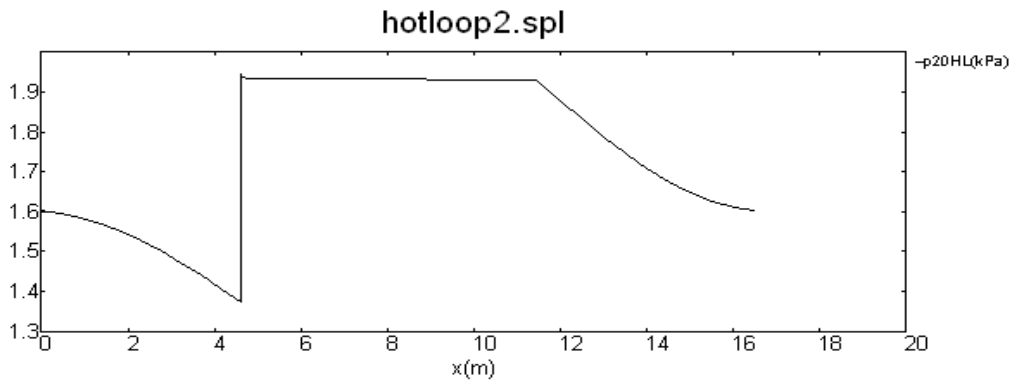


Figure 6.7: Second-order head-loss pressure vs x in the heat-exchange loop. The abrupt rise marks the location of the gas diode.

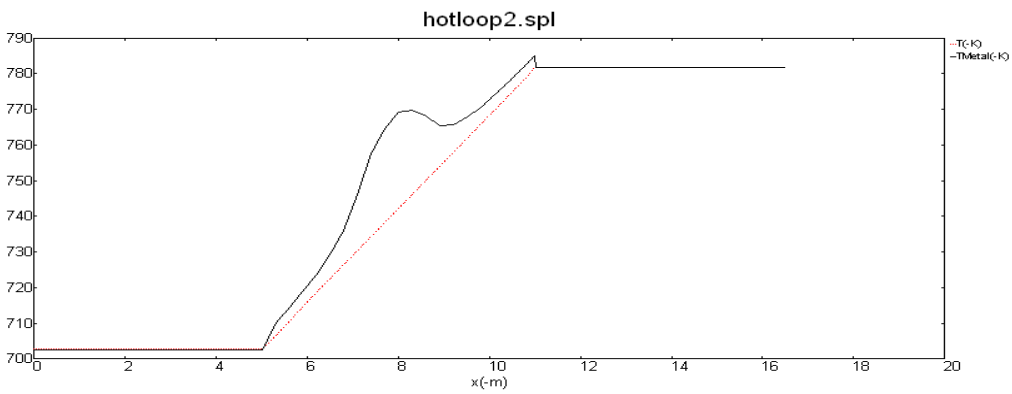


Figure 6.8: Temperatures around the hot loop. Solid black line, solid temperature. Dashed red line, gas temperature. The velocity-dependent gas–solid heat-transfer coefficient in `VXQ1` is evident in the difference between the two curves. (`Nint` = 20 for this plot.)

to model long heat exchangers with shell-side heat transfer such that the solid surfaces are held essentially isothermal. The gas temperature $T_m(x)$ and heat transfer per unit length $\dot{q}(x)$ are obtained numerically in the `VXT**` segments. `VXQ1` and `VXQ2` are intended to model long heat exchangers with shell-side heat transfer such that the heat input per unit length is independent of x. The gas temperature $T_m(x)$ and and solid temperature $T_{\text{solid}}(x)$ are obtained numerically in the `VXQ**` segments. `VXQ2` and `VXT2` allow for two-part heat exchangers, such as heat exchangers with two passes on the shell side. Additional lengths for tubesheets, where no heat transfer occurs, are built into both ends of the `VX**` segments. For more details, see Section 10.7.3. In the present example, Fig. 6.8 shows temperatures around the loop. Outside of the heat exchanger, gas and solid temperatures are equal, as usual. Inside the heat exchanger, T_m increases linearly with x because this is an ideal gas flowing though a `VXQ1` segment, with c_p and $\dot{q}$ independent of x. The difference $T_{\text{solid}} - T_m$ peaks in the center of the `VX`, where $|U_1|$ is the smallest and hence turbulent heat transfer is the least effective at transferring heat from the solid to the gas.

The logistical segments in <`hotloop3.out`> also have a few peculiar features. In the

BEGIN, $\dot{H}_{\rm tot}$ is set to the value that it had just upstream of the regenerator in <tashe5.out>, and $\dot{N}$ is set to a negligible but nonzero number, to turn on $p_{2,0,HL}$ and steady-flow calculations in the rest of the model. $\dot{N}$ is, of course, zero or negligible in the regenerator, since <tashe5.out> properly suppressed Gedeon streaming with its gas diode. However, $\dot{N}_{\rm br}$ in the TBRANCH is large. DeltaEC adjusts this guess to hit the $p_{2,0,HL}$ target at the UNION. We don't know exactly what $\dot{H}_{\rm tot,br}$ should be, because we don't know exactly how much heat might be convected into the loop by $\dot{N}$ here, but the measurements reported in Ref. [8] indicate that any such convected heat is much smaller than what is convected *out* of such a loop at the other end, so we simply set $\dot{H}_{\rm tot,br} = \dot{E}_{\rm br}$ in the TBRANCH. At the UNION, we do not use the temperature target, because this would force the gas temperature at both ends of the loop to be equal, which would contradict the entire purpose of the circulating loop. Instead, we essentially let the 883-K gas coming out of the SOFTEND of the loop pour into the UNION where $T_m = 703$ K. We rely on the energy-conserving feature of DeltaEC's UNION to add the heat picked up in the loop to the $\dot{H}_{\rm tot}$ leaving the UNION.

Next, as shown in Fig. 6.9, we combine <hotloop3.out> and <tashe5.out>, to make a complete DeltaEC model of the TASHE, the hot self-circulating heat-exchange loop, and the resonator. We run <tashe5.out>, Kill its traditional hot heat exchanger, and then copy and paste the appropriate segments from <hotloop3.out>. We fix a few things that DeltaEC did not do correctly, such as renumbering the SOFTEND segment-number pointer in line a of the copied UNION segment. A run fails to converge, so we Restore guessez and then clear all guesses and targets, intending to introduce them one pair at a time. With no guesses and targets, a run shows mostly sensible results, except at the HARDEND at the end of the model, where energy is flowing out but acoustic power is flowing in, in the $-x$ direction. We introduce guesses and targets beginning with the four each that were involved in the hot loop, since they are closest to being on target already. Then we add three of the four UNION targets that were used in <tashe5.out> (skipping the temperature target for the moment), and three corresponding guesses, two $Z_{\rm br}$ in the segment-1 TBRANCH and the third being the slot width in the gas diode in segment 9. At this point, we have 7 targets and 7 guesses, and each change we make is requiring small, gentle steps. We remove the temporary RPN target we had been using in <hotloop3.out> to keep the temperature near ambient at the end of <hotloop3.out>, adding instead the segment-34 UNION temperature target. We add frequency as a guess, and $\mathrm{Im}(1/z_n)$ at the HARDEND as a target. Pause to think. In <tashe5.out>, our perspective was to give the engine a known hot heat input, and observe performance. Here, we are currently guessing the hot heat input, because that is what we did in <hotloop3.out>. Clear that guess, and try guessing the main ambient heat instead. With that change, DeltaEC still tolerates changes to variables reasonably well, given the fact that we have 8 targets and 8 guesses. In <tashe5.out>, we injected 3700 W of hot heat; let's increase the heat in our hot loop to the same value here, for a fair comparison to the earlier model. We begin incremental plotting to higher hot heat, in 5-W steps which are small enough that DeltaEC stays converged at each step. Along the way, we notice that the $\dot{E}$ flowing in at the as-yet-incompletely-targeted HARDEND is steadily decreasing, approaching zero at about 3500 W. We stop there, and add the HARDEND's $\mathrm{Re}[1/z_n]$ to the target list and the BEGIN's $|p_1|$ to the guess list, and resume our increase of $\dot{Q}_H$ up to 3700.

The resulting file, <combo2.out>, appears below. Whereas <tashe5.out> ran at an

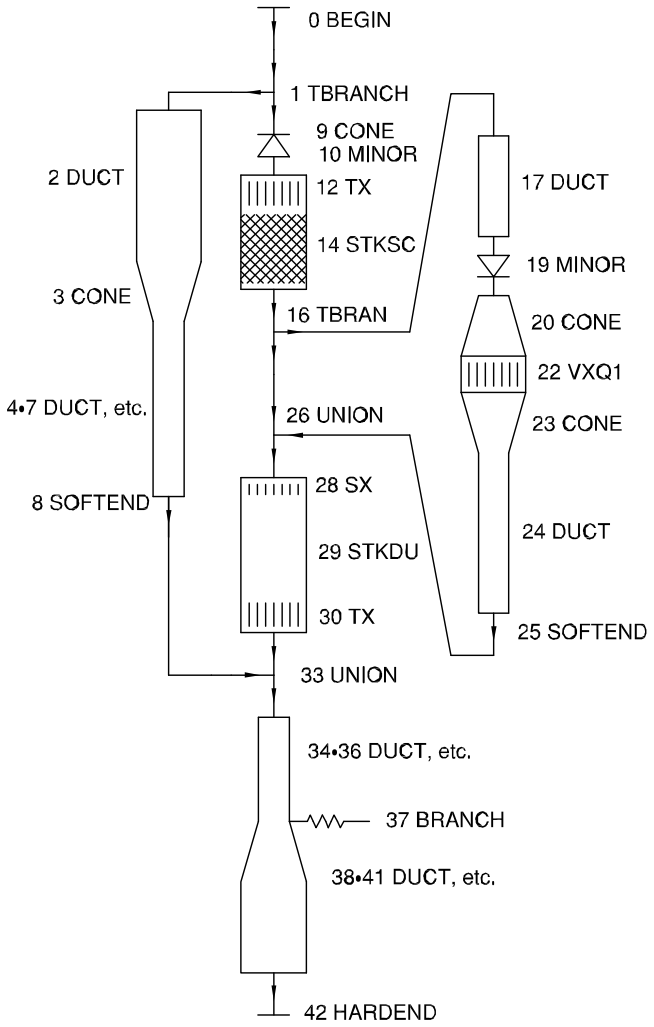


Figure 6.9: Sketch of the <`combo.out`> model. The TASHE feedback loop hangs to the left of the trunk, and the hot self-circulating heat exchanger loop hangs to the right.

amplitude (in segment 0) of 299 kPa, <`combo.out`> runs at 311 kPa. However, <`tashe5.out`> required only $T_{\text{solid},H} = 795$ K; <`combo2.out`> requires solid temperatures from 1079 K to 1171 K.

This example may not be practical, but it does demonstrate the creation of a DeltaEC model for a topologically complicated apparatus with 9 targets and 9 guesses.

```
 GUESS      0b          0d          1a          1b          9d         12e
 name  BEGIN:Freq  BEGIN:|p|   TBRAN:Re(Zb TBRAN:Im(Zb  CONE:AreaF    TX:HeatI
 units  Hz          Pa          Pa-s/m^3    Pa-s/m^3    m^2         W
 value    88.306    3.1131E+05 -1.4974E+07  1.3764E+07  2.1000E-04 -2220.4
 GUESS     16a         16b         16d
 name  TBRAN:Re(Zb TBRAN:Im(Zb TBRAN:Ndotb
 units  Pa-s/m^3    Pa-s/m^3    mol/s
 value  2.8393E+06  1.2187E+07     1.9517
TARGET     26b         26c         26e         33b         33c         33d
 name  UNION:|p|SO UNION:Ph(p) UNION:p20hl UNION:|p|SO UNION:Ph(p) UNION:T_SOF
 units  Pa          deg         Pa          Pa          deg         K
 value  2.5686E+05     1.6651   -0.72644    2.5490E+05     1.2058    325.00
result  2.5686E+05     1.6651   -0.72644    2.5490E+05     1.2058    325.00
TARGET     33e         42a         42b
 name  UNION:p20hl HARDE:R(1/z HARDE:I(1/z
 units  Pa
 value    -0.50122    0.0000      0.0000
result    -0.50122 -1.3244E-14  3.5147E-12


TITLE     Backhaus 1998 TASHE
!->E:\DeltaEC\TeXFiles\examples\SteadyFlow\combo2.out
!Created@17:30:29  25-Dec-2007 with DeltaEC version 6.1a17 under win32,
using Win 5.1.2600 (Service Pack 2) under Python DeltaEC.
!--------------------------------  0 --------------------------------
BEGIN      the setup
 3.1030E+06 a Mean P Pa
   88.306   b Freq   Hz       G
  325.00    c TBeg   K
 3.1131E+05 d |p|    Pa       G
     0.0000 e Ph(p)  deg
     0.0000 f |U|    m^3/s
     0.0000 g Ph(U)  deg
 1.0000E-21 i Ndot   mol/s
helium           Gas type
!--------------------------------  1 --------------------------------
TBRANCH    Split up the flow
-1.4974E+07 a Re(Zb) Pa-s/m^3 G          3.1131E+05 A |p|    Pa
 1.3764E+07 b Im(Zb) Pa-s/m^3 G              0.0000 B Ph(p)  deg
     0.0000 d NdotBr mol/s               1.5306E-02 C |U|    m^3/s
                                         -137.41    D Ph(U)  deg
                                        -1754.0     E HtotBr W
                                        -1754.0     F EdotBr W
                                         1754.0     G EdotTr W
+  2 DUCT       180 bend plus brass connecting flange (pg 27 book 4)
+  3 CONE       4" to 3" Concentric reducer (pg 36 book 4)
+  4 DUCT       3" FB Duct - Length given in concept.skf
+  5 CONE       3.5" to 3" Long radius reducing elbow (pg 36 book 4)
+  6 MINOR      minor loss, small end of elbow
+  7 DUCT       FB connector/part of tee (Pg 55 book 4 concept.skf)
!--------------------------------  8 --------------------------------
SOFTEND    End of feedback branch
     0.0000 a Re(z)                      2.5490E+05 A |p|    Pa
     0.0000 b Im(z)                         1.2058  B Ph(p)  deg
     0.0000 c Htot   W                      0.20875 C |U|    m^3/s
                                          -92.879   D Ph(U)  deg
                                        -1754.0     E Htot   W
                                        -1895.0     F Edot   W
                                        -0.11073    G Re(z)
                                            1.5506  H Im(z)
                                          325.00    I T      K
                                        -0.50122    J p20HL  Pa
!--------------------------------  9 --------------------------------
CONE       gas diode plate
 5.8880E-04 a AreaI  m^2                 3.1278E+05 A |p|    Pa
    0.4210  b PerimI m                  -0.43187    B Ph(p)  deg
 2.5399E-02 c Length m                   1.5039E-02 C |U|    m^3/s
 2.1000E-04 d AreaF  m^2      G            41.660   D Ph(U)  deg
    0.1850  e PerimF m                   1754.0     E Htot   W
 5.0000E-04 f Srough                     1745.2     F Edot   W
ideal            Solid type
!-------------------------------- 10 --------------------------------
MINOR      minor loss here
sameas   9d a Area   m^2                 3.0956E+05 A |p|    Pa
    0.8000  b K+                        -0.97177    B Ph(p)  deg
 7.0000E-02 c K-                         1.5039E-02 C |U|    m^3/s
                                           41.660   D Ph(U)  deg
```

```
                                        1754.0     E Htot   W
                                        1712.5     F Edot   W
!-------------------------------- 11 --------------------------------
DUCT       jetting space
 6.8250E-03 a Area   m^2                3.0962E+05 A |p|    Pa
    0.2930  b Perim  m                 -0.98637    B Ph(p)  deg
 1.9000E-02 c Length m                 1.2515E-02 C |U|    m^3/s
 5.0000E-04 d Srough                     26.993   D Ph(U)  deg
                                        1754.0     E Htot   W
ideal           Solid type              1711.0     F Edot   W
!-------------------------------- 12 --------------------------------
TX         Main room temp water HX (pg 90 book 3)
 6.6580E-03 a Area   m^2                3.0978E+05 A |p|    Pa
    0.2275  b GasA/A                      -1.0673  B Ph(p)  deg
 2.0400E-02 c Length m                 1.1962E-02 C |U|    m^3/s
 1.2700E-03 d radius m                   22.469   D Ph(U)  deg
-2220.4     e HeatIn W        G         -466.4     E Htot   W
  300.00    f SolidT K                  1698.6     F Edot   W
     0.0000 g FracQN                     325.00    G GasT   K
                                         280.87    H SolidT K
stainless           Solid type          -466.4     I H2k    W
!-------------------------------- 13 --------------------------------
DUCT       Regen cold end dead space due to ribs (pg 91 book 3)
 4.9700E-03 a Area   m^2                3.0978E+05 A |p|    Pa
    0.7400  b Perim  m                    -1.0706  B Ph(p)  deg
 3.1750E-03 c Length m                 1.1757E-02 C |U|    m^3/s
                                         20.116   D Ph(U)  deg
                                        -466.4     E Htot   W
stainless           Solid type          1698.0     F Edot   W
!-------------------------------- 14 --------------------------------
STKSCREEN  Regenerator (pg 92 book 3) (Ks frac est:pg 20 book 4)
 6.2070E-03 a Area   m^2                2.5688E+05 A |p|    Pa
    0.7190  b VolPor                       1.6746  B Ph(p)  deg
 7.3000E-02 c Length m                 3.8856E-02 C |U|    m^3/s
 4.2200E-05 d rh     m                  -26.444   D Ph(U)  deg
    0.3000  e ksFrac                    -466.4     E Htot   W
                                        4401.7     F Edot   W
                                         325.00    G TBeg   K
                                        1074.8     H TEnd   K
stainless           Solid type          -466.4     I H2k    W
!-------------------------------- 15 --------------------------------
DUCT       top half of mixing chamber
 6.2000E-03 a Area   m^2      Mstr      2.5686E+05 A |p|    Pa
    0.2791  b Perim  m        15a          1.6651  B Ph(p)  deg
 1.0000E-02 c Length m                 3.9686E-02 C |U|    m^3/s
 6.0000E-04 d Srough                    -28.642   D Ph(U)  deg
                                        -466.4     E Htot   W
stainless           Solid type          4400.3     F Edot   W
!-------------------------------- 16 --------------------------------
TBRANCH    branch into hot loop
 2.8393E+06 a Re(Zb) Pa-s/m^3 G        2.5686E+05 A |p|    Pa
 1.2187E+07 b Im(Zb) Pa-s/m^3 G           1.6651  B Ph(p)  deg
  606.94    c HtotBr W                 2.0526E-02 C |U|    m^3/s
    1.9517  d NdotBr mol/s    G          -75.22    D Ph(U)  deg
                                         606.94    E HtotBr W
                                         598.14    F EdotBr W
                                        3802.1     G EdotTr W
+ 17 DUCT       quarter-wave duct from engine to diode
+ 18 RPN        calculate Borda-Carnot minor loss here
+ 19 MINOR      asymmetric flow at gas diode
+ 20 CONE       expansion to plenum
+ 21 RPN        watch that angle
!-------------------------------- 22 --------------------------------
VXQ1       plenty of room for heat transfer
 1.5000E-02 a Area   m^2                2.4607E+04 A |p|    Pa
    0.4000  b GasA/A                    -174.95    B Ph(p)  deg
    0.1000  c SolA/A                    6.6004E-02 C |U|    m^3/s
 1.0000E-02 d rh     m                    92.200   D Ph(U)  deg
 1.0000E-02 e LenTS1 m                  4306.9     E Htot   W
    6.0000  f Len P1 m                   -40.422   F Edot   W
 1.0000E-02 g LenTS2 m                  1078.9     G TSolBeg P1 K
 3700.0     h HeatP1 W                  1170.6     H TSolEnd P1 K
    1.0000  i FracQN                   -3.0825E+04 I H2k    W
stainless           Solid type
+ 23 CONE       adapt between heat exchanger and quarter-wave tube
+ 24 DUCT       quarter wave connection tube
!-------------------------------- 25 --------------------------------
SOFTEND    end of the branch; this will reconnect to trunk at UNION
     0.0000 a Re(z)                     2.5686E+05 A |p|    Pa
     0.0000 b Im(z)                        1.6651  B Ph(p)  deg
     0.0000 c Htot   W                  1.7790E-02 C |U|    m^3/s
                                         107.09    D Ph(U)  deg
                                        4306.9     E Htot   W
                                        -607.82    F Edot   W
                                          -1.0446  G Re(z)
```

```
                                          -3.7851  H Im(z)
                                         1166.0     I T      K
                                        -0.72644    J p2OHL  Pa
!-------------------------------- 26 --------------------------------
UNION      reconnect the "final" end of the loop here
 25         a SegNum                    2.5686E+05 A |p|    Pa
sameas  25A b |p|Sft Pa       =26A          1.6651  B Ph(p)  deg
sameas  25B c Ph(p)S deg      =26B       3.0191E-02 C |U|    m^3/s
sameas  25I d TSoft  K                     36.195   D Ph(U)  deg
sameas  25J e p2OHLS Pa       =26H       3233.6     E Htot   W
                                         3194.3     F Edot   W
                                         1074.8     G T      K
                                        -0.72644    H p2OHL  Pa
!-------------------------------- 27 --------------------------------
DUCT       bottom half of mixing chamber
sameas  15a a Area   m^2                2.5688E+05 A |p|    Pa
sameas  15b b Perim  m                     1.6581  B Ph(p)  deg
sameas  15c c Length m                  2.9241E-02 C |U|    m^3/s
                                           33.430   D Ph(U)  deg
                                         3233.6     E Htot   W
ideal           Solid type               3192.9     F Edot   W
!-------------------------------- 28 --------------------------------
SX         flow straightener (no heat input)
sameas  27a a Area   m^2                2.5674E+05 A |p|    Pa
    0.6500  b VolPor                       1.6303  B Ph(p)  deg
 5.0000E-03 c Length m                  2.8634E-02 C |U|    m^3/s
 5.0000E-04 d rh     m                     32.328   D Ph(U)  deg
     0.0000 e HeatIn W                  3233.6     E Htot   W
     0.0000 f SolidT K                  3160.6     F Edot   W
     0.0000 g FracQN                    1074.8     G GasT   K
                                        1074.8     H SolidT K
ideal           Solid type              3233.6     I H2k    W
!-------------------------------- 29 --------------------------------
STKDUCT    Straight section of pulse tube (pg 101 bk 4)
 7.0000E-03 a Area   m^2      Mstr       2.5617E+05 A |p|    Pa
    0.2963  b Perim  m        29a           1.3980  B Ph(p)  deg
    0.2400  c Length m                  3.9947E-02 C |U|    m^3/s
 1.1600E-03 d WallA  m^2                  -50.915   D Ph(U)  deg
                                         3233.6     E Htot   W
                                         3128.1     F Edot   W
                                         1074.8     G TBeg   K
                                          325.00    H TEnd   K
stainless           Solid type          3233.6     I H2k    W
!-------------------------------- 30 --------------------------------
RPN        how much heat must be rejected here? Resonator is insulated
     0.0000 a G or T                    -1479.6                    A ChngeMe
22h 12e + ~
!-------------------------------- 31 --------------------------------
TX         Small water Xger
 6.6580E-03 a Area   m^2                2.5567E+05 A |p|    Pa
    0.2690  b GasA/A                       1.3207  B Ph(p)  deg
 1.0160E-02 c Length m                  4.0349E-02 C |U|    m^3/s
 2.2860E-03 d radius m                   -51.394   D Ph(U)  deg
sameas  30A e HeatIn W                  1754.0     E Htot   W
  290.00    f SolidT K                  3124.6     F Edot   W
     0.0000 g FracQN                     325.00    G GasT   K
                                         235.11    H SolidT K
ideal           Solid type              1754.0     I H2k    W
!-------------------------------- 32 --------------------------------
DUCT       PT connector (see pg 55 book 4 and concept.skf)
 6.2070E-03 a Area   m^2                2.5490E+05 A |p|    Pa
    0.2790  b Perim  m                     1.2058  B Ph(p)  deg
 5.1000E-02 c Length m                  4.7540E-02 C |U|    m^3/s
 6.0000E-04 d Srough                      -57.78   D Ph(U)  deg
                                         1754.0     E Htot   W
stainless           Solid type          3121.9     F Edot   W
!-------------------------------- 33 --------------------------------
UNION      Rejoin
  8         a SegNum                    2.5490E+05 A |p|    Pa
sameas   8A b |p|Sft Pa       =33A          1.2058  B Ph(p)  deg
sameas   8B c Ph(p)S deg      =33B          0.24915 C |U|    m^3/s
sameas   8I d TSoft  K        =33G        -86.58    D Ph(U)  deg
sameas   8J e p2OHLS Pa       =33H      -3.2287E-11 E Htot   W
                                         1226.9     F Edot   W
                                          325.00    G T      K
                                        -0.50122    H p2OHL  Pa
+ 34 DUCT       Section of 3.5" tee
+ 35 CONE       3.5" to 4" cone to adapt to resonator (pg 93 book 4)
+ 36 DUCT       Initial section of resonator
+ 37 BRANCH     RC dissipator
+ 38 DUCT       Continuation of resonator
+ 39 CONE       7 degree cone - 10.02" final diameter
+ 40 DUCT       10" duct                    P8
+ 41 SURFACE    end cap of resonator
!-------------------------------- 42 --------------------------------
```

```
HARDEND    end of duct
     0.0000 a R(1/z)          =42G       8.7575E+04 A |p|    Pa
     0.0000 b I(1/z)          =42H       -178.58    B Ph(p)  deg
     0.0000 c Htot   W                   1.5638E-12 C |U|    m^3/s
     0.0000 d Ndot   mol/s                -88.448   D Ph(U)  deg
                                        -3.2287E-11 E Htot   W
                                        -1.6197E-10 F Edot   W
                                        -4.0481E-15 G R(1/z)
                                         1.7114E-12 H I(1/z)
                                             0.0000 I Ndot   mol/s
```

7. Mixture separation

A sound wave in a binary gas mixture in a channel causes time-averaged molar fluxes of the two components of the mixture in opposite directions, and can create a significant mole-fraction gradient along the channel. Oscillating lateral thermal diffusion causes the light and heavy components of the mixture to take turns enriching the central region of the channel, where the oscillating motion is the greatest; the time phasing is best when the oscillating enrichment and oscillating velocity are in phase, causing the velocity oscillations to carry light-enriched gas in one direction and heavy-enriched gas in the other direction [42, 43, 44]. The light component usually flows in the direction of $\dot{E}$.

The thermoacoustic mixture-separation mole flux and mole-fraction gradient are in some ways analogous to the time-averaged enthalpy flux and temperature gradient that a sound wave creates in a stack in a thermoacoustic refrigerator.

DeltaEC can model this effect accurately [10].

7.1. Principles of calculation

Consider a binary gas mixture flowing steadily in the x direction through a channel, with a molar flow rate $\dot{N}$. Let the mole fraction of the lighter component be n_L and let its molar flow rate be $\dot{N}_L$; similarly n_H and $\dot{N}_H$ for the heavy component. It is always true that

$$n_L + n_H = 1 \tag{7.1}$$

and

$$\dot{N}_L + \dot{N}_H = \dot{N}. \tag{7.2}$$

In a simple "bulk" flow of the gas mixture (i.e., flow that does not involve mass diffusion, mole-fraction gradients, or thermoacoustic mixture separation), it is also true that

$$\dot{N}_L = n_L \dot{N}. \tag{7.3}$$

Combining Eqs. (7.1)–(7.3) algebraically shows that

$$\dot{N}_H = n_H \dot{N} \tag{7.4}$$

is also true in a bulk flow, as expected from the symmetry of the equations. Thus, bulk flow in a binary mixture can be uniquely described by assigning values to just *two* variables, which can be chosen as $\dot{N}$ and n_L.

Nontrivial flows arise in binary mixtures whenever mass diffusion or thermoacoustic mixture separation occurs. For example, for $\dot{N} = 0$ and no thermoacoustics, a nonzero $\dot{N}_L$

and an equal and opposite $\dot{N}_H$ arise from ordinary diffusion in the x direction whenever a mole-fraction gradient $dn_L/dx \equiv -dn_H/dx$ exists; Eq. (7.3) is obviously not satisfied. Thus, DeltaEC must routinely handle nontrivial mixture flows, which must be specified by three independent variables. DeltaEC takes those three variables to be $\dot{N}$, $\dot{N}_L$, and n_L.[1]

For $dT_m/dx = 0$, DeltaEC allows mole fraction to evolve with x in segment types `MIXBL` (boundary-layer approximation), `MIXSLAB` (between parallel plates), and `MIXCIRC` (circular channel), according to the theory described in Ref. [10] and references therein. Nonzero dn_L/dx is supported for three of DeltaEC's gas mixtures: He-Ar, He-Xe, and Ne-Xe. The user can create an external file to specify the properties of other gas mixtures, as described in Section 12.15.2. The equations being integrated in these `MIX**` segments are

$$\frac{dp_1}{dx} = \mathcal{F}_{\text{momentum}}(p_1, U_1, T_m;\ p_m, \omega;\ \text{geometry, gas properties, etc.}), \tag{7.5}$$

$$\frac{dU_1}{dx} = \mathcal{F}_{\text{continuity}}(p_1, U_1, T_m;\ p_m, \omega; \text{geometry, gas properties, etc.}), \tag{7.6}$$

$$\frac{dT_m}{dx} = 0, \tag{7.7}$$

$$\frac{dn_L}{dx} = \mathcal{F}_{\text{mix sep}}(p_1, U_1, T_m, \dot{N},\ \dot{N}_L;\ p_m, \omega; \text{geometry, gas properties, etc.}). \tag{7.8}$$

The similar segment types `MIXTBL`, `MIXTSLAB`, and `MIXTCIRC` allow $dT_m/dx \neq 0$, so the equations being integrated in these segments are

$$\frac{dp_1}{dx} = \mathcal{F}_{\text{momentum}}(p_1, U_1, T_m;\ p_m, \omega;\ \text{geometry, gas properties, etc.}), \tag{7.9}$$

$$\frac{dU_1}{dx} = \mathcal{F}_{\text{continuity}}(p_1, U_1, T_m;\ p_m, \omega; \text{geometry, gas properties, etc.}), \tag{7.10}$$

$$\frac{dT_m}{dx} = \mathcal{F}_{\text{energy}}(p_1, U_1, T_m;\ p_m, \omega; \text{geometry, gas properties, etc.}), \tag{7.11}$$

$$\frac{dn_L}{dx} = \mathcal{F}_{\text{mix sep}}(p_1, U_1, T_m, \dot{N},\ \dot{N}_L;\ p_m, \omega; \text{geometry, gas properties, etc.}). \tag{7.12}$$

In DeltaEC's `BEGIN` segment, we specify $\dot{N}$ and n_L. If the "bulk flow" box is checked, DeltaEC calculates $\dot{N}_L$ using Eq. (7.3); if the bulk-flow box is not checked, the user must specify $\dot{N}_L$.

Continuity of n_L between segments is enforced. $\dot{N}$ and $\dot{N}_L$ split up at `TBRANCH`s, add at `UNION`s, and are independent of x in all physical segments.

7.2. Boundary conditions

Deciding how to handle $\dot{N}$, $\dot{N}_L$, and n_L at `BEGIN`s, `SOFTEND`s, etc. is subtle and depends on exactly what the user wants to model. A feedstock with a given n_L might be injected into

[1] DeltaEC focuses on the light component because DeltaEC's first mixture, He–Ar, was set up many years ago with its input parameter representing the helium fraction. This convention causes a minor bookkeeping nuisance now, since Refs. [10, 42, 43, 44] focus on the heavy component.

the middle or end of a mixture separator whose vigorous thermoacoustic processes maintain a locally different n_L; in this case, n_L might be regarded as discontinuous [10], and Eq. (7.3) would not apply if the local thermoacoustic channel's n_L were used, even while the feedstock flow itself is bulk [i.e., the feedstock mole fraction and feedstock flows satisfy Eq. (7.3)]. In other circumstances, one location of the mixture separator might be firmly connected to an essentially infinite bath of feedstock, so n_L would be maintained by *net* molar or partial-molar flows to or from the bath that might not satisfy Eq. (7.3). If a `**END` represents a location where the acoustics stops and an enriched product flows out of the system, Eq. (7.3) must apply to the outflow.

Here are several possible situations, with discussions of how to initialize variables in `BEGIN` and how to choose guesses and targets for each of them in DeltaEC. We only discuss the mixture-specific guesses and targets here; realistic mixture-separation models typically also include guesses and targets related to p_1 and U_1.

Sealed system

The steady-state conditions of Refs. [42, 43] represent sealed systems with $\dot{N} = 0$ everywhere. Those experiments had no steady inflow or outflow, but were constrained by the mole fractions present in the original charge of gas mixture, so getting the right answer from DeltaEC depends on constraining DeltaEC's solution with an integral of n_L over the volume of the apparatus to match the total number of moles of light component that was present in the original charge. Getting the correct answer to that integral would be the target corresponding to the guessed value of n_L in the `BEGIN`. In practice, we might approximate such an integral in DeltaEC with a simple, volumetrically weighted average of values at a small number of locations. The `BEGIN` should have $\dot{N} = 0$ (and, by default, $\dot{N}_L = 0$), because nothing flows in or out there.

Firm connection to feedstock bath, no steady flow

If one end of a mixture-separation channel is firmly connected to a feedstock bath, then the system could be modeled starting at that end with a `BEGIN` having a known mole fraction and a guessed $\dot{N}_L$, and integrate along the channel. If the `BEGIN` is not at a location where the system is firmly connected to a feedstock bath, then n_L could be guessed at the `BEGIN` and the mole fraction could be targeted wherever the system is firmly connected to a feedstock bath.

Inject feedstock at one end, withdraw product at the other end

Now consider a slow injection of feedstock at one end of the acoustic separation channel, and a removal of product at the other end, through capillaries [9]. When the system reaches steady state, the flow variables at the exit— $\dot{N}_L$ and $\dot{N}$ —must equal their incoming values. Yet, if the flow is weak enough, we feel confident that a mole-fraction gradient must exist from end to end. Thus, the flow cannot satisfy $\dot{N}_L = n_L\dot{N}$ at both ends. What happens? The exit flow is bulk, the mole fraction in the exit flow must be the same as the mole fraction

in the feedstock, and the feedstock mixes abruptly into a locally different mole fraction at the feedstock-end of the apparatus.

To model that situation with DeltaEC, the easiest approach is to put the `BEGIN` at the exit end, because the exit flow is bulk so we have fewer unknowns to guess there. Suppose we know the incoming $\dot{N}$ and n_L. We use these values in the `BEGIN` (with a minus sign for the molar flux), check the "bulk flow" box to set $\dot{N}_L$ appropriately, and integrate straightforwardly to the other end of the channel, where we learn what the mole fraction in the channel is where the feedstock mixes into it. This approach is the easiest because it involves only two segments—`BEGIN` and `MIXBL`—and it involves no targets or guesses.

A seemingly more logical and readable approach to this situation is to begin at the feedstock end, but then we need more DeltaEC baggage, to work around the fact that we know less information *a priori* at that end:

0. `BEGIN` with the known p_1, U_1, feedstock $\dot{N}$, and feedstock n_L. Since the feedstock capillary flow is bulk, check the "bulk" box in the `BEGIN` segment to establish the correct $\dot{N}_L$.

1. `RPN` with `1a` a guessed mole fraction at the feedstock end of the separation channel and `1b` having `inp =nL .`

2. `MIXCIRC`.

3. `RPN` expressing the bulk condition, Eq. (7.3), for the exit flow, and target it. This target is associated with the guess in segment 1. For example, put `0.` in `3a` and "`nL Ndot * NLdot - `" in `3b`.

Feedstock in the middle of a long tube, products extracted at both ends

Let's assume that we know the molar flow rate and mole fraction of the feedstock injected into the middle of a long separation tube, and that external control of the capillary flow impedances forces a known fraction of that molar flow out the top and the remainder out the bottom. We'll begin at one of the places where the steady flow goes out, because we can count on bulk flow there. This situation is modeled in Ref. [10] and shown in the example in Section 7.3 below.

0. `BEGIN` with the acoustic conditions at one end of the channel, with guessed n_L. Set $\dot{N}$ equal to the known rate of product extraction here. Check the "bulk" box in the `BEGIN` segment so that $\dot{N}_L$ is automatically assigned the appropriate, bulk value at this product-outflow location.

1. `MIXBL` for the first part of the channel.

2. `RPN` with the helium mole fraction of the injected feedstock in `2a`. (not targeted!)

3. `RPN` with the injected molar flow rate in `3a` and with the following instructions in `3b` to add the correct increment to both $\dot{N}$ and $\dot{N}_L$: `3a Ndot + =Ndot 3a 2a * NLdot + =NLdot`

4. `MIXBL` for the second part of the channel.

At this point, we have the correct $\dot{N}$ and $\dot{N}_L$, but we are not necessarily guaranteed the bulk condition needed for outflow into a capillary. Hence, we need the next segment:

5. `RPN` expressing the bulk condition, Eq. (7.3), and target it. For example, put `0.` in `5a` and "` nL Ndot * NLdot - Ndot / `" in `5b`. (We often multiply the bulk-flow-enforcement

expression $0 = n_L\dot{N} - \dot{N}_L$ by a large number, or divide it by $\dot{N}$, to give DeltaEC a weighty enough target.) This target corresponds to the guessed value of n_L in segment 0.

Firm connection to feedstock bath in the middle of a long tube, products extracted at both ends

This time, let's assume that we know the two exiting molar flow rates and the mole fraction of the feedstock bath in the middle. The feedstock bath must supply $\dot{N}$ and $\dot{N}_L$ in the center, but not necessarily in bulk proportions.

In hardware, how might we make a firm connection to a fixed mole fraction in the middle of a tube? We could have two capillaries there, and pump a huge mole flux in to one of them and a nearly equal mole flux at a slightly different mole fraction out of the other one. The net flow—the sum of these two—is not a bulk flow.

0. `BEGIN` at one end of the channel with a guessed n_L and the known $\dot{N}$. Check the "bulk flow" box to get the known $\dot{N}_L$, because this is a location of product outflow.

1. `MIXBL` for the first part of the channel.

2. `RPN` Target the feedstock bath's mole fraction here, with the target value in `2a` and `nL` in `2b`.

3. `RPN` Guess the incremental $\dot{N}$ in `3a`. Add the incremental $\dot{N}$ and $\dot{N}_L$ to the separation tube in `3b` using `inp Ndot + =Ndot inp 2a * NLdot + =NLdot` .

4. `MIXBL` for the second part of the channel.

5. `RPN` expressing the bulk condition, Eq. (7.3), and target it. For example, put `0.` in `5a` and `nL Ndot * NLdot - Ndot /` in `5b`.

We have 2 targets (segments 2, 5) and 2 guesses (segments 0, 3).

7.3. Continuous He–Ar separator

Guidance for choosing the guesses and targets related specifically to mixture separation was given above. Here is an extended example from Ref. [10] including the targets and guesses relating to the acoustic wave. The apparatus is shown in Fig. 7.1.

First, we use DeltaEC to predict the shape of $n_H(x)$ for steady flow injected at the tube midpoint and extracted at the ends, for one particular operating point, namely

$\dot{N}_{\text{feedstock}} = 1.59\ \mu$mol/s of 50–50 He–Ar at $x = 0.5$ m,
$\dot{N}_{\text{product}} = 20\% \times 1.59\ \mu$mol/s at $x = 0.0$ m,
$\dot{N}_{\text{product}} = 80\% \times 1.59\ \mu$mol/s at $x = 1.0$ m,
$|p_1| = 3.00$ kPa at $x = 0.50$ m,
$z = Ap_1/U_1 = \rho_m a$, purely real, at $x = 0.5$ m.

The DeltaEC model for the hardware dimensions given in Ref. [10] and the conditions listed above uses the mixture-specific guess and target described above under the heading "Feedstock in the middle of a long tube, products extracted at both ends," plus three additional guesses and targets to enforce the acoustic conditions specified in that paper. The <`.out`> file is

```
TITLE     planned first expt in fy 06
!---05081220.out
!Created@14:57:50 28-Oct-06 with DeltaE Vers. 6.0g2 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
```

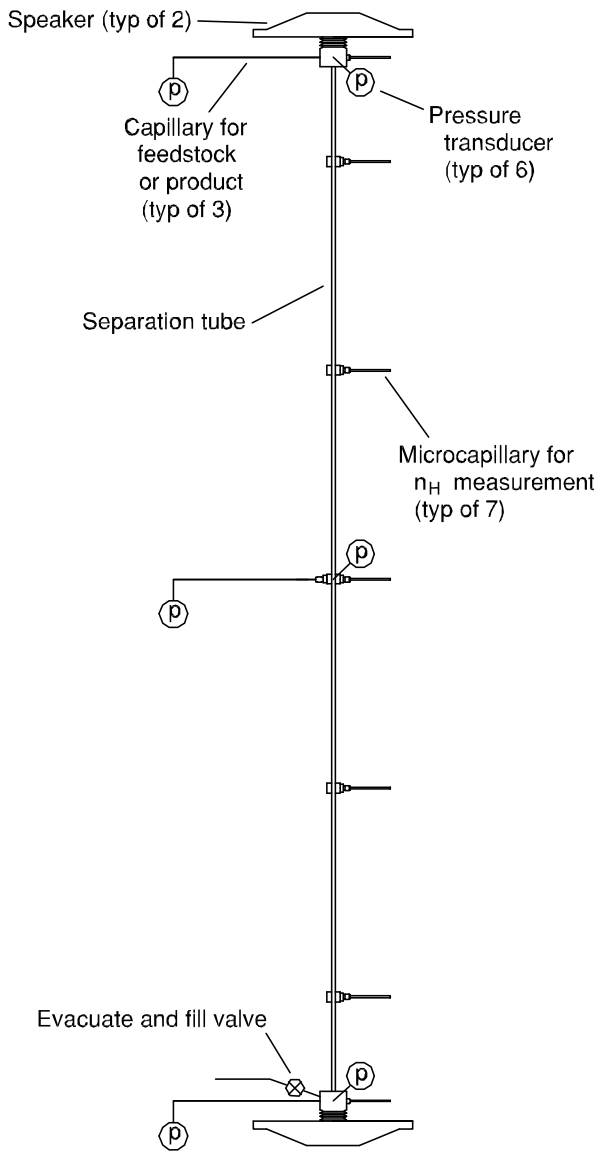


Figure 7.1: Apparatus for helium–argon separation used in Ref. [10] and in this example.

```
 BEGIN      in front of top driver
  7.9907E+04 a Mean P    Pa
   200.00    b Freq      Hz
   298.15    c TBeg      K
  3882.6     d  |p|      Pa      G
     0.0000  e Ph(p)     deg
  9.6980E-05 f  |U|     m^3/s    G
    -0.9501  g Ph(U)     deg     G
 -2.6300E-08 h Htot       W
 -3.1800E-07 i Ndot    mol/s
     0.3406  j nL                G
 HeAr       Gas type
!---------------------------------  1 ---------------------------------
 MIXCIRC    tube #1 (bottom)
  8.5630E-06 a Area     m^2               3000.0     A |p|       Pa
     1.0000  b GasA/A                     -101.23    B Ph(p)     deg
     0.5000  c Length    m                8.1411E-05 C |U|     m^3/s
  1.6510E-03 d radius    m                -101.23    D Ph(U)     deg
                                         -2.6300E-08 E Htot      W
                                             0.1221  F Edot      W
                                             0.3406  G nL Beg
                                             0.4661  H nL End
!---------------------------------  2 ---------------------------------
 RPN        pressure amplitude in center
  3000.0     a G or T          =  2A?     3000.0     A  |p1|
   p1 mag
!---------------------------------  3 ---------------------------------
 RPN        set Re[z]= rho a
     0.0000  a G or T          =  3A?     3.6028E-16 A  null
   rho a * p1 U1 / 1a * real - rho a * /
!---------------------------------  4 ---------------------------------
 RPN        set Im[z] = 0
     0.0000  a G or T          =  4A?     1.8585E-16 A  Im z
   p1 U1 / 1a * imag rho a * /
!---------------------------------  5 ---------------------------------
 RPN        feedstock concentration
     0.5000  a G or T            (t)         0.0000  A nL Feed
   0
!---------------------------------  6 ---------------------------------
 RPN        [a]=injection (micromol/sec)
     1.5900  a G or T            (t)      6.8668E-07 A ChngeMe
                                          1.2720E-06 B ChngeMe
   6a 1e-6 * Ndot + =Ndot 6a 1e-6 * 5a * NLdot + =NLdot
!---------------------------------  7 ---------------------------------
 MIXCIRC    tube #2 (bottom)
 sameas   1a a Area     m^2               2350.2     A |p|       Pa
 sameas   1b b GasA/A                      163.69    B Ph(p)     deg
 sameas   1c c Length    m                6.7249E-05 C |U|     m^3/s
 sameas   1d d radius    m                 163.84    D Ph(U)     deg
                                         -6.1169E-05 E Htot      W
                                          7.9026E-02 F Edot      W
                                             0.4661  G nL Beg
                                             0.5398  H nL End
!---------------------------------  8 ---------------------------------
 RPN        exit bulk condition
     0.0000  a G or T          =  8A?    -3.3295E-16 A null
   nL Ndot * NLdot - Ndot /
```

The <`.sp`> file from that run was imported into a spreadsheet to convert n_L to $n_H = 1 - n_L$ for plotting in Ref. [10]. Figure 7.2 shows the result, compared with measurements of the heavy mole fraction n_H that were made at 7 locations along the separation tube.

Next, an incremental plot was made, with feedstock $\dot{N}$ as the independent variable and with an `RPN` segment maintaining the molar flow rate in the `BEGIN` segment equal to -20% of the feedstock flow. The result is shown in Fig. 7.3.

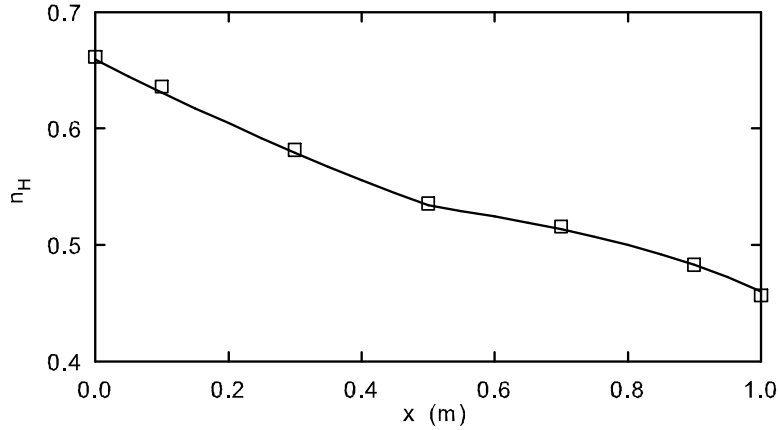


Figure 7.2: Heavy mole fraction n_H vs position in tube for one particular operating condition that is described in the text. The points are measurements from Ref. [10] and the curve is the result of DeltaEC's calculation as described in the text.

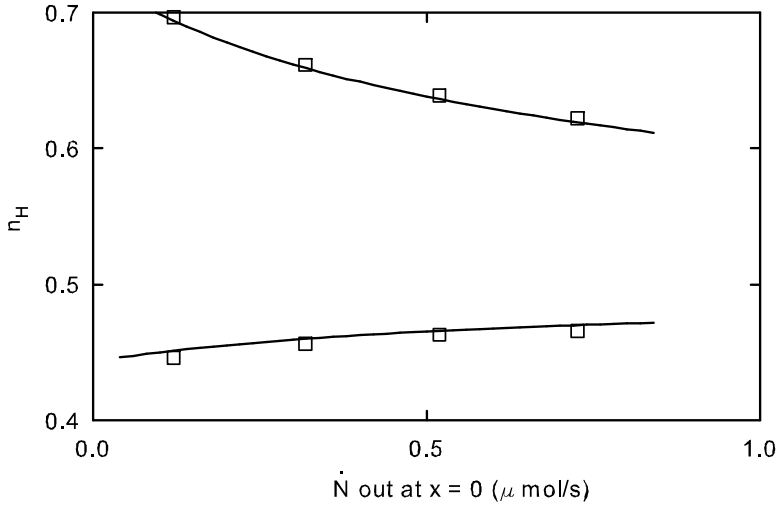


Figure 7.3: Heavy mole fractions n_H at $x = 0$ and $x = 1$ m, at various feedstock molar flows, with one product flow equal to 20% of the feedstock flow and the other product flow equal to 80% of the feedstock flow. The points are measurements from Ref. [10] and the curves are DeltaEC calculations as described in the text.

Part II: Reference

8. General principles

8.1. Integration from BEGIN to **END

Except for trivial cases (e.g., IMPEDANCE) that are handled algebraically, each of DeltaEC's physical segments numerically integrates two or more of the following differential equations from its initial end to its final end:

$$\frac{dp_1}{dx} = \mathcal{F}_{\text{momentum}}(\text{local and global parameters and variables}), \tag{8.1}$$

$$\frac{dU_1}{dx} = \mathcal{F}_{\text{continuity}}(\text{local and global parameters and variables}), \tag{8.2}$$

$$\frac{dT_m}{dx} = \mathcal{F}_{x\text{-energy}}(\text{local and global parameters and variables}), \tag{8.3}$$

$$\frac{dT_{\text{solid}}}{dx} = \mathcal{F}_{\text{solid}}(\text{local and global parameters and variables}), \tag{8.4}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \mathcal{F}_{\text{lateral energy}}(\text{local and global parameters and variables}), \tag{8.5}$$

$$\frac{dp_{2,0,HL}}{dx} = \mathcal{F}_{\text{head loss}}(\text{local and global parameters and variables}), \tag{8.6}$$

$$\frac{dn_L}{dx} = \mathcal{F}_{\text{mix sep}}(\text{local and global parameters and variables}). \tag{8.7}$$

The first of these equations, involving p_1 and U_1, are sufficient for users who are interested only in acoustics, without thermoacoustics. For these users, the other equations can be ignored. Users who are interested in ordinary thermoacoustic and Stirling engines and refrigerators must also concern themselves with T_m, T_{solid}, and $\dot{H}_{\text{tot}}$. Very few users have to consider all seven of these equations simultaneously.

In most segments that employ numerical integration of these equations, Nint steps of fourth-order Runge-Kutta integration [15] are employed. However, HX, TX, SX, and PX, which are most often used for heat exchangers whose lengths are only of the order of $|\xi_1|$, employ only one step of fourth-order Runge-Kutta integration.

For each integration through the segments in the model, the initial values of the integration variables, p_1, U_1, T_m, $\dot{H}_{\text{tot}}$, $p_{2,0,HL}$, and n_L, are set in the BEGIN segment. Except for T_{solid}, all of these variables are taken to be continuous between segments; but T_{solid} is usually discontinuous at the ends of heat exchangers.

A few additional variables, such as $\dot{N}$, are also initialized in the BEGIN segment but are independent of x unless they are changed abruptly at a logistical segment such as a TBRANCH.

The total power flow $\dot{H}_{\text{tot}}$ is often independent of x, but there are important exceptions. In heat exchangers, $\dot{H}_{\text{tot}}$ changes by the amount of heat added (or removed), and in transduc-

ers it changes by the amount of power added (or removed). If the `ANCHOR` logistical segment (described in Section 11.3.1) is used upstream, $\dot{H}_{\text{tot}}$ also changes in every component except `STK**`s, because `ANCHOR` means that acoustic power dissipation is removed locally as heat in each such component.

Usually the initial values of at least some of these integration variables are unknown at the `BEGIN`, so DeltaEC uses a shooting method to find initial values that are consistent with targeted results that are identified by the user elsewhere in the model. The shooting method is described in the next section.

After the shooting method has met its targets, DeltaEC knows the complete solution $p_1(x)$, $U_1(x)$, $T_m(x)$, $T_{\text{solid}}(x)$, $\dot{H}_{\text{tot}}(x)$, $p_{2,0,HL}(x)$, and $n_L(x)$. Three additional useful functions of x are calculated straightforwardly from that solution: the acoustic power $\dot{E}$, the exergy flow $\dot{X}$, and the total second-order time-averaged pressure $p_{2,0,\text{tot}}$:

$$\dot{E}(x) = \frac{1}{2}\text{Re}\left[p_1\widetilde{U}_1\right], \tag{8.8}$$

$$\begin{aligned}\dot{X}(x) &= \frac{T_0}{T_m}\dot{E} + \left(1 - \frac{T_0}{T_m}\right)\dot{H}_{2,k} + \dot{N}\left(mw_m - T_0 ms_m\right) \\ &\quad + \dot{N}_L R_{\text{univ}} T_0 \ln\left(\frac{n_L}{n_{L,0}}\right) + \left(\dot{N} - \dot{N}_L\right) R_{\text{univ}} T_0 \ln\left(\frac{1-n_L}{1-n_{L,0}}\right),\end{aligned} \tag{8.9}$$

$$p_{2,0,\text{tot}}(x) = p_{2,0,\text{rev}} + p_{2,0,HL} \tag{8.10}$$

$$= \frac{|p_1|^2}{4\rho_m a^2} - \frac{\rho_m |U_1|^2}{4A_{\text{gas}}^2} + p_{2,0,HL}. \tag{8.11}$$

The expression for acoustic power is discussed in Ref. [12] and should be familiar to most acousticians.

The expression for $\dot{X}$ is an amalgam of the basic thermoacoustic exergy flow derived in Ref. [12], a steady-flow component derived in Ref. [31], and the $\dot{N} \neq 0$ extension of the thermoacoustic mixture-separation expression derived in Ref. [43]. The reference temperature T_0, which is the ambient temperature at which heat has no value, is set to 300 K by default in `BEGIN` segments and can be changed thereafter by using the operation `=Tzero` in an `RPN` segment. Steady pressure makes no contribution to the steady-flow component because we assume that the reference pressure for exergy is p_m. The second line of Eq. (8.9) is relevant only for mixtures. The zero-exergy mixture mole fraction, $n_{L,0}$, is the mole fraction for which the mixture has no value, typically the feedstock's mole fraction. It is set to 0.500 by default in `BEGIN` segments, and can be changed to another value by using `=nLzro` in an `RPN` segment soon after the `BEGIN`.

The reversible part of the second-order time-averaged pressure, $p_{2,0,\text{rev}}$, is derived in Ref. [35], and measurements of $p_{2,0,\text{tot}}$ are described in Ref. [34].

All ten of these functions of x are available to the user in tabular and graphical forms via the <`.sp`> file, and values of relevant integration variables at the end of each segment are also shown in the <`.out`> file.

8.1.1. Additional details for $\dot{N}$ and $p_{2,0,HL}$

DeltaEC's input parameter `Ndot`, denoted in the Users Guide by $\dot{N}$, represents the true time-averaged molar flow, which is the steady-flow quantity under the most direct experimental control and the quantity that carries steady enthalpy flux

$$\dot{H}_{\dot{N}} = \dot{N} m w_m, \tag{8.12}$$

where m is the molar mass and w_m is the mean enthalpy per unit mass. The total power

$$\dot{H}_{\text{tot}} = \dot{H}_{2,k} + \dot{H}_{\dot{N}} \tag{8.13}$$

is the power that obeys the first law of thermodynamics for open control volumes. The total power $\dot{H}_{\text{tot}}$ is the sum of the second-order time-averaged thermoacoustic enthalpy flux, the ordinary thermal conduction in the x direction, and $\dot{H}_{\dot{N}} = \dot{N} m w_m$.

In DeltaEC, mean enthalpy is referenced to zero at temperature T_0, which is set to 300 Kelvin by default in the first `BEGIN` segment of a model. This default can be overridden by using the `=Tzero` operator in an `RPN` segment. For an ideal gas, Eq. (8.12) can then be regarded as

$$\dot{H}_{\dot{N}} = \dot{N} m c_p \left(T_m - T_0\right). \tag{8.14}$$

To second order,

$$\dot{N} m \simeq \frac{1}{2} \text{Re} \int \left[\rho_1 \widetilde{u}_1\right] \, dA_{\text{gas}} + \rho_m U_{2,0}, \tag{8.15}$$

where m is the molar mass. However, the second-order time-averaged momentum equation shows that second-order time-averaged head-loss gradients are generated by $U_{2,0}$, with no contribution from $\rho_1 \widetilde{u}_1$. Defining $\dot{N}_{2,0} \equiv \rho_m U_{2,0}/m$, substituting $\rho_1 = (\gamma/a^2)\, p_1 - \rho_m \beta T_1$ in Eq. (8.15), and solving for $\frac{1}{2}\text{Re}\int [T_1 \widetilde{u}_1] \, dA_{\text{gas}}$ in the standard thermoacoustic expression for $\dot{H}_{2,k}$, it is easy to show that Eq. (8.15) is equivalent to

$$\dot{N}_{2,0} = \dot{N} + \frac{\beta}{m c_p} \left[\dot{H}_{2,k} + \left(A_{\text{gas}} k + A_{\text{solid}} k_s\right) \frac{dT_m}{dx}\right] - \left(\frac{1}{m a^2} + \frac{\beta}{m c_p}\right) \dot{E}, \tag{8.16}$$

which is the molar flow that is used in most of DeltaEC's $p_{2,0,HL}$ calculations.[1] Note that $\dot{N}_{2,0}$ is often nonzero when $\dot{N} = 0$, which means that interesting second-order, time-averaged head losses arise in many circumstances having $\dot{N} = 0$, such as in the regenerators of TASHEs and pulse-tube refrigerators. Click the Enable Ndot button in the `BEGIN` segment's Optional parameters to "turn on" the display of `p2OHL` even when $\dot{N} = 0$.

8.2. Shooting method

If initial values of all of the integration variables of interest [chosen from among $p_1(x)$, $U_1(x)$, $T_m(x)$, $T_{\text{solid}}(x)$, $\dot{H}_{\text{tot}}(x)$, $p_{2,0,HL}(x)$, and $n_L(x)$] are known at the `BEGIN`ning end

[1] In a few expressions for the dissipative parts of dp_1/dx below, our choice of whether to use $\dot{N}_{2,0}$ or $\dot{N}$ is based as much on history or convenience as on careful thought. In those cases, $\dot{N} - \dot{N}_{2,0}$ is of second order and affects dp_1/dx only to third order.

of the thermoacoustic apparatus, then DeltaEC's numerical integration is straightforward, proceeding from the initial end to the final end. But usually one or more boundary conditions are instead specified at the final end or somewhere in the middle. In such circumstances DeltaEC uses a shooting method [15], by *guess*ing any unknowns at the `BEGIN`, integrating to the `**END`, comparing the results with the *target* boundary conditions imposed at that end and/or elsewhere, and adjusting its guesses until the targets are met. One of DeltaEC's most powerful features is that the guesses and targets are not limited to the conventional choices, which would be the initial values of integration variables at the `BEGIN` as guesses and calculated values of integration variables at the `**END` as targets. *Any* results can be used for targets, and *any* upstream variables that have effects on downstream target variables can be used as guesses. This enables DeltaEC to calculate a resonance frequency, a geometrical dimension, a temperature, or even the concentration in a binary gas mixture in order to satisfy desired boundary conditions.

DeltaEC treats its collection of n guesses and n targets as a system of n nonlinear equations,

$$F_1 = r_1(g_1, g_2, g_3 ... g_n) - t_1, \tag{8.17}$$
$$F_2 = r_2(g_1, g_2, g_3 ... g_n) - t_2, \tag{8.18}$$
$$F_3 = r_3(g_1, g_2, g_3 ... g_n) - t_3, \tag{8.19}$$
$$... \tag{8.20}$$
$$F_n = r_n(g_1, g_2, g_3 ... g_n) - t_n, \tag{8.21}$$

where g_i is the ith guess, t_i is the ith target value, and r_i is the ith calculated result, which we hope can be made equal to the ith target t_i by suitable choices of the n guesses g_i. Thus, DeltaEC must seek values of the n guesses g_i that make $F_i = 0$ for all i. Several methods for finding the zeroes of complicated functions exist. The routine incorporated in DeltaEC is called DNSQ, and it is part of the SLATEC Common Mathematical Library, which is freely available through the internet software repository at http://www.netlib.org. The algorithm used is a modification of the Powell hybrid method [45]. The ordinary DeltaEC user can just as well imagine that DNSQ is the Newton–Raphson method, which is easy to understand and is described in Ref. [15] and many tutorial websites. The description below is based on that slight oversimplification.

A Newton–Raphson algorithm begins with initial "guessed" values of the guesses, and calculates the n values of F_i. If $\sum (F_i)^2$ is not close enough to zero, the algorithm estimates the n^2 local slopes

$$\frac{\partial F_i}{\partial g_j} \simeq \frac{F_i(g_1, g_2, \ ... \ g_j + \delta g_j, \ ... \ g_n) - F_i(g_1, g_2, \ ... \ g_j, \ ... \ g_n)}{\delta g_j}. \tag{8.22}$$

The algorithm forms a "better" set of guesses based on the assumption that these functions F_i are linear, and repeats the process, until it judges the results to be close enough to the targets, according to

$$\sqrt{\sum (w_i F_i)^2} \leq \texttt{Tolerance} \times \sqrt{\sum (w_i t_i)^2}, \tag{8.23}$$

where the weighting factors w_i passed from DeltaEC to DNSQ are given by

$$w_i = \frac{1}{100} \text{ if the target is a phase angle,} \tag{8.24}$$

$$= \frac{1}{|t_i|} \text{ if } |t_i| \geq 10^{-4} \text{ and the target is not a phase angle,} \tag{8.25}$$

$$= 10^4 \text{ if } |t_i| \leq 10^{-4} \text{ and the target is not a phase angle} \tag{8.26}$$

in the default "Mode 1." When `Tolerance` $= 10^{-N}$, this should ensure N digits of accuracy for most targets like temperatures and pressure amplitudes, and values $< 10^{-(N+4)}$ for "zero" targets such as normalized `**END` impedances.

8.3. Numerical options

DeltaEC has many internal parameters that can be altered by the experienced user to control the behavior of DeltaEC's numerical integration and shooting method, and the amount of diagnostic information displayed. These parameters are accessible via Options in the Edit pulldown menu.

If any of the parameters described below are modified from their default values, it is usually desirable to keep the new values for all subsequent runs, even after exiting from DeltaEC and restarting. Therefore, whenever the default settings are changed, all of the tunable parameters are written to a special file when the model is saved. This file has the same base filename as the model, ending with <`.ini`>. Whenever a new model is loaded, DeltaEC checks in the same folder for a <`.ini`> file with a matching base filename and loads these settings if it is found. This file is written in `NAMELIST` format which makes it easy to examine and modify using any text editor.

Frequently, a collection of similar models will reside in a single folder, and these files will share identical custom settings. For these situations, any <`.ini`> file can be copied (or renamed) to <`default.ini`> and DeltaEC will use the settings within <`default.ini`> for any model run from that folder. If a model has its own individual <`.ini`>, the individual settings take precedence over <`default.ini`>.

8.3.1. Integration options

`Runge-Kutta steps.` Default: 10. Recommended range: $1 - 50$. This is an integer `Nint` that determines the number of integration steps used to span each `STK**`, `DUCT`, `CONE`, `MIX**`, and `VX**` segment. It does not affect other segment types. It also determines the resolution in state plots: `Nint` +1 points per integrable segment. Larger values will cause a slower, more accurate computation. Small values will increase speed at the price of integration accuracy and possible convergence problems.

`Minimum Temperature.` Default: 10 K. This is a temperature floor to prevent DeltaEC's shooting method from exploring unphysical temperatures such as negative temperatures. Brave users with special needs at lower temperatures (and with their own, user-defined thermophysical properties files!) can set this floor to a lower value. (Some of DeltaEC's internal gases use a higher temperature floor. See Chapter 12.)

8.3.2. Shooting options

Most of these parameters are described more fully in the online documentation for DNSQ at the SLATEC software repository. More tips on successful shooting in complicated DeltaEC models are in Section 5.4.

`Tolerance.` Default: 10^{-8}. Recommended range: 10^{-8} or smaller. This parameter governs the accuracy at which DeltaEC considers its shooting-method iterations finished. See Eq. (8.23). At 10^{-N}, this should ensure N digits of accuracy for most targets, and values $< 10^{-(N+4)}$ for "zero" targets if the Mode is 1.

`Mode.` Default: 1. In a numerical problem in which the output variables used in the target list differ greatly in magnitudes, a difficulty arises in choosing how much to weigh the errors between each target and corresponding result. This particularly affects `**END` segments. A 0.001 K error in a heat-exchanger temperature is fairly benign to us, but an error of 0.001 in the normalized complex end impedance z_n could leave us with tens of watts of power flow where there must be zero. In the standard Mode (1), DeltaEC uses the target weighting described in Eqs. (8.24)–(8.26) to normalize targets, which usually does a reasonable job. For unusual cases, DeltaEC has a second Mode (2) that weighs zero targets 1000 times more strongly.

`Step Bound Factor.` Default: 100. Recommended range: 0.01–100. This parameter controls the boldness with which DeltaEC first tries to adjust the g_i, after the $\partial F_i/\partial g_j$ have been calculated for the first time. Some difficult cases can benefit by a smaller Step Bound Factor.

`EpsFcn.` Default: 10^{-10}. Recommended range: $> 5 \times 10^{-15}$. To estimate $\partial F_i/\partial g_j$ using Eq. (8.22), DeltaEC sets $\delta g_j = g_j\sqrt{\texttt{EspFcn}}$. (But if $g_j = 0$, then $\delta g_j = \sqrt{\texttt{EspFcn}}$.) A large value of `EpsFcn` can sometimes speed iterations when initial values of guesses are very inaccurate. Too small a value can cause the solver to lose its way completely, as roundoff error in the computer ruins the accuracy with which $\partial F_i/\partial g_j$ is calculated. The optimal value is computer-dependent and model-dependent.

8.4. Error messages

Sometimes exiting from DeltaEC will produce an error message informing you to "See DeltaEC.exe.log for details." If you encounter this message, please look for that file in folder <`d:\ProgramFiles\DeltaEC`> and email it to `ww@lanl.gov` and `swift@lanl.gov`, with a brief explanation of what you were doing.

Sometimes DeltaEC will produce a "wxWidgets Debug Alert." If you see this alert, please copy the cryptic message in the top of the box and send it to `ww@lanl.gov` and `swift@lanl.gov`, with a brief explanation of what you were doing. Please check for a <`.exe.log`> file in folder <`d:\ProgramFiles\DeltaEC`> too.

Most of DeltaEC's other diagnostics are meant to be self-explanatory, but some of the obscure messages may require additional explanation:

Warnings were generated while parsing... This warning message may indicate a problem with the <`.out`> file. This usually occurs when a version-5 model is opened in version 6 for the first time, or when the savvy user has made a minor mistake when editing a <`.out`> file with a text editor instead of using DeltaEC's internal parameter- and segment-modification methods. Examine the <`.err`> file for specific information. When translating a model from version 5 to version 6, DeltaEC gives warnings whenever it is unsure of itself, even though it usually makes the right assumptions about the translation.

Success, Partial convergence, or FAILED. The Run Monitor at the bottom of the main display gives one of these three messages after every run, on a panel of green, yellow, or red, respectively. "Success" indicates that targets were met (or no targets and guesses exist at all). "FAILED" and texttttIteration is complete, but some results may `not be near their targets` indicate that targets have definitely not been met to DeltaEC's satisfaction. If the error occurs on a model that has displayed good convergence under other conditions, this may simply be one pathological point. The user may be able to jump-start it by manually (somewhat intelligently) changing the value of one or two members of the guess list to put the solver on the right track. Otherwise, consider the tactics described in Section 5.4. "Partial convergence" indicates that targets are almost met to DeltaEC's satisfaction, and in most cases the user can also be satisfied. Usually "Partial convergence" can quickly be changed to "Success" simply by running again. If the frequent appearance of "Partial success" becomes a nuisance, try changing the `Tolerance`, described in Section 8.3.2.

Plot non-convergence indicators. When a convergence error occurs during a plotting operation, a '*' is prepended to the plot line for that point, followed by a single digit representing the "info" variable at that point. A typical line in the <`.ip`> file might then look like this:

```
*4   2.990       98.61       528.2       45.36       559.3       3219.
```

The plotter indicates such suspicious and failed convergence by plotting the point in red. Below is a key to interpreting the "info" codes:

`info=`	Significance
1	Partial convergence: The solver considers the iteration successful, but the residual error is suspiciously large.
2	The solver was making progress, but the maximum number of iterations has been reached. Another `run` might make further progress.
4	FAILED: The solver was unable to converge.

There are some cases when the user may want to delete the prepended asterisk and number. For example, if the code is "info=2," and the remaining error that the solver was working on was already quite small, it might be quite valid to use this plot point instead of discarding it.

8.5. Inherent limitations

Users should be aware of fundamental, inherent limitations in DeltaEC's ability to model real hardware:

DeltaEC assumes that p_1 depends only on x, that u_1 depends only on x except within the viscous boundary layer, and that T_m depends only on x. Thus, DeltaEC cannot accurately model flow in short-radius elbows, flow and pressure at abrupt transitions between `DUCT`s of different areas (as shown in Fig. 8.1), or inertial or thermal effects in thermal buffer tubes and pulse tubes suffering from significant Rayleigh streaming. (For example, in `STKDUCT`, $T_m(x)$ and $T_{\text{solid}}(x)$ are equal for all x, which is a very bad approximation if significant Gedeon streaming, Rayleigh streaming, or jet-driven streaming is present.) We often use `MINOR` segments, `RPN` segments, `IMPEDANCE` segments, and off-line hand calculations to try to account for 2- and 3-dimensional effects like these.

The derivations of most thermoacoustic equations rely on the assumption that the gas displacement amplitude $|U_1|/\omega A$ is much smaller than all other relevant dimensions in the x direction. This assumption is often violated in thermoacoustic components, especially heat exchangers, whose length in the x direction is seldom larger than $2\,|U_1|/\omega A$.

The turbulence algorithms in `DUCT`, `CONE`, and `VX**` are based on the quasi-static approximation, namely that the pressure gradient at any instant of time in time-dependent flow is the same as that which would be present for steady flow at the same rate. This is known to be naive [46, 47, 48, 49].

Harmonics are often present in $p(t)$ and $U(t)$, but DeltaEC works exclusively with the fundamentals p_1 and U_1. This limitation usually has little impact on the accuracy of power calculations [12], but experiments being compared to DeltaEC calculations must use instruments that measure the fundamental components of the wave, not rms or peak values.

DeltaEC's shooting method is very efficient at converging to solutions for complicated systems; however, it knows nothing about acoustics, or any other aspect of physical reality. DeltaEC does not know that negative frequencies, negative pressures, or negative lengths are improper; it simply does the math. Thus, the reasonableness of the answers produced by DeltaEC's shooting method often depend on the quality of the initial guesses, which in turn depend on the thermoacoustics knowledge of the user.

`VXQ1` and `VXQ2` rely on assumptions that may be invalid in some circumstances. See Section 10.7.3 for details.

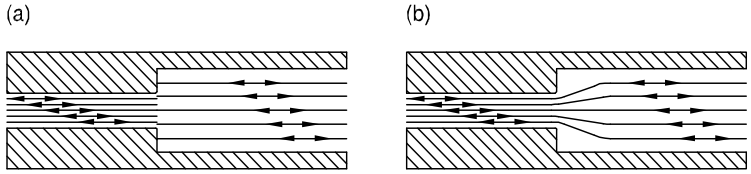


Figure 8.1: (a) Naive and (b) more-realistic sketches of streamlines where two large-diameter `DUCT`s join. DeltaEC performs inherently one-dimensional calculations, so it naively imagines that the streamlines are as shown in (a). In reality, some high-velocity flow extends into the large-diameter duct, adding inertance that DeltaEC does not automatically account for, and resistive minor losses also occur at high Reynolds numbers. These effects can and usually should be modeled with `RPN` calculations, `MINOR` and/or `IMPEDANCE` segments, and human intelligence.

9. Good design habits

When we study fundamental thermoacoustics issues, we try to design experiments to minimize unrelated effects so the issues being studied stand out clearly. In contrast, when we apply what we know about thermoacoustics to design a practical device for a goal-oriented sponsor, we often find ourselves forced by the sponsor's goals into operating conditions with great complexity and significant uncertainty, and yet we must account for *everything* as well as possible. This chapter outlines some habits that we use at Los Alamos, as of 2008, to make complex DeltaEC design models as realistic as possible, so that we and our goal-oriented sponsors are not disappointed when hardware is built and tested.

Our philosophy: (1) Attention to details, because there are many details and their effects add up. (2) Component-by-component conservatism, based on experimental evidence when it differs from "raw" DeltaEC results. Ignoring these considerations can lead to factor-of-two overprediction of performance.

We try to be honest with ourselves about these issues: If there are enough such issues present in a given design, then any uncertainties about these issues should average out, so we'll end up with hardware that works approximately like we predict, with equal probability of working a little better or a little worse. In contrast, neglecting these issues early in a design process can produce misleading "optimization" that does not represent reality, and neglecting them in the final stages of a design usually leads to disappointment when hardware is tested.

Note that some of these details can be neglected at low amplitudes.

(Thanks to Scott Backhaus for contributions to this chapter.)

9.1. Attention to details

9.1.1. Imperfect external heat exchange

We pay attention to the interfaces between DeltaEC-modeled heat exchangers and their environment. Even though DeltaEC is not set up to model anything outside the "skin" of the solid that is in contact with the thermoacoustic gas in heat exchangers, many effects beyond that boundary are important, and should be modeled, either using `RPN` segments within DeltaEC or using calculations elsewhere (e.g., spreadsheets). For example, in a water-cooled heat exchanger, non-negligible temperature differences usually exist across the water boundary layer (due to the finite thermal conductivity of the water) and between the inlet and outlet streams (due to the finite specific heat and flow rate of the water). Our sponsors usually care about the water inlet temperature, not DeltaEC's heat-exchanger internal temperature. Similar considerations apply in a drilled metal-block heat exchanger

that relies on conductivity of metal to transfer heat between the thermoacoustic gas in the drilled channels and an external heat source or sink at the perimeter of the block.

9.1.2. Internal 2-D effects from imperfect external heat exchange

DeltaEC assumes that p_1, U_1, and T_m are independent of coordinates perpendicular to x. Sometimes the low heat capacity and/or low flow rate of an external heat-transfer fluid (especially combustion products from a burner flowing through an engine's hot heat exchanger) ruin this laterally-uniform-T_m assumption in a cross-flow heat exchanger, thereby ruining the same assumption in an adjacent stack or regenerator, which in turn can cause lateral nonuniformity in U_1 and/or internal streaming in the stack or regenerator. DeltaEC is fundamentally ill suited to modeling such issues, yet we must consider them. To get a rough idea of the impact of such nonuniformity, we have sometimes used `TBRANCH` and `UNION` to split a hx-regen-hx (or hx-stack-hx) package into two (or even three) parallel subsets, each with half (or a third) of the total area, allowing different $T_m(x)$ in each. RPN calculations can be used to keep the heat-exchanger temperatures consistent with the heat-transfer fluid's flow rate and heat capacity. (Such split models usually require so many guesses and targets that other guesses and targets in the model must be disabled while this issue is being explored.)

9.1.3. Heat leaks

We include axial thermal conduction of the pressure vessel surrounding any `STK**` components that have nonzero axial temperature gradients. This is built in to `STKDUCT` and `STKCONE` segments, but it is not built in to the segments used for stacks and regenerators. In `STKSCREEN` regenerators, we often tie `ksFrac` to an RPN calculation that knows the vessel wall geometry, so the vessel wall conduction shows up directly in `HX` or other `*X` segments' results. If the screens and the pressure vessel are made of the same material, the expression is

$$k_{s,\text{Frac,effective}} = k_{s,\text{Frac,Lewis}} + \frac{A_{\text{wall}}}{(1-\phi)\,A_{\text{screen}}}, \tag{9.1}$$

where $k_{s,\text{Frac,Lewis}} = 0.1$ is recommended by Lewis et al. [27] for stacked screens. In other cases, we use `RPN` segments to add and subtract appropriate conduction estimates to `HX` and other `*X` heats, yielding "net" heats as `RPN` results. In the early stages of a design, such `RPN`s can keep wall thickness proportional to `STK**` diameter, making this heat leak track the reality of ASME pressure-vessel design.

We model black-body radiation heat leaks along pulse tubes and thermal buffer tubes using `RPN`s, adding them to DeltaEC's built-in `*X` calculations to obtain "net" heats. We assume that the walls of such tubes are fairly good infrared reflectors and the flow straighteners at the ends are fairly good black-body surfaces, so we simply use $\sigma_{\text{Stefan-Boltzmann}} A\left(T_H^4 - T_C^4\right)/2$ where A is the inside area of the `STKDUCT` and the factor of 2 accounts very roughly for the "fairly good" surfaces. We neglect black-body radiation in `STKSCREEN`s, which are optically opaque. We still debate whether and how to account for black-body radiation in standing-wave stacks, where the long aspect ratio of the channels gives radiation ample opportunity for absorption and re-emission at intermediate temperatures.

We also account for unavoidable heat flow through insulation layers wrapped around components, with `RPN`s or separate spreadsheets.

In systems with toroidal topology (e.g., TASHE [39, 5]), we suppress Gedeon streaming, or account for it with nonzero $\dot{N}$.

9.1.4. Geometry

As a design evolves towards its final state, we add more and more geometrical detail to the DeltaEC model. Even including the short gaps between regenerators and heat exchangers, or accounting for weld shrinkage that shortens such gaps, is not too much attention to detail. When shop drawings are being prepared, we modify the DeltaEC model to reflect machine-shop realities (e.g., inch-based dimensions; available drill sizes). While assembling parts, we often check measured dimensions against the DeltaEC model one more time.[1]

9.1.5. Actual operating conditions

Actual operating conditions are seldom identical to the final "design" condition, especially for ambient temperature. When comparing experimental results with DeltaEC calculations to decide whether hardware is working acceptably well, we always use actual, experimental conditions for parameters like ambient temperature. (See also Section 5.3 in this Users Guide, and Section 9.4 in Ref. [12], to consider what variables to regard as "given.")

In high-power systems, different parts of an "ambient"-temperature resonator can have significantly different temperatures. We often use RPN expressions like "`inp =Tm`" or "`Tm inp + =Tm`" to make our DeltaEC models follow this reality.

9.1.6. Thermophysical properties

DeltaEC's built-in thermophysical properties may not be accurate in the temperature (or pressure) range of interest for a new design. If we are going to work far from room temperature and/or with a new gas or solid, we check DeltaEC's gas and solid properties against some reliable reference.

9.2. Component conservatism

We rely on best-estimate "headroom" associated with every segment in the DeltaEC models wherever our previous experience shows that experimental performance falls a little short of the built-in first-principles calculations.

9.2.1. Turbulence

We habitually use a value of 5×10^{-4} for the relative surface roughness in `DUCT`s and `CONE`s, even when it is greater than the actual roughness, because this gives results that fit a lot of

[1] If a heat exchanger's `GasA/A` is small and its pores are large compared with those of the adjacent `STK**`, a gap is absolutely necessary to prevent U_1 from getting crowded into a reduced area in the beginning of the `STK**`. DeltaEC's naive one-dimensional calculations do not show the user the need for such gaps.

$\dot{N} = 0$ data over the past 15 years. When steady flow is superimposed, we use the "average" turbulence setting, which is consistent with halfway in between the two lines in Fig. 9 in Ref. [41].

We use `MINOR` segments generously wherever appropriate, e.g., corner turning, `TBRANCH`s and `UNION`s, and area changes (except very gently tapered cones), with Idelchik's book [50] for estimates of the K coefficients. We even check the need for `MINOR` segments at transitions between heat exchangers and open ducts, where abrupt changes in `GasA/A` occur.

9.2.2. Adiabatic–isothermal mixing

We use `JOIN` segments wherever appropriate, e.g., on both ends of pulse tubes, thermal buffer tubes, and hx-regen-hx packages. `JOIN` segments are usually negligible around hx-stack-hx packages in standing-wave systems, but we always check anyway.

We know that some of the same dissipative physics is present if the gap between a heat exchanger and an adjacent regenerator or stack is an appreciable fraction of the volume-flow-rate amplitude and is spatially stratified, so we strive to keep such gaps small and/or well-mixed by jets.

9.2.3. Stacked screen regenerators

We determine the "nominal" hydraulic radius r_h of a stacked-screen regenerator using the manufacturer's specified wire diameter, the volumetric porosity determined by weighing the complete regenerator, and Eq. (10.116). We have never seen a pressure drop across a stacked-screen regenerator that is smaller than DeltaEC predicts using this nominal r_h Our measured pressure drops have ranged from 0% to 15% above DeltaEC predictions using the nominal value. Thus, taking the middle of that range, and noting that the pressure drop goes like r_h^{-2}, we usually set the DeltaEC r_h equal to 0.93–0.95 times the nominal value.

We set `ksFrac` equal to 0.1 to account for screen-to-screen conduction, as recommended by Ref. [27], unless we are using this parameter to account for pressure-vessel-wall conduction [see Eq. (9.1)] as well as the screen-to-screen conduction.

9.2.4. Pulse tubes and thermal buffer tubes

We know that jet-driven streaming in pulse tubes and thermal buffer tubes can carry a lot of heat, so we always use screen flow straighteners on both ends, using `SX` segments to model their acoustic characteristics in DeltaEC and using the guidance near Eq. (7.86) in Ref. [12] to choose the screen size and number of layers. Since this design process does not fill us with confidence, we sometimes double the number of layers, because it is often easier to remove unnecessary layers later than to add layers later.

Rayleigh streaming can also carry a lot of heat in these tubes. In the experiment described in Ref. [30], very careful tuning of the phase of Z in a pulse tube could bring that streaming heat leak to zero, but we generally don't expect to keep pulse tubes so accurately tuned at all times. Thus, we try to work as close as possible to the zero-Rayleigh-streaming condition described in Ref. [28] while avoiding cone angles that might create flow separation, and even then we usually deduct 10% of $\dot{E}$ at the cold heat exchanger from DeltaEC's raw prediction

of cooling power in pulse tube refrigerators, and add 10% of $\dot{E}$ at the hot heat exchanger to the heater power in engines having thermal buffer tubes. (This correction also affects the ambient heat exchangers at the other ends of the tubes.)

Large-aspect-ratio refrigerator regenerators can suffer from regenerator internal streaming, as described in Ref. [51], which must be accounted for, even if only approximately.

(See also "Adiabatic–isothermal mixing" and "Heat leaks—black-body radiation" above.)

9.2.5. Standing-wave engine stacks

At low amplitudes, standing-wave engines have worked in agreement with DeltaEC predictions, so we trust that the calculations are correct to lowest order (i.e., energy expressions are correct to second order in acoustic amplitude). However, at higher amplitude, measurements on standing-wave engines are always worse than calculations: higher hot temperature T_H and higher heater power $\dot{Q}_H$ for a given p_1 and Z. Thus, until research brings understanding to this situation, we build some "headroom" into our DeltaEC designs of standing-wave engines.

Figures 2 and 3 in Ref. [24] show that the extra, unaccounted-for heater power is proportional to $|p_1|^3$ and the extra, unaccounted-for hot temperature is proportional to $|p_1|^2$. In the absence of any evidence to the contrary, we assume that these dependences on $|p_1|$ are applicable to all standing-wave engines.

Figure 9.1 summarizes our experience with several standing-wave engines, with all data extrapolated to $|p_1|/p_m = 0.10$ at the hot end of the stack using the pressure dependences described in the previous paragraph. The honeycomb data (small symbols) are more reliable than the parallel-plate data (large triangles), so we trust the honeycomb fit (dashed curves) more than the parallel-plate fit (solid curves). We believe that putting some headroom in standing-wave designs as suggested by Fig. 9.1 and the pressure dependences described above is much better than putting in no headroom at all.

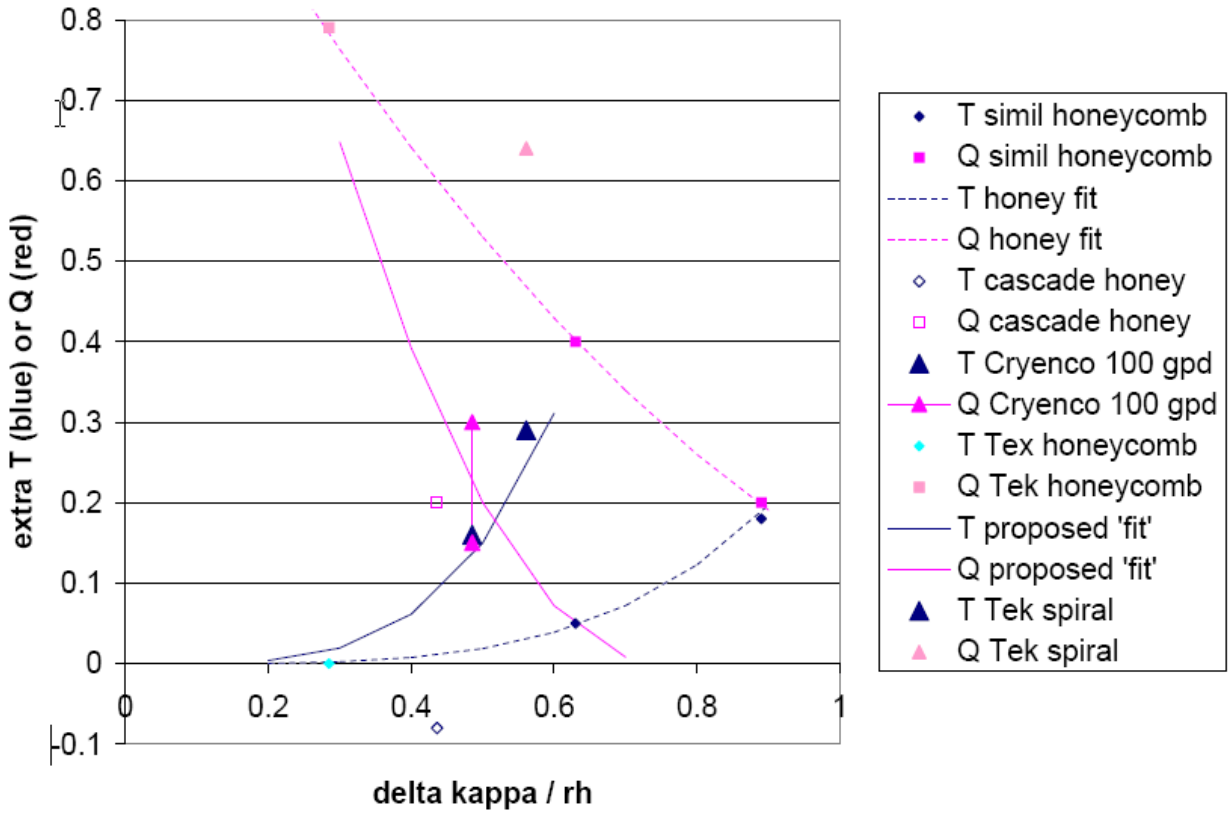


Figure 9.1: The vertical axis shows the experimentally observed extra T_H (blue) or $\dot{Q}_H$ (pink), in excess of DeltaEC's calculations, divided by DeltaEC's values, for standing-wave engines that we have used. The horizontal axis is δ_κ at the ambient end of the stack, divided by r_h. All data are scaled to $|p_1|/p_m = 0.10$ at the hot end of the stack, using $|p_1|^3$ scaling for extra $\dot{Q}_H$ and $|p_1|^2$ scaling for extra T_H. Data from three honeycomb (`STKCIRC`) stacks are shown: "simil honeycomb" is Ref. [24], "cascade honey" is Ref. [18], and "Tex honeycomb" is Ref. [52]. Unpublished data from two spiral (`STKSLAB`) stacks are shown: "Cryenco 100 gpd" is from the engine shown in Fig. 4 of Ref. [53], corroborated by measurements on a half-scale model at Los Alamos, and with the range of data for heater power indicating low acoustic load (more excess heat) vs. heavy acoustic load (less excess heat), and "Tek spiral" is from the engine described in Ref. [52] but with the honeycomb stacks replaced by spiral stacks as shown in Fig. 1.11 in Ref. [12].

10. Physical segments

All of DeltaEC's physically meaningful segment types are listed in this chapter, organized by function. Logistical and management segments appear in the next chapter. An alphabetical index of all segment types is in the Segment Index at the end of the Users Guide.

10.1. Ducts and cones

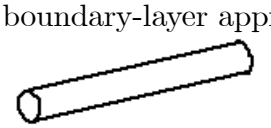
10.1.1. `DUCT`

General description:

Use `DUCT` for circular ducts of any diameter, and for large ducts (i.e., with $r_h \gg \delta_\nu$, δ_κ) of any cross-sectional shape by giving suitable area and perimeter. Mean temperature is independent of x, while p_1 and U_1 evolve with x.

DeltaEC's turbulence algorithm is controlled through optional parameter `d`, the relative roughness ε, whose definition for steady flow can be found in fluid-mechanics textbooks [54]: roughness height divided by pipe diameter. We have found that $\varepsilon = 0.0005$ works well in high-amplitude acoustics, even if the geometrical roughness is smaller than this value. Omitting this parameter forces the calculation to be laminar at all Reynolds numbers, no matter how large.

(See also `STKDUCT` in Section 10.6.1, which allows a temperature gradient along a duct in boundary-layer approximation.)



Input variables:

Area (m^2) Cross-sectional area A available to the gas, based on inside dimensions of the `DUCT`.

Perim (m) Perimeter Π of the inside cross-section described above.

Length (m) The length Δx of the `DUCT`.

Laminar\turbulent If the laminar box is checked, the calculations of dp_1/dx and $dp_{2,0,HL}/dx$ in the `DUCT` are based on noninteracting flow, using laminar flow for dp_1/dx and either laminar or turbulent flow for $dp_{2,0,HL}/dx$, depending on the steady-flow Reynolds number. The laminar condition is identified in the <`.out`> file as a blank line where input `d` would otherwise be.

Srough This input variable appears if the turbulent box is checked. Srough is ε, the surface roughness inside the `DUCT`, relative to the diameter. This is called "relative roughness" in many fluid-mechanics books, for steady flow. However, for oscillating flow, we usually regard Srough as a fitting parameter, with a typical value of $\varepsilon = 5 \times 10^{-4}$. Larger values yield higher turbulent dissipation of $\dot{E}$.

Optimistic/Normal/Pessimistic These choices appear if the turbulence box is checked and if the "Steady flow" box is checked in the `BEGIN` segment. These choices control the nonlinear interaction between oscillating flow and steady flow in the `DUCT`. "Optimistic" and "Pessimistic" refer to the two calculation schemes described in Ref. [41], and "Normal" is the average of the two. For the optimistic case, the turbulent steady and turbulent oscillating flows are assumed to be independent, simply superimposed. For the pessimistic case, the turbulent steady flow and turbulent oscillating flow are assumed to interact with each other and to increase each other's resistive effects according to the quasi-steady approximation. In the <`.out`> file, Optimistic/Normal/Pessimistic is recorded with `SrOptm`, `Srough`, or `SrPess` to indicate the name of ε.

Solid The solid material on the inside surface of the `DUCT`. (This is used only in the calculation of ϵ_s, which is seldom significant in `DUCT`s.)

Potential targets:

None.

Master–slave links:

The perimeter of a `DUCT` can be slaved to its area to keep the cross-sectional shape the same when area is varied.

The length of a `DUCT` can be slaved to that of another segment to keep the total length constant.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples:

Many examples of `DUCT` occur throughout Chapters 2, 5, and 6.

Calculation of $p_1(x)$ and $U_1(x)$ with $\dot{N} = 0$:

In `DUCT`s with no superimposed steady flow, p_1 and U_1 evolve according to

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A}U_1, \tag{10.1}$$

$$\frac{dU_1}{dx} = -\frac{iA\omega}{\rho_m a^2}\left(1+\frac{\gamma-1}{1+\epsilon_s}f_\kappa\right)p_1, \tag{10.2}$$

which is equivalent to

$$\left(1+\frac{\gamma-1}{1+\epsilon_s}f_\kappa\right)p_1+\frac{a^2}{\omega^2}\left(1-f_\nu\right)\frac{d^2p_1}{dx^2}=0. \tag{10.3}$$

In laminar DUCTs of length Δx, these are equivalent to

$$\begin{aligned} p_{1,\text{out}} &= p_{1,\text{in}}\cos k\Delta x-\frac{i\omega\rho_m}{(1-f_\nu)kA}U_{1,\text{in}}\sin k\Delta x, \\ U_{1,\text{out}} &= U_{1,\text{in}}\cos k\Delta x-\frac{i(1-f_\nu)kA}{\omega\rho_m}p_{1,\text{in}}\sin k\Delta x, \end{aligned} \tag{10.4}$$

with complex wave number k given by

$$k=\frac{\omega}{a}\sqrt{\frac{1+(\gamma-1)f_\kappa/(1+\epsilon_s)}{1-f_\nu}}. \tag{10.5}$$

In DUCTs, we define the effective radius to be $r_0=2A/\Pi$. (For circular cross sections, this is indeed the radius of the circle. For other cross sections, it is twice the hydraulic radius.) In narrow DUCTs, for $r_0/\delta<25$, f_κ and f_ν are calculated using complex Bessel functions

$$f_\kappa=\frac{2J_1[(i-1)r_0/\delta_\kappa]}{(i-1)(r_0/\delta_\kappa)J_0[(i-1)r_0/\delta_\kappa]},\quad f_\nu=\frac{2J_1[(i-1)r_0/\delta_\nu]}{(i-1)(r_0/\delta_\nu)J_0[(i-1)r_0/\delta_\nu]}.$$

Where $r_0/\delta>30$, the boundary-layer approximation is used:

$$f_\kappa=(1-i)\Pi\delta_\kappa/2A,\quad f_\nu=(1-i)\Pi\delta_\nu/2A. \tag{10.6}$$

For intermediate values, linear interpolation is used to make a smooth match between the two regimes. While the narrow-duct solution assumes a circular cross-section, the shape of the duct is irrelevant in the boundary-layer approximation. A square duct with dimensions much larger than the penetration depth can be modeled simply by choosing $\Pi=4\sqrt{A}$, for example.

When turbulent, p_1 and U_1 are numerically integrated according to Eqs. (10.1) and (10.2) above, with f_ν and f_κ modified as described below to account for the turbulence. DeltaEC's turbulence algorithm is described in detail in Chapter 7.2 of Ref. [12] and is based on the quasi-steady approximation: that turbulent oscillatory flow is described by the Moody friction factor at each instant of time during the oscillatory flow. (See any engineering fluid mechanics textbook, such as Ref. [54], to review the Moody friction factor as a function of Reynolds number and relative roughness of the pipe wall.) This assumption must be excellent in the low-frequency limit, in which $r_0/\delta_\nu\to 0$. This limit is approached in many inertances for pulse tube refrigerators. We have learned by experience with a few large systems that the assumption is reasonable for large r_0/δ_ν, which is of interest in the resonators of standing-wave thermoacoustic systems. For experimental validation of the assumption for intermediate r_0/δ_ν, see Ref. [14].

To account for turbulence, DeltaEC increases the resistive component of the pressure gradient, and hence the viscous power dissipation, by

$$m = \frac{\delta_\nu^2 N_R}{24\pi r_h^2} \frac{\left[f_M - (1 - 9\pi/32)\, N_R\, df_M/dN_R\right]}{\mathrm{Re}\left[i/\left(1 - f_\nu\right)\right]}. \tag{10.7}$$

DeltaEC evaluates the Moody friction factor f_M and its derivative with respect to Reynolds number, df_M/dN_R, as functions of Reynolds number and ε using the iterative expression

$$\frac{1}{\sqrt{f_M}} = 1.74 - 2\,\log_{10}\left(2\varepsilon + \frac{18.7}{N_R\sqrt{f_M}}\right), \tag{10.8}$$

which is a remarkably good approximation [55]. (In our own work at Los Alamos, we have set roughness equal to 5×10^{-4} for many years, as representative of most turbulent acoustic losses in ducts, even when the actual surface roughness over pipe diameter is smaller than this value.) DeltaEC decreases the inertial pressure gradient by

$$m' = \left(\frac{1 - \delta_\nu/r_0}{1 - \delta_\nu/mr_0}\right)^2 \tag{10.9}$$

to correct approximately for the steeper velocity gradient at the wall, which increases the effective area open to gas that contributes to inertial effects. It also multiplies the thermal penetration depth by m, in an attempt to account very approximately for changes in thermal relaxation losses due to increased heat transfer.

At low enough velocities, $m \to 1$ and DeltaEC reverts to a laminar calculation. The $m = 1$ boundary between the laminar and turbulent zones in DeltaEC occurs roughly at

$$N_R \simeq 2000 \text{ for } r_0/\delta_\nu < 2, \tag{10.10}$$

$$\frac{N_R}{2r_h/\delta_\nu} \simeq 1000 \text{ for } r_0/\delta_\nu > 2. \tag{10.11}$$

Whether laminar or turbulent, and in both small and large `DUCT`s, the boundary-layer approximation is used for the effect of the solid on thermal-hysteresis loss:

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s \rho_s c_s}\right)^{1/2}. \tag{10.12}$$

Calculation of $p_1(x)$ and $U_1(x)$ with $\dot{N} \neq 0$:

Nonzero $\dot{N}$ in a `DUCT` creates a Doppler-shift effect on p_1 and U_1, an increased resistive effect on p_1 if the flows are turbulent, and a time-averaged resistive head loss. Whether the flows are treated as laminar or as one of three varieties of turbulent is controlled by the user. The pressure and volume flow rate change with x according to

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{A(1-f_\nu)}U_1 + \frac{i\omega m\dot{N}}{\rho_m a^2 A}\frac{1 + (\gamma-1)f_\kappa/(1+\epsilon_s)}{(1-f_\nu)}p_1\ , \tag{10.13}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A}{\rho_m a^2}\frac{1 + (\gamma-1)f_\kappa}{1+\epsilon_s}p_1 + \frac{i\omega m\dot{N}}{\rho_m a^2 A(1-f_\nu)}U_1, \tag{10.14}$$

which can be derived using standard results of thermoacoustics combined with Eqs. (11.1.3) in Morse and Ingard's *Theoretical Acoustics*, Ref. [32]. (In CONEs, A is a function of x.)

Reference [41] describes two approaches to the calculation of the resistive effects on f_ν, f_κ, and $dp_{2,0,HL}/dx$ if the flow is turbulent. These two approaches, which we call "optimistic" and "pessimistic," bracket the experimental results in that paper. In DELTAEC the user can select either of these two approaches, or their average. Until more physics is learned, we recommend selecting the average by checking the "Normal" box, because the experimental results in Ref. [41] were roughly the average of the optimistic and pessimistic calculations.

For the optimistic case, the turbulent steady and turbulent oscillating flows are assumed to be independent, simply superimposed. Turbulent f_ν and f_κ are calculated with Eqs. (10.7)–(10.9).

For the pessimistic case, the turbulent steady flow and turbulent oscillating flow are assumed to interact with each other and to increase each other's resistive effects, as described in the appendix of Ref. [41]. In this case, f_ν and f_κ are calculated according to the procedure described near Eqs. (10.7)–(10.9), but with the turbulent multiplier m enhanced by the steady flow according to

$$m = \frac{\delta_\nu N_{R,\max}}{16 r_h}\left[|\varepsilon_{2,0}|\, f_{M,\max} + \frac{(3-2\,|\varepsilon_{2,0}|)\,(1-2\,|\varepsilon_{2,0}|)}{8(1+|\varepsilon_{2,0}|)}\frac{df_M}{dN_R}\,N_{R,\max}\right], \quad |\varepsilon_{2,0}| \geq 1\,, \tag{10.15}$$

$$= \frac{\delta_\nu N_{R,\max}}{12\pi r_h}\left\{\left[\left(1+\frac{\varepsilon_{2,0}^2}{2}\right)\sqrt{1-\varepsilon_{2,0}^2} + \frac{3}{2}\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0}\right] f_{M,\max} - \left[\left(1+\frac{\varepsilon_{2,0}^2}{2}\right)\sqrt{1-\varepsilon_{2,0}^2} + \frac{3}{2}\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0} - \frac{9\pi}{32}\frac{1+4\varepsilon_{2,0}^2}{1+|\varepsilon_{2,0}|}\right]\frac{df_M}{dN_R}\,N_{R,\max}\right\}, \quad |\varepsilon_{2,0}| \leq 1, \tag{10.16}$$

where the friction factor and its slope are based on the peak Reynolds number

$$N_{R,\max} = \frac{\left(m|\dot{N}_{2,0}|/\rho_m + |U_1|\right)4r_h\rho_m}{A\mu} \tag{10.17}$$

and $\varepsilon_{2,0} = m\dot{N}_{2,0}/\rho_m\,|U_1|$. The derivations of these expressions can be found in the appendix of Ref. [41].

Calculation of $p_{2,0,HL}(x)$ with $\dot{N} \neq 0$:

How the flow is treated depends on the roughness parameter in the DUCT. There are three cases: optimistic, pessimistic, and "normal," which is the average of the other two.

For the optimistic case, the turbulent steady and turbulent oscillating flows are assumed to be independent, simply superimposed. The steady-flow pressure drop is calculated using

$$\frac{dp_{2,0,HL}}{dx} = -\mathrm{sign}(\dot{N}_{2,0})\frac{1}{2}\rho_m\left(\frac{m\dot{N}_{2,0}}{\rho_m A}\right)^2\frac{f_{\mathrm{Moody,steady}}}{4r_h}. \tag{10.18}$$

The steady-flow Moody friction factor $f_{\mathrm{Moody,steady}}$ is calculated with a Reynolds number based on the steady velocity:

$$N_{R,2,0} = \frac{m|\dot{N}_{2,0}|4r_h}{A\mu}. \tag{10.19}$$

For the pessimistic case, the turbulent steady flow and turbulent oscillating flow are assumed to interact with each other and to increase each other's resistive effects, as described in the appendix of Ref. [41] ignoring any dependence on p_1 and in Chapter 14 including the lowest-order dependence on p_1. From Chapter 14, the steady-flow pressure drop is calculated using

$$\begin{aligned}
\frac{dp_{2,0}}{dx} &= -\operatorname{sign}(\dot{N}_{2,0})\frac{m^2\dot{N}_{2,0}^2}{8\rho_m r_h A^2}\left[\left(1+\frac{1}{2\varepsilon_{2,0}^2}+\frac{|p_1|}{\gamma p_m}\frac{\cos\phi}{\varepsilon_{2,0}}\right)f_{M,\max}\right. \\
&\quad \left.-\frac{2\varepsilon_{2,0}^2-2|\varepsilon_{2,0}|+1}{2\varepsilon_{2,0}^2(1+|\varepsilon_{2,0}|)}\frac{df_M}{dN_R}N_{R,\max}\right], \qquad |\varepsilon_{2,0}|\geq 1, \qquad (10.20)\\
&= -\frac{\rho_m|U_1|^2}{8\pi r_h A^2}\left\{\left[\left(1+2\varepsilon_{2,0}^2\right)\sin^{-1}\varepsilon_{2,0}+3\varepsilon_{2,0}\sqrt{1-\varepsilon_{2,0}^2}\right.\right. \\
&\quad \left.+\frac{2|p_1|}{\gamma p_m}\cos\phi\left(\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0}+\frac{2+\varepsilon_{2,0}^2}{3}\sqrt{1-\varepsilon_{2,0}^2}\right)\right]f_{M,\max} \\
&\quad \left.+\left[\pi\varepsilon_{2,0}\frac{3/2+\varepsilon_{2,0}^2}{1+|\varepsilon_{2,0}|}-3\varepsilon_{2,0}\sqrt{1-\varepsilon_{2,0}^2}-\left(1+2\varepsilon_{2,0}^2\right)\sin^{-1}\varepsilon_{2,0}\right]\frac{df_M}{dN_R}N_{R,\max}\right\}, \\
&\qquad |\varepsilon_{2,0}|\leq 1, \qquad (10.21)
\end{aligned}$$

where $\varepsilon_{2,0} = m\dot{N}_{2,0}/\rho_m|U_1|$.

Calculation of $T_m(x)$ and $\dot{H}$:

T_m is not affected by `DUCT`s.

The exiting energy flow is left as $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}}$ in insulated mode, described in Section 11.1.2. This essentially assumes that the side walls are thermally insulated, so that the heat generated by acoustic power dissipation finds its way to a heat exchanger somewhere else. If several `DUCT`s (and/or `CONE`s) are strung together in insulated mode, the power dissipated in all of them must show up at one or two nearby heat exchangers.

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{E}_{\text{out}}$ in "anchored" mode, described in Section 11.1.2. This essentially assumes that the duct wall is thermally anchored, e.g., by a water bath, so heat generated by acoustic-power dissipation is carried away locally. Thermoacoustic heat transport in the boundary layer along the perimeter, which in fact contributes a small difference between $\dot{H}_{2,k}$ and $\dot{E}$ in ducts, is neglected in "anchored" mode.

10.1.2. `CONE`

General description:

Use `CONE` to find the evolution of p_1 and U_1 with x in tapered channels of any cross-sectional shape (*e.g.*, square, circular) by giving suitable areas and perimeters. The perimeter is linearly interpolated between its initial and final values, and the area is quadratically interpolated between its initial and final values. Use this segment only when T_m is independent

of x. (See STKCONE in Section 10.6.1, to allow nonzero dT_m/dx.) The calculation is identical to that for a DUCT if the initial and final areas are equal and the initial and final perimeters are equal. As discussed below, the laminar calculation is accurate for circular cones of any diameter, and for large cones (i.e., with $r_0 \gg \delta_\nu$, δ_κ) of any cross-sectional shape.

DELTAEC's turbulence algorithm is controlled through optional parameter f, the relative roughness ε, whose definition for steady flow can be found in fluid-mechanics textbooks [54]: roughness height divided by pipe diameter. We have found that $\varepsilon = 0.0005$ works well in high-amplitude acoustics, even if the geometrical roughness is smaller than this value. Omitting this parameter forces the oscillatory calculation to be laminar at all Reynolds numbers.

At high enough amplitudes, we believe that a steeply tapered CONE experiences separation during the half of the cycle when the flow goes from small toward large radius, just like the separation at high Reynolds number in a too-steeply-tapered diffuser. DELTAEC does not warn of this condition. To account for such separation, the user could include minor loss, described in Section 10.2.4.

Input variables:

AreaI (m^2) Cross-sectional area A_I available to the gas at the initial end of the CONE, based on inside dimensions.

PerimI (m) Perimeter Π_I of the initial inside-cross-section described above.

Length (m) The length Δx of the CONE.

AreaF (m^2) Cross-sectional area A_F available to the gas at the final end of the CONE, based on inside dimensions.

PerimF (m) Perimeter Π_F of the final inside-cross-section described above.

Laminar\turbulent If the laminar box is checked, the calculations of dp_1/dx and $dp_{2,0,HL}/dx$ in the CONE are based on noninteracting, laminar flow. The laminar condition is identified in the <.out> file as a blank line where parameter f would otherwise be.

Srough This input parameter appears if the turbulent box is checked. Srough is ε, the surface roughness inside the CONE, relative to the diameter. This is called "relative roughness" in many fluid-mechanics books, for steady flow. However, for oscillating flow, we often regard Srough as a fitting parameter, with a typical value of 5×10^{-4}. Larger values yield higher turbulent dissipation of $\dot{E}$.

Optimistic/Normal/Pessimistic These choices appear if the turbulence box is checked and if the "Steady flow" box is checked in the BEGIN segment. These choices control the nonlinear interaction between oscillating flow and steady flow in the CONE. "Optimistic" and "Pessimistic" refer to the two calculation schemes described in Ref. [41], and "Normal" is the average of the two. For the optimistic case, the turbulent steady and turbulent oscillating flows are assumed to be independent, simply

superimposed. For the pessimistic case, the turbulent steady flow and turbulent oscillating flow are assumed to interact with each other and to increase each other's resistive effects according to the quasi-steady approximation. In the <`.out`> file, Optimistic/Normal/Pessimistic is recorded with `SrOptm`, `Srough`, or `SrPess` to indicate the name of ε.

Solid The solid material on the inside surface of the `CONE`. (This is used only in the calculation of ϵ_s, which is seldom significant in `CONE`s.)

Potential targets:

None.

Master–slave links:

The initial perimeter of a `CONE` can be slaved to the initial area, to keep the cross-sectional shape the same when area is changed.

The final perimeter of a `CONE` can be slaved to the final area to keep the cross-sectional shape the same when area is changed.

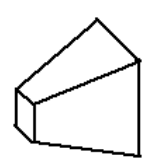
Both perimeters of a `CONE` can be slaved to their respective areas to keep the cross-sectional shapes the same when either or both areas are changed.

The length and both perimeters of a `CONE` can be slaved to its two areas, to keep the cross-sectional shapes of the ends *and* the wall taper angle constant when either or both areas are changed.

The length of a `CONE` can be slaved to that of another segment to keep the total length constant.

(For an introduction to master–slave links, and a complete listing of possible links for all segments, see Section 2.7.2.)

Format and examples:



Examples of the use of `CONE` appear throughout Chapters 2, 5, and 6.

Calculation of $p_1(x)$ and $U_1(x)$ with $\dot{N} = 0$:

In `CONE`s, p_1 and U_1 evolve according to

$$\begin{aligned} \frac{dp_1}{dx} &= -\frac{i\omega\rho_m}{(1-f_\nu)A}U_1, \\ \frac{dU_1}{dx} &= -\frac{iA\omega}{\rho_m a^2}\left(1+\frac{\gamma-1}{1+\epsilon_s}f_\kappa\right)p_1, \end{aligned} \tag{10.22}$$

which are equivalent to the Webster horn equation [56] with losses included:

$$\left(1+\frac{\gamma-1}{1+\epsilon_s}f_\kappa\right)p_1 + \frac{a^2}{\omega^2}\frac{1}{A}\frac{d}{dx}\left[(1-f_\nu)\,A\frac{dp_1}{dx}\right] = 0. \tag{10.23}$$

The perimeter varies linearly from its initial value to its final value, and area varies quadratically:

$$\Pi(x) = \Pi_I + (\Pi_F - \Pi_I)\,\frac{x - x_o}{\Delta x}, \tag{10.24}$$

$$A(x) = \left[\sqrt{A_I} + \left(\sqrt{A_F} - \sqrt{A_I}\right)\frac{x - x_o}{\Delta x}\right]^2, \tag{10.25}$$

where x_o is the x coordinate of the initial end of the `CONE`. Hence, `CONE`s that are circular at both ends have circular cross-sections everywhere in between, with diameter varying linearly with axial position.

In `CONE`s, we define the effective radius to be $r_0(x) = 2A/\Pi$. (For circular cross sections, this is indeed the radius of the circle. For other cross sections, it is twice the hydraulic radius.) For $r_0/\delta < 25$, f_κ and f_ν are calculated using complex Bessel functions:

$$f_\kappa = \frac{2J_1[(i-1)r_0/\delta_\kappa]}{(i-1)(r_0/\delta_\kappa)J_0[(i-1)r_0/\delta_\kappa]}, \quad f_\nu = \frac{2J_1[(i-1)r_0/\delta_\nu]}{(i-1)(r_0/\delta_\nu)J_0[(i-1)r_0/\delta_\nu]}.$$

Where $r_0/\delta > 30$, the boundary-layer approximation is used:

$$f_\kappa = (1-i)\delta_\kappa/r_0, \quad f_\nu = (1-i)\delta_\nu/r_0. \tag{10.26}$$

For intermediate values, linear interpolation is used to make a smooth match between the two regimes. While the narrow solution assumes a circular cross-section, the shape of the duct is irrelevant in the boundary-layer approximation.

In all cases, boundary-layer approximation is used for the effect of the solid:

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s\rho_s c_s}\right)^{1/2}. \tag{10.27}$$

Calculation of $p_1(x)$, $U_1(x)$, and $p_{2,0,HL}(x)$ with $\dot{N} \neq 0$:

In `CONE`, these calculations are exactly the same given above for `DUCT`, except that area A is a function of x in the equations for `CONE`.

Calculation of T_m and $\dot{H}$:

T_m is not affected by `CONE`. (But see also `STKCONE` in Section 10.6.1, which allows nonzero dT_m/dx.)

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}}$ in insulated mode, described in Section 11.1.2. This essentially assumes that the side walls are thermally insulated, so that the heat generated by acoustic power dissipation is deposited in an adjacent heat exchanger. If several `CONE`s (and/or `DUCT`s) are strung together in insulated mode, the power dissipated in all of them should show up in a nearby heat exchanger.

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{E}_{\text{out}}$ in anchored mode, described in Section 11.1.2. This mode essentially assumes that the cone wall is thermally anchored, e.g., by a water bath, so heat generated by acoustic-power dissipation is carried away externally. Thermoacoustic heat transport along the perimeter, which in fact contributes a small difference between $\dot{H}_{2,k}$ and $\dot{E}$ in `CONE`s, is neglected.

10.2. Lumped elements

10.2.1. COMPLIANCE

General description:

A COMPLIANCE is a lumped acoustic volume element, with adiabatic volumetric compressibility and surface thermal-hysteresis dissipation.

Input variables:

Area (m^2) The surface area S inside the COMPLIANCE.

Volume (m^3) The volume V inside the COMPLIANCE.

Solid The solid material on the inside surface of the COMPLIANCE. (This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and specific heat in COMPLIANCEs.)

Potential targets:

None.

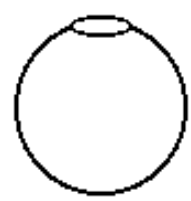
Master–slave links:

The surface area of a compliance can be slaved to its volume, so the shape remains the same when the volume is changed.

(For an introduction to master–slave links, and a complete listing of link options for all segments, see Section 2.7.2.)

Format and examples:

```
COMPLIANCE  this one is a sphere, R = 0.01 m
0.1257   m2  Area
4.189e-3 m3  Volume
nickel      solid
```



Other examples of COMPLIANCE can be found in Sections 5.2 and 5.8.2.

Calculations:

Pressure amplitude and mean temperature are unchanged by COMPLIANCE. Volume flow changes according to

$$U_{1,\mathrm{out}} = U_{1,\mathrm{in}} - i\frac{\omega p_1}{\rho_m a^2}\left[V - i\frac{\gamma-1}{1+\epsilon_s}S\frac{\delta_\kappa}{2}\right], \tag{10.28}$$

with the thermal-hysteresis contribution of the solid calculated in boundary-layer approximation:

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s\rho_s c_s}\right)^{1/2}. \tag{10.29}$$

The exiting energy flow is computed as $\dot{H}_{2,k,\mathrm{out}} = \dot{H}_{2,k,\mathrm{in}}$ in insulated mode. This essentially assumes that the component is thermally insulated, so that the heat generated by acoustic power dissipation must somehow be deposited elsewhere, either upstream or downstream, such as in an adjacent heat exchanger (or flowing out through a `BEGIN` or `**END` segment).

The exiting energy flow is computed as $\dot{H}_{2,k,\mathrm{out}} = \dot{E}$ in anchored mode. This is intended to model a component that is thermally anchored, e.g., by a water bath, so heat generated by acoustic power dissipation is carried away locally.

`COMPLIANCE` behaves the same whether $\dot{N} = 0$ or not. Since T_m is unchanged in `COMPLIANCE`, $\dot{H}_{\dot{N},\mathrm{out}} = \dot{H}_{\dot{N},\mathrm{in}}$.

10.2.2. `SURFACE`

General description:

A `SURFACE` is a surface area with thermal-hysteresis dissipation. It always absorbs acoustic power. It has no volume. It is often used at the end of a `DUCT`, before a `HARDEND`. (See Fig. 10.1 for another typical use for `SURFACE`.)

Input variables:

Area (m^2) The surface area S exposed to oscillating pressure in the `SURFACE` segment, not accounted for in other segments nearby.

Solid The solid material on the exposed surface. (This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and specific heat on `SURFACE`s.)

Potential targets; Master–slave links:

None.

Format and examples:

```
SURFACE    a surface with thermal hysteresis dissipation
1.134e-3 m2  Area
ideal        solid
```

Examples of `SURFACE` also occur throughout Chapter 2 and Sections 5.2 and 5.5.

Calculations:

A `SURFACE` does not affect temperature or pressure amplitude. Volume flow changes according to

$$U_{1,\mathrm{out}} = U_{1,\mathrm{in}} - \frac{\omega p_1}{\rho_m a^2} \frac{\gamma - 1}{1 + \epsilon_s} S \frac{\delta_\kappa}{2}. \tag{10.30}$$

(This is the same calculation as would occur in a `COMPLIANCE` with zero volume.) The influence of the solid on thermal hysteresis is calculated in boundary-layer approximation:

$$\epsilon_s = \left(\frac{k \rho_m c_p}{k_s \rho_s c_s} \right)^{1/2} . \tag{10.31}$$

The exiting energy flow is computed as $\dot{H}_{2,k,\mathrm{out}} = \dot{H}_{2,k,\mathrm{in}}$ in insulated mode. This essentially assumes that the component is thermally insulated, so that the heat generated by acoustic power dissipation must somehow be deposited elsewhere, either upstream or downstream, such as in an adjacent heat exchanger (or flow out through a `BEGIN` or `**END` segment).

The exiting energy flow is computed as $\dot{H}_{2,k,\mathrm{out}} = \dot{E}_{\mathrm{out}}$ in anchored mode. Thus, in a series of such anchored segments, heat generated by acoustic power dissipation is carried away locally.

`SURFACE` behaves the same whether $\dot{N} = 0$ or not. Since T_m is unchanged in `SURFACE`, $\dot{H}_{\dot{N},\mathrm{out}} = \dot{H}_{\dot{N},\mathrm{in}}$.

10.2.3. `IMPEDANCE`

General description:

An `IMPEDANCE` is a lumped series complex impedance.

Input variables:

Re(Z) (Pa·s/m^3) The real part of the series acoustic impedance Z. If the impedance has resistance, this number is nonzero.

Im(Z) (Pa·s/m^3) The imaginary part of the series acoustic impedance Z. If the impedance has inertance L, this number is equal to ωL.

Potential targets:

None.

Master–slave links:

In an `IMPEDANCE`, Im[Z] can be slaved to Re[Z] so that $|Z|$ remains constant when Re[Z] is changed. Note, however, that this becomes impossible if Re[Z] is changed to a value that is too large.

Alternatively, Im[Z] can be slaved to Re[Z] so that phase(Z) remains constant when Re[Z] is changed.

(For an introduction to master–slave links and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples:

```
IMPEDANCE    just a lumped series impedance, i.e., resistance plus inertance
1.0 Pa-s/m3   Re(Z)
0.2 Pa-s/m3  Im(Z)
```

Linked to one or more RPN calculations, IMPEDANCE gives the user the freedom to create nonlinear impedances. The following example shows an impedance with its resistance proportional to the square of the flow and its inertance proportional to the flow:

```
!-------------2
RPN  real part of impedance
12.0
U1 mag sqrd inp *
!-------------3
RPN  imaginary part of impedance
3.0E-4
U1 mag inp *
!-------------4
IMPEDANCE
sameas 2A
sameas 3A
```

Another example of IMPEDANCE, in the context of the orifice of a pulse-tube refrigerator, is in Section 5.8.3.

Calculations:

At an IMPEDANCE, volume flow rate and temperature are unchanged. Pressure changes according to

$$p_{1,\text{out}} = p_{1,\text{in}} - ZU_1. \tag{10.32}$$

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}}$ in insulated mode, described in Section 11.1.2. This assumes that the component is thermally insulated, so that the heat generated by acoustic power dissipation must somehow be deposited elsewhere, either upstream or downstream, such as in a nearby heat exchanger (or flow out through a BEGIN or **END segment).

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{E}_{\text{out}}$ for anchored mode, described in Section 11.1.2. This essentially assumes that the component is thermally anchored, such as by a water bath, so heat generated by acoustic power dissipation is carried away locally.

If $\dot{N} \neq 0$ in IMPEDANCE, the steady flow and the oscillating flow are assumed to be independent, simply superimposed, as if the flows are laminar. Thus, the difference in p_1 across an IMPEDANCE is unaffected by the presence of the steady flow, and the change in $p_{2,0,HL}$ is given by

$$p_{2,0,HL,\text{out}} - p_{2,0,HL,\text{in}} = \text{Re}[Z]\dot{N}_{2,0}m/\rho_m. \tag{10.33}$$

Since an IMPEDANCE has no defined area, $p_{2,0,\text{rev}}$ is calculated based on the area of the most recent upstream segment having area.

10.2.4. MINOR

General description:

Segment type MINOR accounts conveniently for minor-loss effects. The basic equation for a minor loss for steady flow [54, 50] is simply

$$\Delta p = -\frac{1}{2}Ku^2, \tag{10.34}$$

where K is the minor-loss coefficient and u is the velocity. DeltaEC's treatment of minor loss in `MINOR` is based on this equation.

Equations (10.35)–(10.42) were derived using the quasi-steady approximation, in which it is assumed that Eq. (10.34) is obeyed at each instant of time as u varies in time, with K independent of time except for its dependence on the direction of flow. This approximation is very good when $|\xi_1| \gg r_h$. For $|\xi_1| \sim r_h$ or $|\xi_1| \ll r_h$, actual pressure drops may differ significantly from values calculated in `MINOR`.

Since DeltaEC uses U_1 to characterize flow, `MINOR` has an area for converting U_1 to velocity. Because DeltaEC treats oscillating flow, `MINOR` has two inputs for Ks: K_+ for the minor-loss coefficient in the $+x$ direction, and K_- for the minor-loss coefficient in the $-x$ direction. The coefficient in the $\dot{N} = 0$ equations below differs from the 1/2 in Eq. (10.34) because of the sinusoidal time dependence of the velocity.

As shown in the second example below, the gas diodes in Ref. [41] can be modeled as a `MINOR` and a `CONE` in series.

We often obtain K_+ and K_- from Ref. [50], even though that reference is for steady flow. This approximation should be fairly accurate when the Reynolds number is large and $|\xi_1|$ is much greater than any dimensions of the component.

Input variables:

Area (m^2) The area A used in the equations below. This is the area on which the minor-loss coefficients are based. Conventionally, this is usually the smallest area for area-changing geometries such as diffusers, but it is sometimes the nominal pipe area for valves and fittings.

K+ Minor-loss coefficient K_+ for flow in the positive-x direction.

K− Minor-loss coefficient K_- for flow in the negative-x direction.

Potential targets:

None.

Master–slave links:

None. (Use `sameas` if you want to keep K_+ and K_- equal to one another.)

Format and examples:

This is an example of a small orifice plate mounted in a pipe, as illustrated in Fig. 10.1(a):

```
!--------------------------------  9 ---------------------------------
 DUCT       Pre-orifice pipe
  1.8000E-02 a Area      m^2     S= -2    2.2139E+04 A |p|       Pa
     0.4756  b Perim      m    Fn(  9a)   -141.30    B Ph(p)     deg
     0.1500  c Length     m               2.3148E-02 C |U|     m^3/s
                                           128.66    D Ph(U)     deg
                                          4.1756E-09 E Htot      W
 ideal      Solid type                      -0.1923  F Edot      W
!--------------------------------- 10 ---------------------------------
 RPN        calculate area difference (m2)
     0.0000  a G or T            (t)      1.6000E-02 A ChngeMe
```

```
   9a 12a -
!---------------------------------- 11 ----------------------------------
 SURFACE    surface area on top of orifice plate
 sameas  10A a Area      m^2              2.2139E+04 A |p|       Pa
                                          -141.30    B Ph(p)     deg
                                          2.3148E-02 C |U|     m^3/s
                                           128.65    D Ph(U)     deg
                                          4.1756E-09 E Htot      W
 ideal      Solid type                      -0.2320  F Edot      W
!---------------------------------- 12 ----------------------------------
 MINOR      minor loss in orifice plate
  2.0000E-03 a Area      m^2              2.2140E+04 A |p|       Pa
     1.0000  b   K+                       -140.59    B Ph(p)     deg
     1.0000  c   K-                       2.3148E-02 C |U|     m^3/s
                                           128.65    D Ph(U)     deg
                                          4.1756E-09 E Htot      W
                                            -3.3973  F Edot      W
!---------------------------------- 13 ----------------------------------
 SURFACE    surface area on bottom of orifice plate
 sameas  11a a Area      m^2              2.2140E+04 A |p|       Pa
                                          -140.59    B Ph(p)     deg
                                          2.3148E-02 C |U|     m^3/s
                                           128.64    D Ph(U)     deg
                                          4.1756E-09 E Htot      W
 ideal      Solid type                      -3.4370  F Edot      W
!---------------------------------- 14 ----------------------------------
 DUCT       Post-orifice pipe
 sameas   9a a Area      m^2              1.9565E+04 A |p|       Pa
 sameas   9b b Perim      m               -140.52    B Ph(p)     deg
 sameas   9c c Length     m               4.4525E-02 C |U|     m^3/s
                                           129.01    D Ph(U)     deg
                                          4.1756E-09 E Htot      W
 ideal      Solid type                      -3.6358  F Edot      W
```

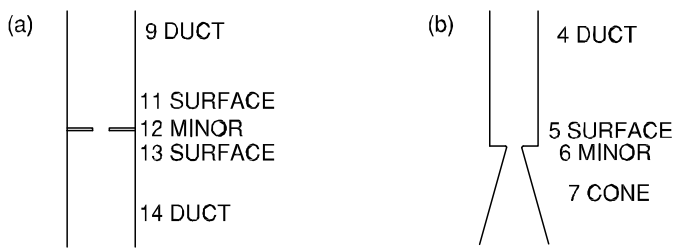


Figure 10.1: Examples of the use of `MINOR`. (a) Orifice plate in a channel. (b) Conical gas diode.

This is an example of a conical gas diode, such as is described in Ref. [41] and illustrated in Fig. 10.1(b).

```
!---------------------------------  4 ----------------------------------
 DUCT       quarter-wave duct from engine to diode
  7.0000E-04 a Area      m^2     S= -2    7705.2     A |p|       Pa
  9.3748E-02 b Perim      m    Fn(  4a)   -161.41    B Ph(p)     deg
     4.6000  c Length     m               5.4080E-02 C |U|     m^3/s
  9.0500E-02 d Srough                      -87.804   D Ph(U)     deg
                                           354.15    E Htot      W
 ideal      Solid type                     58.812   F Edot      W
!---------------------------------  5 ----------------------------------
 RPN        surface area of adapter plate
     0.0000  a G or T            (t)      3.0000E-04 A ChngeMe
   4a 8a -
!---------------------------------  6 ----------------------------------
 SURFACE    surface area of adapter plate
 sameas   5A a Area      m^2              7705.2     A |p|       Pa
                                          -161.41    B Ph(p)     deg
                                          5.4080E-02 C |U|     m^3/s
                                           -87.804   D Ph(U)     deg
                                           354.15    E Htot      W
 ideal      Solid type                     58.812   F Edot      W
!---------------------------------  7 ----------------------------------
 RPN        calculate Borda-Carnot minor loss here
     0.0000  a G or T            (t)         0.1837  A ChngeMe
   1 8a 4a / - sqrd
```

```
!--------------------------------  8 ---------------------------------
 MINOR      asymmetric flow at gas diode
  4.0000E-04 a Area      m^2              7402.7     A |p|       Pa
  5.0000E-02 b   K+                       -174.70    B Ph(p)     deg
 sameas   7A c   K-                       5.4080E-02 C |U|     m^3/s
                                           -87.804   D Ph(U)     deg
                                           354.15    E Htot      W
                                            10.829   F Edot      W
!--------------------------------  9 ---------------------------------
 CONE       expansion to plenum
 sameas   8a a AreaI     m^2     S= -9    1.7471E+04 A |p|       Pa
  7.0899E-02 b PerimI     m    Fn(  9b)   -177.49    B Ph(p)     deg
     0.4000  c Length     m               4.9949E-02 C |U|     m^3/s
  1.5000E-02 d AreaF     m^2     S= -9     -87.809   D Ph(U)     deg
     0.4342  e PerimF     m    Fn(  9d)    354.15    E Htot      W
  5.0000E-04 f Srough                        2.4219  F Edot      W
 ideal      Solid type
```

Other examples of `MINOR` are found in Sections 6.3 and 6.4.

Pressure calculations for $\dot{N} = 0$:

In the $\dot{N} \to 0$ limit, changes in p_1 across segment type `MINOR` are given by

$$p_{1,\text{out}} - p_{1,\text{in}} = -\left(K_- + K_+\right) \frac{2\rho_m \left|U_1\right|}{3\pi A^2} U_1, \tag{10.35}$$

and changes in `p20HL` are given by

$$p_{2,0,HL,\text{out}} - p_{2,0,HL,\text{in}} = \left(K_- - K_+\right) \frac{\rho_m \left|U_1\right|^2}{8A^2}. \tag{10.36}$$

Pressure calculations for $\dot{N} \neq 0$:

For $\dot{N} \neq 0$, changes in p_1 across segment type `MINOR` are given by

$$\begin{aligned} p_{1,\text{out}} - p_{1,\text{in}} &= \varepsilon K_- \frac{\rho_m \left|U_1\right|}{A^2} U_1 \text{ for } \varepsilon \leq -1, && (10.37) \\ &= -\left[\left(1 + \frac{\varepsilon^2}{2}\right)\sqrt{1-\varepsilon^2} + \frac{3}{2}\varepsilon \sin^{-1}\varepsilon - \frac{K_- - K_+}{K_- + K_+}\frac{3\pi\varepsilon}{4}\right]\left(K_- + K_+\right) \\ &\quad \times \frac{2\rho_m \left|U_1\right|}{3\pi A^2} U_1 \text{ for } |\varepsilon| \leq 1, && (10.38) \\ &= -\varepsilon K_+ \frac{\rho_m \left|U_1\right|}{A^2} U_1 \text{ for } \varepsilon \geq 1, && (10.39) \end{aligned}$$

where $\varepsilon = m\dot{N}/\rho_m |U_1|$. Changes in `p2OHL` are given by

$$
\begin{aligned}
p_{2,0,HL,\text{out}} - p_{2,0,HL,\text{in}} &= K_- \frac{m^2 \dot{N}_{2,0}^2}{2\rho_m A^2} \left(1 + \frac{1}{2\varepsilon_{2,0}^2} + \frac{|p_1|}{\gamma p_m} \frac{\cos\phi}{\varepsilon_{2,0}}\right) \text{ for } \varepsilon_{2,0} \le -1, \quad (10.40) \\
&= \frac{\rho_m |U_1|^2}{8A^2} \left\{ (K_- - K_+) \left(1 + 2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0} \frac{|p_1|}{\gamma p_m} \cos\phi\right) \right. \\
&\quad - (K_+ + K_-) \frac{2}{\pi} \left[\left(1 + 2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0} \frac{|p_1|}{\gamma p_m} \cos\phi\right) \sin^{-1} \varepsilon_{2,0} \right. \\
&\quad \left. \left. + \left(3\varepsilon_{2,0} + \frac{4}{3}\left[1 + \varepsilon_{2,0}^2/2\right] \frac{|p_1|}{\gamma p_m} \cos\phi\right) \sqrt{1 - \varepsilon_{2,0}^2} \right] \right\} \\
&\qquad \text{for } |\varepsilon_{2,0}| \le 1, \quad (10.41) \\
&= -K_+ \frac{m^2 \dot{N}_{2,0}^2}{2\rho_m A^2} \left(1 + \frac{1}{2\varepsilon_{2,0}^2} + \frac{|p_1|}{\gamma p_m} \frac{\cos\phi}{\varepsilon_{2,0}}\right) \text{ for } \varepsilon_{2,0} \ge 1, \quad (10.42)
\end{aligned}
$$

where $\varepsilon_{2,0} = m\dot{N}_{2,0}/\rho_m |U_1|$ and ϕ is the phase angle between p_1 and U_1. Derivations of these equations neglecting the terms in p_1 are given in Ref. [41]; and derivations including the p_1 terms appear in Chapter 14.

Other calculations

Segment `MINOR` leaves T_m and U_1 unchanged.

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}}$ in insulated mode, described in Section 11.1.2. This assumes that the component is thermally insulated, so that the heat generated by acoustic power dissipation must somehow be deposited elsewhere, either upstream or downstream, such as in a nearby heat exchanger (or can flow out through a `BEGIN` or `**END` segment). The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{E}_{\text{out}}$ for anchored mode, described in Section 11.1.2. This essentially assumes that the component is thermally anchored, such as by a water bath, so heat generated by acoustic power dissipation is carried away locally.

10.3. Series transducers

10.3.1. `VESPEAKER`, `IESPEAKER`, `VEDUCER`, and `IEDUCER`

General description:

The `*EDUCER`s and `*ESPEAKER`s are electroacoustic transducers with both ends "Enclosed" in the DeltaEC model, so that they are in series between the previous segment and the subsequent segment.

`*ESPEAKER`s let the user specify mass, spring constant, force constant, resistance, and inductance, so that frequency-dependent (even resonant) electrodynamic transducers (e.g., linear motors, linear alternators) can be modeled easily. These `*ESPEAKER` segments can be used to model moving-magnet transducers as well as moving-coil transducers; in the former

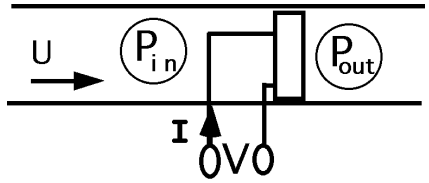


Figure 10.2: Enclosed *EDUCER or *ESPEAKER.

case, set the Bl product equal to the moving-magnet transducer's force constant, expressed in newtons per amp.

*EDUCERs have specified transduction coefficients, which can be linked to the results of RPN calculations upstream to create the frequency or amplitude dependences of piezoelectric or any other type of transducer.

With IEDUCER and IESPEAKER, the user specifies the complex current, and each pass of DeltaEC calculates the complex voltage; with VEDUCER and VESPEAKER, the user specifies complex voltage, and DeltaEC computes complex current. VEDUCER and VESPEAKER cannot be used with zero electrical impedance because this would lead to a division by zero (see below). Hence, use IEDUCER or IESPEAKER for "electrically ideal" transducers.

*ESPEAKERs incorporate thermal-hysteresis losses on their upstream and downstream areas, as if they included two SURFACEs of equal areas but generally different pressure amplitudes. The *EDUCER segments, which have no area parameter, have no built-in thermal-hysteresis losses.

See also Chapter 14 for a discussion of the saturation of magnetic flux in *ESPEAKERs.

See also the side-branch transducers described in Section 10.4.2 below, which have their backsides hanging outside the DeltaEC model.

Input variables, VESPEAKER and IESPEAKER:

Area (m^2) The surface area A of the moving part of the *ESPEAKER. An area A is exposed to oscillating pressures on both the back side and the front side. The velocity of the moving element is U_1/A.

R (ohm) The electrical resistance R_e of the coil.

L (henry) The electrical inductance L of the coil when it is prevented from moving.

BLProd (tesla·m, which is equivalent to N/amp) For a traditional moving-coil loudspeaker, the product of the magnetic field B and the wire length l that is immersed in B. For other electrodynamic transducers, the force in newtons exerted when one amp of current is applied and nothing is allowed to move.

M (kg) The moving mass M of the transducer.

K (N/m) The spring constant K of the transducer. This should include mechanical springs and zero-current magnetic forces, but not gas-pressure forces.

Rm (N·s/m) The mechanical resistance R_m of the transducer.

|V| or |I| (volts or amps) In `VESPEAKER`, the amplitude of the oscillating voltage, $|V_1|$. In `IESPEAKER`, the amplitude of the oscillating current, $|I_1|$.

ph(V) or ph(I) (degrees) In `VESPEAKER`, the phase of the oscillating voltage V_1. In `IESPEAKER`, the phase of the oscillating current I_1.

Solid The solid material on the leading and trailing surfaces A. (This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and infinite specific heat in `*SPEAKER`s.)

Input variables, `VEDUCER` and `IEDUCER`:

Re(Ze) (ohm) The real part of the transducer's electrical impedance Z_e when it is prevented from moving.

Im(Ze) (ohm) The imaginary part of the transducer's electrical impedance Z_e when it is prevented from moving.

Re(T1) (V·s/m^3) The real part of the complex transduction coefficient τ in the equations below.

Im(T1) (V·s/m^3) The imaginary part of the complex transduction coefficient τ in the equations below.

Re(T2) (Pa/A) The real part of the complex transduction coefficient τ' in the equations below.

Im(T2) (Pa/A) The imaginary part of the complex transduction coefficient τ' in the equations below.

Re(Zm) (Pa·s/m^3) The real part of the mechanical impedance Z_m of the transducer.

Im(Zm) (Pa·s/m^3) The imaginary part of the mechanical impedance Z_m of the transducer.

|V| or |I| (volts or amps) In `VEDUCER`, the amplitude of the oscillating voltage V_1. In `IEDUCER`, the amplitude of the oscillating current I_1.

ph(V) or ph(I) (degrees) In `VEDUCER`, the phase of the oscillating voltage V_1. In `IEDUCER`, the phase of the oscillating current I_1.

Potential targets:

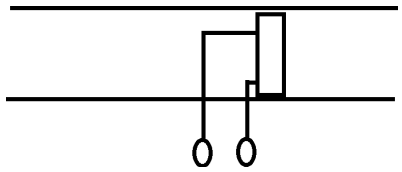
None.

Master–slave links:

None. The use of `sameas` is sometimes convenient in `*EDUCER`s, to keep $\tau' = \tau$.

Format and examples:

```
VEDUCER      Enclosed driver
  1.000E-09 a Re(Ze)   ohms
   .000     b Im(Ze)   ohms
  1.000E+04 c Re(T1) V-s/m^3
   .000     d Im(T1) V-s/m^3
 -1.000E+04 e Re(T2)   Pa/A
   .000     f Im(T2)   Pa/A
  1.000E-09 g Re(Zm) Pa-s/m^3
  1.000E-09 h Im(Zm) Pa-s/m^3
  10.0      i  Vin      V
  45.0      j Ph(Vin)  deg
```



`IEDUCER`: same as `VEDUCER`, except that current appears in lines `i` and `j` instead of voltage.

```
 VESPEAKER
  6.000E-04 a Area      m^2
   6.00     b   R      ohms
   .000     c L         H
   8.00     d B x L    T-m
  5.000E-03 e   M      kg
  1.000E+04 f   K      N/m
   .000     g   Rm    N-s/m
  62.0      h  Vin      V
 -37.2      i Ph(Vin)  deg
kapton      Solid type
```

`IESPEAKER`: same as `VESPEAKER`, except that current appears in line `h` and `i` instead of voltage.

Other examples of series `*ESPEAKER`s are in Sections 5.2 and 5.8.2.

Calculations:

An Enclosed transducer `IEDUCER`, `VEDUCER`, `IESPEAKER`, `VESPEAKER` is an object attached in series with other segments, as shown in Fig. 10.2. Volume flow rate U_1 is nearly unchanged (except for surface thermal-hysteresis losses—see below), but pressure is changed by the force exerted by the transducer, obeying the complex canonical equations

$$V_1 = Z_e I_1 - \tau U_1, \tag{10.43}$$

$$p_{1,\text{out}} - p_{1,\text{in}} = \tau' I_1 - Z_m U_1. \tag{10.44}$$

- If current I_1 is specified, then use `IESPEAKER` or `IEDUCER`, and find $p_{1,\text{out}} = p_{1,\text{in}} + \tau' I_1 - Z_m U_1$ and $V_1 = Z_e I_1 - \tau U_1$.
- If voltage V_1 is specified, then use `VESPEAKER` or `VEDUCER`, and find $I_1 = (V_1 + \tau U_1)/Z_e$ and $p_{1,\text{out}} = p_{1,\text{in}} + \tau' I_1 - Z_m U_1$.
- If a fixed electrical load impedance Z_{ext} is attached to the transducer, it can be modeled using an `IMPEDANCE` segment, with $Z_{\text{imp}} = Z_m - \tau\tau'/(Z_e + Z_{\text{ext}})$.

For `**DUCER`s, the complex electrical impedance $Z_e = \text{Re}[Z_e] + i\text{Im}[Z_e]$ and the complex mechanical impedance $Z_m = \text{Re}[Z_m] + i\text{Im}[Z_m]$ are given explicitly by the user.

In the case of `**SPEAKER`s, $Z_e = R_e + i\omega L$; $\tau = -\tau' = Bl/A$; $Z_m = R_m/A^2 + i(\omega M - K/\omega)/A^2$. Thermal surface losses are computed for area A using the same approach as for a `SURFACE`, but Enclosed speakers have area A exposed to $p_{1,\text{in}}$ and area A exposed to $p_{1,\text{out}}$, because typically both sides of the speaker experience oscillatory pressure. This

thermal hysteresis on those two surfaces manifests itself as a small change in volume flow rate, according to

$$U_{1,\text{out}} = U_{1,\text{in}} - \frac{\omega}{\rho_m a^2} \frac{\gamma - 1}{1 + \epsilon_s} \frac{\delta_\kappa}{2} A \left(p_{1,\text{in}} + p_{1,\text{out}} \right), \tag{10.45}$$

and U_1 in Eqs. (10.43) and (10.44) is actually the value after the *first* thermal-hysteresis loss:

$$U_1 = U_{1,\text{in}} - \frac{\omega}{\rho_m a^2} \frac{\gamma - 1}{1 + \epsilon_s} \frac{\delta_\kappa}{2} A p_{1,\text{in}}. \tag{10.46}$$

The `Px` outputs for these segments give the complex pressure difference across the transducer.

None of these segments affects T_m.

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}} + \dot{W}_{\text{elec}}$ in insulated mode, described in Section 11.1.2. This assumes that the component is thermally insulated, so that the electrical energy added at the transducer must flow out of the segment, either upstream or downstream. The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{E}_{\text{out}}$ for thermally anchored mode, described in Section 11.1.2.

It is hard for us to anticipate what a user might intend if nonzero $\dot{N}$ were to flow through a series transducer, so these segments behave the same whether $\dot{N} = 0$ or not.

10.4. Side-branch transducers and side-branch impedances

10.4.1. `BRANCH`, `OPNBRANCH`, and `PISTBRANCH`

General description:

`BRANCH`, `OPNBRANCH`, and `PISTBRANCH` are side branches with specified impedances. With `BRANCH`, the user specifies the real and imaginary parts of the impedance, assumed independent of frequency (unless linked by `sameas` to results of frequency-dependent RPN calculations). `OPNBRANCH` and `PISTBRANCH` incorporate the frequency dependence of radiation impedance. Radiation impedance at the end of an open tube radiating into 4π solid angle can be modeled as an `OPNBRANCH`, while `PISTBRANCH` approximates the radiation impedance of a baffled piston of the given radius.

(If a side-branch transducer has nonzero $\dot{N}$ flowing past it, the user might consider what fraction of the energy added by the transducer goes into raising the temperature of the steady flow passing by and what fraction is added to the thermoacoustic wave. In a heat-exchanger segment, $F_{\dot{Q}\dot{N}}$, (`FracQN`) determines these fractions, but with a side-branch transducer an `RPN` segment can be used, with user-specified assignment of these fractions using `=Tm` and `=H2k`.)

Input variables, `BRANCH`:

Re(Z) (Pa·s/m^3) The real part of the branch impedance Z_{br}.

Im(Z) (Pa·s/m^3) The imaginary part of the branch impedance Z_{br}.

HtotBr (W) How much power $\dot{H}$ flows out of the trunk into the branch.

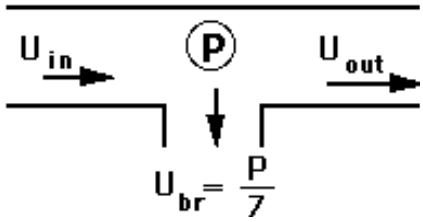


Figure 10.3: BRANCH, OPNBRANCH, or PISTBRANCH.

Input variables, OPNBRANCH:

Re(Z)/k^2 (Pa·s/m) The real part of the branch impedance $Z_{\rm br}$, divided by the square of the wave number $k = \omega/a$.

Im(Z)/k (Pa·s/m^2) The imaginary part of the branch impedance $Z_{\rm br}$, divided by the wave number $k = \omega/a$.

HtotBr (W) How much power $\dot{H}_{\rm tot}$ flows out of the trunk into the branch.

Input variables, PISTBRANCH:

Radius (m) The radius r of the circular, baffled radiating surface.

Potential targets:

None.

Master–slave links:

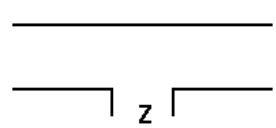
The imaginary part of a BRANCH impedance can be slaved to the real part, to hold the magnitude constant or to hold the phase constant when the real part is changed.

No master–slave options are available for OPNBRANCH and PISTBRANCH.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

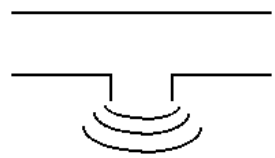
Format and examples:

```
BRANCH
1  Pa-s/m3  Re(Z)
1.  Pa-s/m3  Im(Z)
12.5  W      HtotBr
```




```
OPNBRANCH
0.05  Pa-s/m  Re(Z)/k^2
0.20  Pa-s/m2 Im(Z)/k
12.5   W      HtotBr
```

```
PISTBRANCH     Baffled Piston
0.05  Radius   m
```



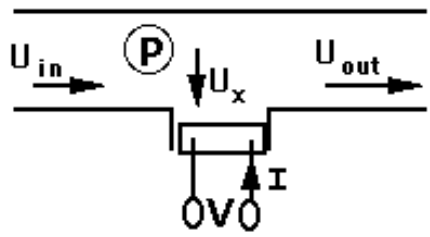


Figure 10.4: The side-branched transducers: `IDUCER`, `ISPEAKER`, `VDUCER`, or `VSPEAKER`.

Calculations:

A `BRANCH` is a side branch with complex impedance $Z_{\rm br}$. Pressure is unchanged, but volume flow rate changes according to $U_{1,\rm out} = U_{1,\rm in} - p_1/Z_{\rm br}$.

For an `OPNBRANCH`, the numbers in lines `a` and `b` are multiplied by $(\omega/a)^2$ and ω/a, respectively, to obtain the impedance $Z_{\rm br}$.

For a baffled piston of radius r where the wave number is $k = \omega/a$ locally, the `PISTBRANCH` radiation impedance is given by

$$Z_{\rm br} = \frac{\rho_m a}{A}\left(1 - \frac{2J_1(2kr)}{2kr} + i\begin{cases}\left(\frac{4/\pi}{2kr} + \frac{\sqrt{8/\pi}\sin(2kr-3\pi/4)}{(2kr)^{3/2}}\right) & \text{if } 2kr > 2.68,\\ \frac{(4/\pi)2kr}{3}\left(1 - \frac{(2kr)^2}{15}\right) & \text{otherwise.}\end{cases}\right) \tag{10.47}$$

Note the implicit assumption that the outside and inside gases have the same density and sound speed.

Output `EdotBr` gives the acoustic power $\dot{E}_{\rm br}$ flowing into the branch.

None of these segments affects T_m. In `BRANCH` and `OPNBRANCH`, the exiting energy flow is computed as $\dot{H}_{2,k,\rm out} = \dot{H}_{2,k,\rm in} - \dot{H}_{\rm br}$ in insulated mode, described in Section 11.1.2. The exiting energy flow is computed as $\dot{H}_{2,k,\rm out} = \dot{H}_{2,k,\rm in} - \dot{E}_{\rm br}$ in `PISTBRANCH` in insulated mode. In thermally anchored mode, described in Section 11.1.2, $\dot{H}_{2,k,\rm out} = \dot{E}_{\rm out}$ in all three of these segments.

The behavior of these segments is independent of $\dot{N}$.

10.4.2. `VSPEAKER`, `ISPEAKER`, `VDUCER`, and `IDUCER`

General description:

The `*DUCER`s and `*SPEAKER`s without "`E`" as their second character are electroacoustic transducers mounted as side branches to the apparatus. `*DUCER`s have frequency-independent parameters; `*SPEAKER`s let the user specify mass, spring constant, force constant, resistance, and inductance, so that frequency-dependent (even resonant) transducers can be modeled. With `IDUCER` and `ISPEAKER`, the user specifies the (real) current, and each pass of DeltaEC calculates the complex voltage; with `VDUCER` and `VSPEAKER`, the user specifies the (real) voltage, and DeltaEC computes complex current. `IDUCER` and `ISPEAKER` cannot be used with zero mechanical impedance because this would lead to a division by zero (see below). Hence, use `VDUCER` or `VSPEAKER` for lossless resonant or massless-and-springless transducers.

*SPEAKER segments incorporate thermal-hysteresis losses on their area as if they included a SURFACE, but *DUCER segments, which have no area parameter, do not.

Side-branched transducer elements, which accept only the magnitude of voltage or current applied as an input, but not the phase, effectively fix the overall time phase of the entire DeltaEC model by enforcing zero time phase for that voltage or current. The phase of pressure or volume flow rate (or both, if they are both nonzero), as given in the BEGIN statement, must usually be allowed to vary (i.e., guessed) to accommodate this condition. This effectively limits a model to only one V or ISPEAKER (or *DUCER), unless they are wired exactly in phase (or 180° out of phase, which can be accommodated with a minus sign).

See also Chapter 14 for a discussion of the saturation of magnetic flux in *SPEAKERs.

See also the series transducers described in Section 10.3.1 above.

Input variables, VSPEAKER and ISPEAKER:

Area (m^2) The surface area S of the moving part of the *SPEAKER. An area S is exposed to oscillating pressure on the front side. The velocity of the moving element is U_1/S.

R (ohm) The electrical resistance R_e of the coil.

L (henry) The electrical inductance L of the coil when it is prevented from moving.

BLProd (tesla·meter, which is equivalent to newton/amp) For a traditional moving-coil loudspeaker, the product of the magnetic field B and the wire length l immersed in B. For other electrodynamic transducers, the force in newtons exerted when one amp of current is applied and nothing is allowed to move.

M (kg) The moving mass M of the transducer.

K (N/m) The spring constant K of the transducer. This should include mechanical springs and magnetic forces, but not gas-pressure forces.

Rm (N·s/m) The mechanical resistance R_m of the transducer.

|V| or |I| (volts or amps) In VSPEAKER, the amplitude of the oscillating voltage V_1, which is taken to be real. In ISPEAKER, the amplitude of the oscillating current I_1, which is taken to be real.

Solid The solid material on the surface S. (This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and infinite specific heat in *SPEAKERs.)

Input variables, VDUCER and IDUCER:

Re(Ze) (ohm) The real part of the transducer's electrical impedance Z_e when it is prevented from moving.

Im(Ze) (ohm) The imaginary part of the transducer's electrical impedance Z_e when it is prevented from moving.

Re(T1) (V·s/m^3) The real part of the complex transduction coefficient τ in the equations below.

Im(T1) (V·s/m^3) The imaginary part of the complex transduction coefficient τ in the equations below.

Re(T2) (Pa/A) The real part of the complex transduction coefficient τ' in the equations below.

Im(T2) (Pa/A) The imaginary part of the complex transduction coefficient τ' in the equations below.

Re(Zm) (Pa·s/m^3) The real part of the mechanical impedance Z_m of the transducer.

Im(Zm) (Pa·s/m^3) The imaginary part of the mechanical impedance Z_m of the transducer.

|V| or |I| (volts or amps) In `VDUCER`, the amplitude of the oscillating voltage V_1, which is taken to be real. In `IDUCER`, the amplitude of the oscillating current I_1, which is taken to be real.

Potential targets:

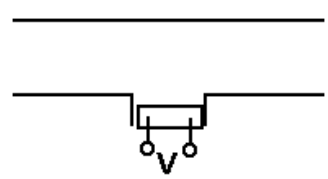
None.

Master–slave links:

None. The use of `sameas` is sometimes convenient in `*DUCER`s, to keep $\tau' = \tau$.

Format and examples:

```
VDUCER
  1.000E-09 a Re(Ze)   ohms
   .000     b Im(Ze)   ohms
  1.000E+04 c Re(T1) V-s/m^3
   .000     d Im(T1) V-s/m^3
 -1.000E+04 e Re(T2)   Pa/A
   .000     f Im(T2)   Pa/A
  1.000E-09 g Re(Zm) Pa-s/m^3
  1.000E-09 h Im(Zm) Pa-s/m^3
  10.0      i AplVol    V
```




```
VSPEAKER
  6.000E-04 a Area      m^2
   6.00     b   R      ohms
   .000     c L         H
   8.00     d B x L    T-m
  5.000E-03 e   M      kg
   .000     f   K      N/m
   .000     g   Rm    N-s/m
  -22.5     h AplVol    V
ideal      Solid type
```

Another example of `VSPEAKER` is in Section 5.2.

Calculations:

A side-branch transducer IDUCER, VDUCER, ISPEAKER, VSPEAKER is an object attached as shown Fig. 10.4 like a branch impedance, but obeying the complex equations $V_1 = Z_e I_1 + \tau U_x$, $p_1 = \tau' I_1 + Z_m U_x$. Pressure is unchanged, but volume flow rate changes according to $U_{1,\text{out}} = U_{1,\text{in}} - U_x$. The volume flow rate of the transducer, U_x, is displayed in the output column.

- If current I_1 is specified (note! its phase must be real), then use ISPEAKER or IDUCER, and find $U_x = (p_1 - \tau' I_1)/Z_m$ and $V_1 = Z_e I_1 + \tau U_x$.
- If voltage V_1 is specified (note! its phase must be real), then use VSPEAKER or VDUCER, and find $I_1 = (Z_m V_1 - \tau p_1)/(Z_e Z_m - \tau\tau')$ and $U_x = (V_1 - Z_e I_1)/\tau$.
- If a fixed electrical load impedance Z_{ext} is attached to the transducer, it can be modeled using a BRANCH segment, with $Z_{\text{br}} = p_1/U_x = Z_m - \tau\tau'/(Z_e + Z_{\text{ext}})$.

In the case of speakers, $Z_e = R_e + i\omega L$; $\tau = -\tau' = Bl/S$; $Z_m = R_m/S^2 + i(\omega M - K/\omega)/S^2$. Side-branch speakers are assumed to have area S exposed to the oscillating pressure. Thermal-hysteresis surface losses are computed for that area S using the same formula as for a SURFACE, Eqs. (10.30) and (10.31). As described in Section 10.2.2, thermal-hysteresis losses manifest themselves as a small change in volume flow rate.

Note that IDUCER and ISPEAKER will crash if Z_m is zero, so it is best to use VDUCER or VSPEAKER for mechanically ideal or lossless resonant transducers.

None of these segments affects T_m.

The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}} + \dot{W}_{\text{elec}}$ in insulated mode, described in Section 11.1.2. This assumes that the component is thermally insulated, so the electrical energy added at the transducer must flow out of the segment, either upstream or downstream. The exiting energy flow is computed as $\dot{H}_{2,k,\text{out}} = \dot{E}_{\text{out}}$ for thermally anchored mode, described in Section 11.1.2.

These segments behave the same whether $\dot{N} = 0$ or not.

10.5. Stacks and regenerators

The first three letters STK indicate a segment[1] that can develop a nonzero dT_m/dx. These segments are used to model the stacks of standing-wave thermoacoustic engines and refrigerators, the regenerators of Stirling engines and refrigerators (including pulse-tube refrigerators), the thermal buffer tubes of thermoacoustic-Stirling hybrid engines and refrigerators, and the pulse tubes of pulse-tube refrigerators. They are collected into three groups here: the smooth porous media STKSLAB, STKRECT, STKCIRC, and STKPIN, which can be used for either stacks and regenerators and are described in Section 10.5.1, the tortuous porous media STKSCREEN and STKPOWERLW for modeling regenerators, described in Section 10.5.2, and the boundary-layer STKDUCT and STKCONE segments, mostly used for pulse tubes and thermal buffer tubes and described in Section 10.6.1. If pore size or plate separation is much greater than thermal and viscous penetration depths, use STKDUCT or STKCONE.

[1] Segments MIXT**, described in Section 10.9.2, also develop nonzero dT_m/dx.

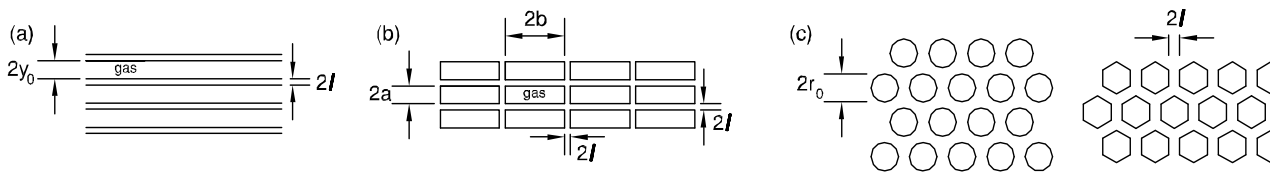


Figure 10.5: Common smooth-stack geometries, with the x axis perpendicular to the page. (a) STKSLAB. (b) STKRECT. (c) STKCIRC.

10.5.1. STKSLAB, STKCIRC, STKRECT, and STKPIN

General description of smooth stacks and regenerators:

Use STKSLAB for parallel-plate or spiral stacks or regenerators. Use STKRECT for square pores or rectangular pores whose aspect ratio is not large enough to justify the use of STKSLAB (see Ref. [57]). Use STKCIRC for circular or hexagonal pores. Use STKPIN for stacks comprised of pin arrays (see Ref. [58] for geometrical details of pin arrays).

In STKSLABs, STKRECTs, STKCIRCs, and STKPINs, the "Area" (the first line of the segment) is the total cross sectional area of the stack assembly, including both gas cross section and solid cross section.

In STKSLABs, STKRECTs, and STKCIRCs, $A_{\text{gas}} = (\texttt{Area}) \times (\texttt{GasA/A})$ and $A_{\text{solid}} = (\texttt{Area}) \times (1 - \texttt{GasA/A})$. Plate half thickness (the 4th line of the segment) is used only for computing ϵ_s, not for computing heat conduction along x or what fraction of the Area is available to the gas. This allows separate accounting for area blocked by "ideal" fins and by support struts or other structure as shown in Fig. 2.10(e). In most cases, ϵ_s is near 0, so plate thickness need not be specified with much accuracy; GasA/A is far more important.

Because of the need to compute specialized functions, STKCIRCs compute more slowly than STKSLABs or STKDUCTs; STKPINs are slower still, and STKRECTs are very slow, especially for large aspect ratios. Hence, in the latter case, it may be convenient to use STKSLABs, at least until a design is almost finalized.

Input variables, STKSLAB:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$.

GasA/A The volumetric porosity $\phi = A_{\text{gas}}/A = y_0/(y_0 + l)$, where A_{gas} is the thermoacoustic area. The solid fraction $1 - \phi$ is used for the calculation of axial thermal conduction by the solid.

Length (m) The length Δx of the stack or regenerator.

y0 (m) Half the plate spacing is y_0. Thus, $2y_0$ is the gap available to the gas—not the center-to-center spacing of the plates. See Fig. 10.5(a).

Lplate (m) Half the thickness of the solid plate is l. Thus, $2l$ is the plate thickness. See Fig. 10.5(a). This variable is used in the calculation of ϵ_s to account for any imper-

fection in the imposition of the isothermal boundary condition imposed on the gas by the solid.

Solid The solid of which the plates are made.

Input variables, `STKRECT`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$.

GasA/A The volumetric porosity $\phi = A_{\text{gas}}/A$. The solid fraction $1 - \phi$ is used for the calculation of axial thermal conduction by the solid. Often, the user intends $\phi = ab/\left(ab + al + bl + l^2\right)$; this can be enforced with a master–slave link (see below).

Length (m) The length Δx of the stack or regenerator.

a (m) Half of pore width. Thus, $2a$ is the small dimension of the rectangular pore available to the gas. See Fig. 10.5(b).

Lplate (m) Half the thickness of the solid plate. Thus, $2l$ is the plate thickness. This variable is used for the calculation of ϵ_s to account for any imperfection in the imposition of the isothermal boundary condition imposed on the gas by the solid. See Fig. 10.5(b).

b (m) Half of pore breadth $2b$. Thus, $2a \times 2b$ is the cross-sectional area available to the gas in a single pore. See Fig. 10.5(b).

Solid The solid of which the stack is made.

Input variables, `STKCIRC`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$.

GasA/A The volumetric porosity $\phi = A_{\text{gas}}/A$. The solid fraction $1 - \phi$ is used for the calculation of axial thermal conduction by the solid.

Length (m) The length Δx of the stack or regenerator.

radius (m) The radius r_0 of the circular pore. Thus, πr_0^2 is the cross-sectional area available to the gas in a single pore. (When we use `STKCIRC` to model hexagonal "honeycomb" pores, we choose r_0 so that the pore has the correct cross-sectional area: $r_0 = \left(\sqrt[4]{3}/\sqrt{2\pi}\right) b \simeq 0.525038b$, where b is the flat-to-flat distance across the hexagonal pore.)

Lplate (m) Half the thickness of the solid sheet between pores. Thus, $2l$ is the plate thickness. This variable is used in the calculation of ϵ_s to account for any imperfection in the imposition of the isothermal boundary condition imposed on the gas by the solid.

Solid The solid of which the sheets are made.

Input variables, `STKPIN`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$, but usually not intended to include the cross-sectional area of the pressure-vessel wall.

2y0 (m) $2y_0$ is the center-to-center, nearest-neighbor pin-to-pin distance in the hexagonal lattice of pins. See Ref. [58].

Length (m) The length Δx of the stack or regenerator.

R pin (m) The radius r_i of each pin.

Solid The solid of which the pins are made.

Potential targets:

None.

Master–slave links:

The porosity ϕ (= `GasA/A`) of some of these segments can be slaved to other parameters:

In `STKSLAB`, the porosity ϕ can be slaved to other parameters through the equation

$$\phi = c\frac{y_0}{y_0 + l} \ , \tag{10.48}$$

where c is calculated when the link is first enabled, using

$$c = \phi\frac{y_0 + l}{y_0} \ . \tag{10.49}$$

If $c = 1$, this simply means that all of the cross section is full of gas and plate, with no struts, support members, or pressure-vessel wall. Otherwise, $c \neq 1$ indicates that a fraction of the area is occupied by such structure; the link keeps that fraction a constant.

In `STKRECT`, the porosity ϕ can be slaved to other parameters through the equation

$$\phi = \frac{ab}{(a + l)(b + l)} \ , \tag{10.50}$$

so the geometry looks exactly as shown in Fig. 10.5. No constant c is used.

In `STKCIRC`, the porosity ϕ can be slaved to other parameters through the equation $\phi = \min(cr_0^2/A, 1)$, with c calculated when the link is first enabled using $c = \phi A/r_0^2$. This link maintains a constant number of holes, but with the porosity clipped at a maximum value of 1.

The length of any of these segments can be slaved to the length of another segment to keep the total length constant.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples:

```
STKSLAB   parallel-plate stack
sameas 1  Area
0.724     GasA/A
7.85e-2 m Length
1.8e-4 m  y0 (half the plate spacing)
4.0e-5 m  Lplate (half the plate thickness)
kapton    Solid
```

```
STKRECT  rectangular-pore stack
sameas 1  Area
0.694     GasA/A
7.85e-2 m Length
2.0e-4 m   a (half of pore width)
4.0e-5 m  Lplate (half the plate thickness)
4.0e-4 m   b  (pore area is 2a times 2b)
stainless Solid

STKCIRC approximates hexagonal honeycomb stack
sameas 1  (m^2) total area
0.81     gas area/total area
0.279    (m) length
0.50e-3  (m) radius of circular pore
0.05e-3  (m) L:half of sheet thickness
stainless     stack material

STKPIN  Muller/Keolian pinstack invention
sameas 2a a area   m^2
3.2e-4    b 2y0     m   2y0 = nearest-neighbor center-to-center distance
!                             in the hexagonal lattice
0.1       c Length  m
4.e-5     d R pin   m   pin radius
stainless
```

Another example of `STKSLAB` is in Section 5.2. `STKCIRC` is used in Section 5.5, and `STKRECT` is used in Section 6.2.

Calculations in smooth stacks and regenerators for $\dot{N} = 0$:

Pressure propagates according to Rott's wave equation, written in the form

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\rm gas}}U_1, \tag{10.51}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\rm gas}}{\rho_m a^2}\left(1+\frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1 + \frac{\beta(f_\kappa-f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}U_1, \tag{10.52}$$

subject to the condition that the total energy flow $\dot{H}_{2,k}$ is independent of x, which imposes the following condition on $T_m(x)$:

$$\frac{dT_m}{dx} = \frac{\dot{H}_{2,k} - \frac{1}{2}\mathrm{Re}\left[p_1\tilde{U}_1\left(1-\frac{T_m\beta(f_\kappa-\tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_m c_p|U_1|^2}{2\omega A_{\rm gas}(1-\sigma)|1-f_\nu|^2}\mathrm{Im}\left[\tilde{f}_\nu+\frac{(f_\kappa-\tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] - A_{\rm gas}k - A_{\rm solid}k_s}. \tag{10.53}$$

For `STKSLAB`,

$$f_\kappa = \frac{\tanh[(1+i)y_0/\delta_\kappa]}{(1+i)y_0/\delta_\kappa}, \quad f_\nu = \frac{\tanh[(1+i)y_0/\delta_\nu]}{(1+i)y_0/\delta_\nu}, \tag{10.54}$$

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s\rho_s c_s}\right)^{1/2}\frac{\tanh[(1+i)y_0/\delta_\kappa]}{\tanh[(1+i)\ell/\delta_s]}. \tag{10.55}$$

For STKRECT,

$$f_\kappa = 1 - \frac{64}{\pi^4}\sum_{\substack{m,n\\ \text{odd}}} \frac{1}{m^2 n^2 Y_{mn}(\delta_\kappa)}, \quad f_\nu = 1 - \frac{64}{\pi^4}\sum_{\substack{m,n\\ \text{odd}}} \frac{1}{m^2 n^2 Y_{mn}(\delta_\nu)}, \tag{10.56}$$

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s \rho_s c_s}\right)^{1/2} \frac{f_\kappa (1+i) ab/\delta_\kappa (a+b)}{\tanh[(1+i)\ell/\delta_s]}, \tag{10.57}$$

$$\text{where } Y_{mn}(\delta) = 1 - i\frac{\pi^2\delta^2}{8a^2b^2}(b^2m^2 + a^2n^2). \tag{10.58}$$

For STKCIRC,

$$f_\kappa = \frac{2J_1[(i-1)r_0/\delta_\kappa]}{(i-1)(r_0/\delta_\kappa)J_0[(i-1)r_0/\delta_\kappa]}, \quad f_\nu = \frac{2J_1[(i-1)r_0/\delta_\kappa]}{(i-1)(r_0/\delta_\kappa)J_0[(i-1)r_0/\delta_\kappa]}, \tag{10.59}$$

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s \rho_s c_s}\right)^{1/2} \frac{f_\kappa (1+i) r_0/2\delta_\kappa}{\tanh[(1+i)\ell/\delta_s]}. \tag{10.60}$$

For STKPIN,

$$f_\nu = -\frac{\delta_\nu}{(i-1)}\frac{2r_i}{r_o^2 - r_i^2}\frac{Y_1[(i-1)r_o/\delta_\nu]J_1[(i-1)r_i/\delta_\nu] - J_1[(i-1)r_o/\delta_\nu]Y_1[(i-1)r_i/\delta_\nu]}{Y_1[(i-1)r_o/\delta_\nu]J_0[(i-1)r_i/\delta_\nu] - J_1[(i-1)r_o/\delta_\nu]Y_0[(i-1)r_i/\delta_\nu]},$$
$$f_\kappa = -\frac{\delta_\kappa}{(i-1)}\frac{2r_i}{r_o^2 - r_i^2}\frac{Y_1[(i-1)r_o/\delta_\kappa]J_1[(i-1)r_i/\delta_\kappa] - J_1[(i-1)r_o/\delta_\kappa]Y_1[(i-1)r_i/\delta_\kappa]}{Y_1[(i-1)r_o/\delta_\kappa]J_0[(i-1)r_i/\delta_\kappa] - J_1[(i-1)r_o/\delta_\kappa]Y_0[(i-1)r_i/\delta_\kappa]}, \tag{10.61}$$

and

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s \rho_s c_s}\right)^{1/2} \frac{J_0(\sqrt{-i\omega/\kappa_s}r_i)}{J_1(\sqrt{-i\omega/\kappa_s}r_i)} f_\kappa \sqrt{-i\omega/\kappa}\frac{r_o^2 - r_i^2}{2r_i}, \tag{10.62}$$

where $\kappa = k/\rho_m c_p$, $\kappa_s = k_s/\rho_s c_s$, r_i is pin radius, and $r_o^2 = \left(\sqrt{3}/2\pi\right)(2y_o)^2$ with $2y_o$ being the center-to-center distance between nearest-neighbor pins in the hexagonal array of pins.

Calculations in smooth stacks and regenerators for $\dot{N} \neq 0$:

To turn on steady-flow calculations, check the appropriate box in the BEGIN segment. In the affected STK** segments, the dependence of T_m on x can be changed significantly by the nonzero $\dot{N}$.

The display of information is slightly changed. H2k is the "acoustic"-plus-longitudinal-conduction part of the total energy flux—not including the steady-flow part of the total energy flux, $\dot{N}mw_m$, where w_m is the enthalpy per unit mass. Htot is the sum of H2k and $\dot{N}mw_m$.

Nonzero $\dot{N}$ changes the temperature profile in smooth stacks according to Refs. [37, 38, 12]: The temperature gradient in stacks is computed using constancy of total energy flux

$\dot{H}_{\rm tot}$. In other words, we solve this equation for dT_m/dx :

$$\begin{aligned}\dot{H}_{\rm tot} &= \frac{1}{2}{\rm Re}\left[p_1\widetilde{U}_1\left(1-\frac{T_m\beta\left(f_\kappa-\widetilde{f}_\nu\right)}{(1+\epsilon_s)(1+\sigma)\left(1-\widetilde{f}_\nu\right)}\right)\right]\\ &\quad+\frac{\rho_m c_p}{2A_{\rm gas}\omega(1-\sigma)\left|1-f_\nu\right|^2}\frac{dT_m}{dx}\left|U_1\right|^2{\rm Im}\left[\widetilde{f}_\nu+\frac{(f_\kappa-\widetilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right]\\ &\quad-(A_{\rm gas}k+A_{\rm solid}k_s)\frac{dT_m}{dx}+\dot{N}mw_m \qquad (10.63)\end{aligned}$$

and DELTAEC uses $\dot{H}_{\rm tot}$ = constant to integrate with respect to x through a stack. The only $\dot{N}\neq 0$ feature in this equation is the final term, $\dot{N}mw_m$, where w_m is the mean enthalpy per unit mass. The mean enthalpy is referenced to zero at the environment temperature T_0, which is 300 K by default and can be set to another value in an `RPN` segment. As usual, the value of $\dot{H}_{\rm tot}$ is determined by conditions in segments other than the stack, such as the value of $\dot{N}$ set in the `BEGIN` segment and the heat flows in adjacent heat exchangers.

The stack energy equation, Eq. (10.63), is reasonable only if $\dot{N}$ is essentially of second order, and hence only if $\dot{N}$ is formally negligible in the first-order momentum and energy equations [37, 38, 12]. Hence, we have not implemented any Doppler effect on dp_1 and dU_1 in stacks.

In the smooth stacks, $dp_{2,0,HL}/dx$ is calculated with simple laminar expressions based on $\dot{N}_{2,0}$:

$$\frac{dp_{2,0,HL}}{dx} = -\frac{8\mu m\dot{N}_{2,0}}{\rho_m A_{\rm gas}r_0^2}\ \text{for `STKCIRC`}, \qquad (10.64)$$

$$= -\frac{3\mu m\dot{N}_{2,0}}{\rho_m A_{\rm gas}y_0^2}\ \text{for `STKSLAB`}, \qquad (10.65)$$

$$= -\frac{3\mu m\dot{N}_{2,0}}{\rho_m A_{\rm gas}a^2F_{\rm shape}}\ \text{for `STKRECT`}, \qquad (10.66)$$

$$= -\frac{8\mu m\dot{N}_{2,0}}{\rho_m A_{\rm total}r_o^2\left[3+r_i^4/r_o^4-4r_i^2/r_o^2-4\ln\left(r_i/r_o\right)\right]}\ \text{for `STKPIN`}, \qquad (10.67)$$

where the `STKRECT`'s shape factor in Eq. (10.66) is given by

$$F_{\rm shape} = 1-\frac{192}{\pi^5}\frac{a}{b}\sum_{n=1,3,5,\ldots}\frac{\tanh(n\pi b/2a)}{n^5} \qquad (10.68)$$

$$\simeq 1-\frac{192}{\pi^5}\frac{a}{b}\left[\tanh\left(\pi b/2a\right)+\frac{\tanh(3\pi b/2a)}{3^5}+\frac{\tanh(5\pi b/2a)}{5^5}+0.000088536\right]\ \text{for } a\leq b. \qquad (10.69)$$

10.5.2. `STKSCREEN` and `STKPOWERLW`

(For a general description of all of the `STK**` segments, see the first paragraph of Section 10.5.)

General description of tortuous regenerators:

Use `STKSCREEN` for stacked-screen regenerators (see Ref. [59]). Use `STKPOWERLW` for any other regenerator for which friction factor and heat-transfer coefficients follow power laws in Reynolds number. The algorithms for these segments assume reasonably good internal thermal contact, so do not trust these segments for $r_h \sim \delta_k$ and certainly not for $r_h > \delta_k$. DeltaEC has no capability to model the physics of oscillating flow through tortuous porous media except in the small-pore, nearly isothermal limit.

Input variables, `STKSCREEN`:

Area (m^2) The total cross-sectional area $A = A_{\mathrm{gas}} + A_{\mathrm{solid}}$, including the screens but not including the cross-sectional area of the pressure-vessel wall.

VolPor The volumetric porosity ϕ. This is the ratio of the volume of gas in the regenerator to the total volume of the regenerator. For plain square-weave screen with m wires per unit length and wire diameter d_{wire}, we use

$$\phi \simeq 1 - \frac{\pi m d_{\mathrm{wire}}}{4}, \tag{10.70}$$

as recommended by Organ [26], or we weigh the regenerator and use the known density of its wire material to calculate ϕ.

Length (m) The length Δx of the regenerator.

rh (m) The hydraulic radius r_h of the regenerator. This is the ratio of the gas volume to the gas–solid contact area. For plain square-weave screen with wire diameter d_{wire}, we use

$$r_h \simeq d_{\mathrm{wire}} \frac{\phi}{4\,(1-\phi)}, \tag{10.71}$$

as recommended by Organ [26].

ksFrac If the solid material in the regenerator were arranged in smooth columns parallel with the x axis, as in the smooth stacks described above in Section 10.5.1, then the contribution of ordinary thermal conduction in the solid to $\dot{H}_{2,k}$ would be $(1-\phi)\,A\,dT_m/dx$. However, in `STKSCREEN` the solid conduction is reduced by the random point contacts between wires. `ksFrac` is the reduction factor. Reference [27] recommends a value of 0.1. We often use a larger value, in order to include the conductance of the pressure-vessel wall around the regenerator.

Solid The solid of which the wires are made.

Input variables, `STKPOWERLW`:

Area (m^2) The total cross-sectional area $A = A_{\mathrm{gas}} + A_{\mathrm{solid}}$, not including the cross-sectional area of the pressure-vessel wall.

VolPor The volumetric porosity ϕ. This is the ratio of the volume of gas in the regenerator to the total volume of the regenerator.

Length (m) The length Δx of the regenerator.

rh (m) The hydraulic radius r_h of the regenerator. This is the ratio of the gas volume to the gas–solid contact area.

ksFrac If the solid material in the regenerator were arranged in smooth columns parallel with the x axis, as in the smooth stacks described above in Section 10.5.1, then the contribution of ordinary thermal conduction in the solid to $\dot{H}_{2,k}$ would be $(1-\phi)\,A\,dT_m dx$. However, in `STKSCREEN` the solid conduction is reduced by the random point contacts between wires. `ksFrac` is the reduction factor. Reference [27] recommends a value of 0.1. We often use a larger value, in order to include the conductance of the pressure-vessel wall around the regenerator.

f con The friction-factor parameter f_{con} in the equations below.

f exp The friction-factor parameter f_{exp} in the equations below.

h con The heat-transfer parameter h_{con} in the equations below.

h exp The heat-transfer parameter h_{exp} in the equations below.

Solid The solid with which the regenerator is made.

Potential targets:

None.

Master–slave links:

The length of either of these segments can be slaved to the length of another segment to keep the sum of the two lengths constant.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples of tortuous regenerators:

```
STKSCREEN      a screen regenerator
sameas  1a  a Area      m^2  cross section of regenerator
   .673     b VolPor         volumetric porosity
  5.500E-02 c Length     m
  1.830E-05 d   rh       m   hydraulic radius
   .300     e ksFrac         fudge factor F for solid conduction
stainless  Solid type

STKPOWERLW      an etched foil regenerator
sameas  1a  a Area      m^2  cross section of regenerator
   .700     b VolPor         volumetric porosity
   0.04     c Length     m
  40.e-6    d   rh       m   hydraulic radius
   .300     e ksFrac         fudge factor for solid conduction
  36.       f f_con
  1.0       g f_exp
  24.       h h_con
  0.8       i h_exp
stainless  Solid type
```

Other examples of the use of STKSCREEN are in Sections 5.8 and 6.3.

Calculations in tortuous regenerators for $\dot{N} = 0$:

In STKSCREEN regenerators, the pressure, volume flow rate, and mean temperature evolve according to

$$\frac{dp_1}{dx} = -i\omega\rho_m \left[1 + \frac{(1-\phi)^2}{2(2\phi-1)}\right] \langle u_1 \rangle - \frac{\mu}{r_h^2}\left(\frac{c_1(\phi)}{8} + \frac{c_2(\phi)N_{R,1}}{3\pi}\right)\langle u_1 \rangle\,, \tag{10.72}$$

$$\begin{aligned}\frac{d\langle u_1 \rangle}{dx} &= -\frac{i\omega\gamma}{\rho_m a^2}p_1 + \beta\frac{dT_m}{dx}\langle u_1 \rangle + \\ &\quad i\omega\beta\left[\frac{T_m\beta}{\rho_m c_p}\frac{\epsilon_s + (g_c + e^{2i\theta_p}g_v)\epsilon_h}{1+\epsilon_s+(g_c+e^{2i\theta_T}g_v)\epsilon_h}p_1 - \frac{1}{i\omega}\frac{dT_m}{dx}\frac{\epsilon_s+(g_c-g_v)\epsilon_h}{1+\epsilon_s+(g_c+e^{2i\theta_T}g_v)\epsilon_h}\langle u_1 \rangle\right],\end{aligned} \tag{10.73}$$

$$\begin{aligned}\frac{dT_m}{dx} &= \left\{\mathrm{Re}\left[\left(T_m\beta\frac{\epsilon_s+\epsilon_h(g_c+e^{2i\theta_p}g_v)}{1+\epsilon_s+\epsilon_h(g_c+e^{2i\theta_T}g_v)} + 1 - T_m\beta\right)p_1\widetilde{\langle u_1 \rangle}\right] - \frac{2\dot{H}_{2,k}}{\phi A}\right\} \\ &\quad / \left\{\frac{\rho_m c_p}{\omega}\mathrm{Im}\left[\frac{\epsilon_s+\epsilon_h(g_c-g_v)}{1+\epsilon_s+\epsilon_h(g_c+e^{2i\theta_T}g_v)}\right]\langle u_1 \rangle\,\widetilde{\langle u_1 \rangle} + 2\left[k_{s,\mathrm{eff}}\frac{1-\phi}{\phi} + k\right]\right\},\end{aligned} \tag{10.74}$$

using

$$c_1(\phi) = 1268 - 3545\phi + 2544\phi^2, \quad c_2(\phi) = -2.82 + 10.7\phi - 8.6\phi^2, \tag{10.75}$$

$$b(\phi) = 3.81 - 11.29\phi + 9.47\phi^2, \tag{10.76}$$

$$N_{R,1} = 4\,|\langle u_1 \rangle|\, r_h\rho_m/\mu, \tag{10.77}$$

$$\epsilon_s = \phi\rho_m c_p/(1-\phi)\rho_s c_s, \quad \epsilon_h = 8ir_h^2/b(\phi)\sigma^{1/3}\delta_\kappa^2, \tag{10.78}$$

$$\delta_\kappa^2 = 2k/\omega\rho_m c_p, \tag{10.79}$$

$$\theta_p = \mathrm{phase}(\langle u_1 \rangle) - \mathrm{phase}\,(p_1)\,, \quad \theta_T = \mathrm{phase}(\langle u_1 \rangle) - \mathrm{phase}\left(\langle T \rangle_{u,1}\right), \tag{10.80}$$

$$g_c = \frac{2}{\pi}\int_0^{\pi/2}\frac{dz}{1+N_{R,1}^{3/5}\cos^{3/5}(z)}, \quad g_v = -\frac{2}{\pi}\int_0^{\pi/2}\frac{\cos(2z)\,dz}{1+N_{R,1}^{3/5}\cos^{3/5}(z)}. \tag{10.81}$$

These expressions were derived [59] with the assumption that viscous and thermal penetration depths are much larger than r_h. The spatial average oscillatory velocity $\langle u_1 \rangle = \langle U_1 \rangle / \phi A$, where ϕ is volumetric porosity and A is regenerator cross sectional area; and $k_{\mathrm{eff}} = (\texttt{ksFrac})\,k_s$ where ksFrac is a factor to reduce thermal conduction along x due to the poor thermal contact between adjacent screen layers (Radebaugh [27] recommends ksFrac ≈ 0.1). ksFrac can also be used to account for conduction in the case surrounding the regenerator. The trigonometric integrals in Eqs. (10.81) are not evaluated by DeltaEC;

these integrals were performed once, off-line, to obtain simple functional fits that DeltaEC uses during computation.

STKPOWERLW segments are calculated in the same manner as STKSCREEN, with a few exceptions. The steady-flow friction factor and heat transfer coefficients are parameterized by

$$f = f_{\rm con} N_R^{-f_{\rm exp}}, \tag{10.82}$$

$$N_{St}\,\sigma^{2/3} = h_{\rm con} N_R^{-h_{\rm exp}}, \tag{10.83}$$

where N_{St} is the Stanton number and the Reynolds number N_R is defined in the usual way as

$$N_R = \frac{4Ur_h\rho}{\phi A\mu}. \tag{10.84}$$

[Note: Equation (10.82) is Fanning friction factor, the friction factor used by Kays and London [60], so that instantaneously $dp/dx = -(f/r_h)\,\frac{1}{2}\rho u^2$.] The pressure equation is replaced by

$$\frac{dp_1}{dx} = -i\omega\rho_m \langle u_1\rangle - I_f \frac{\mu}{8r_h^2} f_{\rm con} N_{R,1}^{1-f_{\rm exp}} \langle u_1\rangle\,, \tag{10.85}$$

where

$$I_f = \frac{2}{\pi}\int_0^{\pi} \sin^{3-f_{\rm exp}}(z)dz. \tag{10.86}$$

In the volume-flow-rate and mean-temperature equations, these parameters are redefined for the power-law stack:

$$g_c = N_{R,1}^{h_{\rm exp}-1}\frac{2}{\pi}\int_0^{\pi/2} \cos^{h_{\rm exp}-1}(z)dz, \tag{10.87}$$

$$g_v = -N_{R,1}^{h_{\rm exp}-1}\frac{2}{\pi}\int_0^{\pi/2} \cos 2z\cos^{h_{\rm exp}-1}(z)dz, \tag{10.88}$$

$$b(\phi) = h_{\rm con}. \tag{10.89}$$

The trigonometric integrals in Eqs. (10.87) and (10.88) are not evaluated by DeltaEC; these integrals were performed once, off-line, to obtain simple functional fits that DeltaEC uses during computation.

Calculations in tortuous regenerators for $\dot{N} \neq 0$:

The temperature gradient in STKSCREEN and STKPOWERLW is computed using constancy of total energy flux $\dot{H}_{\rm tot} = \dot{H}_{2,k} + \dot{N}mw_m$. In other words, we solve this equation for dT_m/dx :

$$\begin{aligned}\dot{H}_{\rm tot} = {} & \frac{1}{2}{\rm Re}\left[p_1\widetilde{U_1}\left(1 - \frac{T_m\beta(f_\kappa - \widetilde{f_\nu})}{(1+\epsilon_s)(1+\sigma)(1-\widetilde{f_\nu})}\right)\right] \\ & + \frac{\rho_m c_p}{2A_{\rm gas}\omega(1-\sigma)\left|1-f_\nu\right|^2}\frac{dT_m}{dx}\left|U_1\right|^2\ {\rm Im}\left[\widetilde{f_\nu} + \frac{(f_\kappa - \widetilde{f_\nu})(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] \\ & - (A_{\rm gas}k + A_{\rm solid}k_s)\frac{dT_m}{dx} + \dot{N}mw_m,\end{aligned} \tag{10.90}$$

and DeltaEC uses $\dot{H}_{\text{tot}}=$ constant to find $T_m(x)$ through the regenerator. The only term with nonzero $\dot{N}$ in this equation is the final one, $\dot{N}mw_m$. As usual, the value of $\dot{H}_{\text{tot}}$ is determined by conditions in segments other than the regenerator, such as the heat flows in adjacent heat exchangers.

The first-order pressure gradient dp_1/dx is unchanged by nonzero $\dot{N}$, and $d\langle u_1\rangle/dx$ is changed only indirectly through its dependence on dT_m/dx.

In STKSCREEN, the expression for head-loss gradient is a straightforward result of using the quasi-steady approximation, as outlined in Chapter 14:

$$\frac{dp_{2,0}}{dx} = -\frac{\mu m\dot{N}_{2,0}}{8r_h^2\rho_m A_{\text{total}}\phi}\left[c_1(\phi)+c_2(\phi)N_{R,2,0}\left(1+\frac{1}{2\varepsilon_{2,0}^2}+\frac{|p_1|}{p_m}\frac{\cos\phi}{\varepsilon_{2,0}}\right)\right], \quad |\varepsilon_{2,0}|\geq 1, \tag{10.91}$$

$$= -\frac{\mu m\dot{N}_{2,0}}{8r_h^2\rho_m A_{\text{total}}\phi}\left\{c_1(\phi)+\frac{c_2(\phi)N_{R,2,0}}{\pi\varepsilon_{2,0}^2}\left[\left(1+2\varepsilon_{2,0}^2+2\varepsilon_{2,0}\frac{|p_1|}{p_m}\cos\phi\right)\sin^{-1}|\varepsilon_{2,0}|\right.\right.$$
$$\left.\left.+\operatorname{sign}(\varepsilon_{2,0})\left(3\varepsilon_{2,0}+\frac{2}{3}\left(2+\varepsilon_{2,0}^2\right)\frac{|p_1|}{p_m}\cos\phi\right)\right]\sqrt{1-\varepsilon_{2,0}^2}\right\}, \quad |\varepsilon_{2,0}|\leq 1, \tag{10.92}$$

where the steady-flow Reynolds number is

$$N_{R,2,0} = \frac{m\left|\dot{N}_{2,0}\right|4r_h}{A_{\text{total}}\phi\mu} \tag{10.93}$$

and $\varepsilon_{2,0} = m\dot{N}_{2,0}/\rho_m|U_1|$.

In STKPOWERLW, the expression for head-loss gradient is the same as the pessimistic case in turbulent DUCTs, but with f_M and df_M/dN_R given by the STKPOWERLW expression, Eq. (10.82), and with $|p_1|/\gamma p_m$ replaced by $|p_1|/p_m$ to model isothermal oscillations.

10.6. Pulse tubes and thermal buffer tubes

(For a general description of all of the STK** segments, see the first paragraph of Section 10.5.)

10.6.1. STKDUCT and STKCONE

General description:

If $dT_m/dx \neq 0$ and pore size or plate separation is much greater than thermal and viscous penetration depths, use STKDUCT or STKCONE. These segments are used mostly for pulse tubes and thermal buffer tubes.

In a STKCONE, the variable fwall (parameter f) specifies the wall thickness. If fwall<1, the wall thickness is constant, but if fwall>1, the wall thickness is proportional to the local cone radius (so that the wall stress can be constant, which is the minimum-weight, minimum-thermal-conductance design).

The one-dimensional nature of DeltaEC calculations can be particularly unrealistic in STKDUCT and STKCONE, because DeltaEC assumes that $T_m(x)$ and $T_{\text{solid}}(x)$ are equal at all x, despite the fact that these are likely to be very different if any type of streaming occurs.

Input variables, `STKDUCT`:

Area (m^2) Cross-sectional area A_{gas} available to the gas, based on inside dimensions of the `STKDUCT`.

Perim (m) Perimeter Π of the inside-cross-section described above.

Length (m) The length Δx of the `STKDUCT`.

WallA (m^2) The cross-sectional area A_{solid} of the wall material.

Solid The solid material comprising the walls.

Input variables, `STKCONE`:

AreaI (m^2) Cross-sectional area $A_{gas,I}$ available to the gas at the initial end of the `STKCONE`, based on inside dimensions.

PerimI (m) Perimeter Π_I of the initial inside-cross-section described above.

Length (m) The length Δx of the `STKCONE`.

AreaF (m^2) Cross-sectional area $A_{gas,F}$ available to the gas at the final end of the `STKCONE`, based on inside dimensions.

PerimF (m) Perimeter Π_F of the final inside-cross-section described above.

f_wall For `fwall` < 1, wall thickness = `fwall` $\times$ initial perimeter, and is independent of x. For `fwall` > 1, wall thickness = local perimeter/`fwall`, so local wall cross-sectional area = (local perimeter)^2/`fwall`.

Solid The solid material comprising the walls.

Potential targets:

None.

Master–slave links:

In `STKDUCT`, the perimeter can be slaved to the area to maintain the shape when the area is changed.

In `STKDUCT`, the perimeter *and* the area can also be slaved to the length, to maintain a constant volume and constant shape when the length is changed.

In `STKCONE`, the initial perimeter can be slaved to the initial area, the final perimeter can be slaved to the final area, or both perimeters can be slaved to both areas, so that shapes are maintained when areas are changed.

The length and both perimeters of a `STKCONE` can be slaved to its two areas, to keep the cross-sectional shapes of the ends *and* the wall taper angle constant when either or both areas are changed.

The length of either of these segments can be slaved to the length of another segment to keep the sum of the two lengths constant.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples:

```
STKDUCT  boundary-layer approx
0.01 m2 area of gas
0.4  m perimeter (this duct is square)
1. m length
0.001 m2 wall material's cross-sectional area
stainless

STKCONE  boundary-layer w/ taper
0.01  m2 area of gas
0.35  m  perimeter
1. m length
sameas 8a
sameas 8b
0.001    fwall
stainless
```

Other examples of STKDUCT are in Sections 5.8.3 and 6.3.

Calculations for $\dot{N} = 0$:

In STKDUCT and STKCONE, pressure propagates according to Rott's wave equation, written in the form

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\text{gas}}}U_1, \tag{10.94}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\text{gas}}}{\rho_m a^2}\left(1+\frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1 + \frac{\beta(f_\kappa - f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}U_1, \tag{10.95}$$

subject to the condition that energy flow $\dot{H}_{2,k}$ is independent of x, which imposes the following condition on $T_m(x)$:

$$\frac{dT_m}{dx} = \frac{\dot{H}_{2,k} - \frac{1}{2}\text{Re}\left[p_1\tilde{U}_1\left(1-\frac{T_m\beta(f_\kappa-\tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_m c_p|U_1|^2}{2\omega A_{\text{gas}}(1-\sigma)|1-f_\nu|^2}\text{Im}\left[\tilde{f}_\nu + \frac{(f_\kappa-\tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] - A_{\text{gas}}k - A_{\text{solid}}k_s}. \tag{10.96}$$

For STKDUCT or STKCONE,

$$f_\kappa = (1-i)\Pi\delta_\kappa/2A, \quad f_\nu = (1-i)\Pi\delta_\nu/2A,$$

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s\rho_s c_s}\right)^{1/2}\frac{1}{\tanh[(1+i)\ell/\delta_s]}\text{, where } \ell = \frac{\text{wall cross-sect area}}{\text{perimeter}}, \tag{10.97}$$

so long as $2A/\Pi\delta_\nu > 30$. For $2A/\Pi\delta_\nu < 25$, the functions are the same as for STKCIRC, with $r_0(x) = 2A(x)/\Pi(x)$. In between, a linear combination is used.

Calculation with $\dot{N} \neq 0$:

STKDUCT treats nonzero $\dot{N}$ in the same way as STKCIRC. STKCONE is similar, with the additional variation of A with x.

10.7. Heat exchangers

DeltaEC heat exchangers are used to inject or remove heat. The first law of thermodynamics insists that this heat must equal the difference between upstream and downstream $\dot{H}_{\text{tot}}$. DeltaEC heat exchangers necessarily have surface area, so they experience both viscous and thermal dissipation of acoustic power.

Heat exchangers are collected into three groups here. The smooth, short `HX` and `TX` have parallel-plate and tubular geometry, respectively, and are described in Section 10.7.1. The short, tortuous porous heat exchangers `SX` and `PX`, for screens and power-law structures, respectively, are described in Section 10.7.2. The long, smooth, variable-temperature and variable-heat-density exchangers `VXT1`, `VXT2`, `VXQ1`, and `VXQ2` are described in Section 10.7.3.

10.7.1. `HX` and `TX`

General description:

For a general description of all DeltaEC heat exchangers, see the initial paragraphs of the heat-exchanger documentation, Section 10.7.

In `HX` the thermoacoustic working gas is between parallel plates; in `TX` it is inside cylindrical tubes. Heat $\dot{Q}$ is positive when it flows from the solid into the thermoacoustic gas, adding to $\dot{H}_{\text{tot}}$. The temperature difference between the solid temperature T_{solid} and the gas temperature T_m is proportional to $\dot{Q}$. The proportionality constant is not well validated, either theoretically or experimentally; we find that it seems to be accurate within a factor of 2.

`HX` and `TX` are assumed to be so short that the numerical integration from one end to the other is performed with a single fourth-order Runge-Kutta step. (Most other integrated segments, i.e., `DUCT`, `CONE`, `MIX**`, `STK**`, and `VX**`, are integrated with `Nint` fourth-order Runge-Kutta steps.) See `VX**` in Section 10.7.3 below for longer tubular heat exchangers.

Input variables, `HX`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$.

GasA/A The volumetric porosity $\phi = A_{\text{gas}}/A$.

Length (m) The length Δx of the heat exchanger.

y0 (m) The plate spacing is $2y_0$. Thus, $2y_0$ is the gap available to the gas—not the center-to-center spacing of the plates.

HeatIn The heat $\dot{Q}$ added to the thermoacoustic gas in the heat exchanger. Positive $\dot{Q}$ indicates heat flow from solid to gas.

FracQN If "Enable Ndot" is checked in an upstream `BEGIN`, this variable, $F_{\dot{Q}\dot{N}}$, determines what fraction of the heat $\dot{Q}$ is added to $\dot{H}_{\dot{N}} = \dot{N} m w_m$. (The remainder is added to $\dot{H}_{2,k}$.)

Solid The solid material on the inside surface of the `HX`. This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and specific heat.

Input variables, `TX`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$.

GasA/A The volumetric porosity $\phi = A_{\text{gas}}/A$.

Length (m) The length Δx of the heat exchanger.

radius (m) The radius of each tube, r_0.

HeatIn The heat $\dot{Q}$ added to the thermoacoustic gas in the heat exchanger. Positive $\dot{Q}$ indicates heat flow from solid to gas.

FracQN If "Enable Ndot" is checked in an upstream `BEGIN`, this variable, $F_{\dot{Q}\dot{N}}$, determines what fraction of the heat $\dot{Q}$ is added to $\dot{H}_{\dot{N}} = \dot{N} m w_m$. (The remainder is added to $\dot{H}_{2,k}$.)

Solid The solid material on the inside surface of the `TX`. This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and specific heat.

Potential targets:

TSolid If used as a target, DELTAEC's shooting method compares T_{solid} to this value.

Master–slave links:

In `TX`, the porosity ϕ can be slaved to other parameters via the equation $\phi = \min(cr_0^2/A, 1)$, with c calculated when the link is first enabled via $c = \phi A/r_0^2$. This link maintains a constant number of holes, up to a maximum porosity of 1.

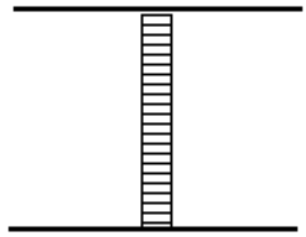
The length of `HX` or `TX` can be slaved to the length of another segment to keep the sum of the two lengths constant.

(For an introduction to master–slave links, and a complete list of linking options for all segments, see Section 2.7.2.)

Format and examples:

```
HX        parallel-plate heat exchanger
sameas 1  Area
0.600     GasA/A
6.35e-3 m Length
1.9e-4  m y0 = half of plate spacing
-20.0   W HeatIn
300.    K Est-T   Target
copper    solid
```

```
TX     tube-in-shell heat exchanger
    0.2     a Area     m^2
   .188     b GasA/A
   .400     c Length    m
  6.350E-03 d radius    m   (radius of each tube)
  1.818E+05 e HeatIn    W
 possible target
nickel     Solid type
```



Other examples of HX appear in Chapter 5.

Calculations:

In HX and TX, wave propagation is calculated using

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\text{gas}}}U_1, \tag{10.98}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\text{gas}}}{\rho_m a^2}\left(1+\frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1. \tag{10.99}$$

HX uses parallel plate geometry in computing f_κ, f_ν, and ϵ_s:

$$f_\kappa = \frac{\tanh[(1+i)y_0/\delta_\kappa]}{(1+i)y_0/\delta_\kappa}, \quad f_\nu = \frac{\tanh[(1+i)y_0/\delta_\nu]}{(1+i)y_0/\delta_\nu}, \tag{10.100}$$

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s\rho_s c_s}\right)^{1/2}. \tag{10.101}$$

Similarly, TX uses cylindrical geometry in computing f_κ, f_ν, and ϵ_s: For $r_0/\delta < 25$, f_κ and f_ν are calculated using complex Bessel functions

$$f_\kappa = \frac{2J_1[(i-1)r_0/\delta_\kappa]}{(i-1)(r_0/\delta_\kappa)J_0[(i-1)r_0/\delta_\kappa]}, \quad f_\nu = \frac{2J_1[(i-1)r_0/\delta_\kappa]}{(i-1)(r_0/\delta_\kappa)J_0[(i-1)r_0/\delta_\kappa]}. \tag{10.102}$$

For $r_0/\delta > 30$, the boundary-layer approximation is used:

$$f_\kappa = (1-i)\delta_\kappa/r_0, \quad f_\nu = (1-i)\delta_\nu/r_0. \tag{10.103}$$

For intermediate values, linear interpolation is used to make a smooth match between the two regimes. In both cases, ϵ_s is calculated using

$$\epsilon_s = \left(\frac{k\rho_m c_p}{k_s\rho_s c_s}\right)^{1/2}. \tag{10.104}$$

In TX, the radius r_0 is that of one circular pore, so that for a heat exchanger comprised of N circular pores, the total cross-sectional area available to the working gas is $N\pi r_0^2 =$ (Area)(GasA/A).

HX and TX have no turbulence algorithms.

For $\dot{N} = 0$, temperature and power change according to

$$\frac{dT_m}{dx} = 0, \tag{10.105}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \frac{d\dot{H}_{2,k}}{dx} = \frac{\dot{Q}}{\Delta x}, \tag{10.106}$$

$$T_{\text{solid}} = T_m + \frac{\dot{Q}}{h\ A_{\text{gas}} x_{\text{eff}}/r_h}, \tag{10.107}$$

where Δx is the heat exchanger length and $\dot{Q}$ is the heat added to the heat exchanger. Positive heat $\dot{Q}$ (parameter `e`) flows into the apparatus. The heat flow $\dot{Q}$ is an input for each pass of DeltaEC, and it is often used as a guess for the shooting method described in Section 2.5. The heat-transfer coefficient h is estimated with

$$h = \frac{k}{y_{\text{eff}}}, \tag{10.108}$$

and

$$x_{\text{eff}} = \min[2\,|\xi_1|\,,\ \Delta x] \tag{10.109}$$

$$y_{\text{eff}} = \min[\delta_\kappa, r_h], \tag{10.110}$$

with hydraulic radius r_h equal to y_0 for `HX` and equal to $r_0/2$ for `TX`. These expressions may be quite inaccurate, but we believe they are more useful than nothing. A little experimental evidence for $T_{\text{solid}} - T_m$ is presented in Ref. [22]. A user who mistrusts these expressions can use the gas temperature (available through `RPN`, and as an output in the adjacent stack segment) instead of the solid temperature for plotting or targeting, and/or use `RPN`s to construct custom expressions instead of Eqs. (10.107)–(10.110).

For $\dot{N} \neq 0$, the steady and oscillating flows are assumed to be independent and simply superimposed. Equations (10.98) and (10.99) are unchanged by $\dot{N} \neq 0$, and the gradients in $p_{2,0,HL}$ are calculated using Eq. (10.65) for `HX` and Eq. (10.64) for `TX`. Temperature and power change according to

$$\frac{dT_m}{dx} = F_{\dot{Q}\dot{N}} \frac{\dot{Q}/\Delta x}{\dot{N} m c_p}, \tag{10.111}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \frac{\dot{Q}}{\Delta x}, \text{ so } \frac{d\dot{H}_{2,k}}{dx} = \left(1 - F_{\dot{Q}\dot{N}}\right) \frac{\dot{Q}}{\Delta x}, \tag{10.112}$$

$$T_{\text{solid}} = \max\left[T_m(0), T_m(\Delta x)\right] + \frac{\dot{Q}}{h\ A_{\text{gas}} x_{\text{eff}}/r_h} \text{ if } \dot{Q} > 0, \tag{10.113}$$

$$= \min\left[T_m(0), T_m(\Delta x)\right] + \frac{\dot{Q}}{h\ A_{\text{gas}} x_{\text{eff}}/r_h} \text{ if } \dot{Q} < 0, \tag{10.114}$$

where $F_{\dot{Q}\dot{N}}$ = `FracQN` is input parameter `g`, which establishes what fraction of $\dot{Q}$ heats the steady flow. $F_{\dot{Q}\dot{N}}$ is often guessed, to give DeltaEC enough flexibility to meet nearby temperature and energy boundary conditions. It can also be set equal to zero without serious

consequences in many models, or set equal to a simple expression such as $\left(1+\rho_m\left|U_1\right|/|\dot{N}|m\right)^{-1}$ in an RPN, supplying an experimentally unsupported but plausible estimate of how power divides itself between the steady flow and the acoustic flow.

10.7.2. SX and PX

General description:

For a general description of all DELTAEC heat exchangers, see the initial paragraphs of the heat-exchanger documentation, Section 10.7.

In SX, the thermoacoustic working gas flows through a pile of woven screens. The PX segment allows the user to specify heat-exchanger parameters for which the steady-flow friction factor and heat transfer coefficient are known power laws in Reynolds number. The SX calculations assume fairly good thermal contact between gas and solid, so these calculations are valid only for hydraulic radius smaller than thermal and viscous penetration depths. There is no warning if this condition is not met.

Heat $\dot{Q}$ is positive when it flows from the solid into the thermoacoustic gas, adding to $\dot{H}_{\text{tot}}$. In SX, typically the screen material has high thermal conductivity and the area is small, because typically the heat must flow radially through the screen wires, from the pressure vessel around the screens to the thermoacoustic gas.

The temperature difference between the solid temperature T_{solid} and the gas temperature T_m is proportional to $\dot{Q}$. The proportionality constant is not well supported, either theoretically or experimentally; we find that it seems to be accurate within a factor of 2.

SX and PX are assumed to be so short that the numerical integration from one end to the other is performed with a single fourth-order Runge-Kutta step. (Most other integrated segments, i.e., DUCT, CONE, MIX**, STK**, and VX**, are integrated with Nint fourth-order Runge-Kutta steps.)

Input variables, SX:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$.

VolPor The volumetric porosity ϕ. This is the ratio of the volume of gas in the SX to the total volume of the heat exchanger. For plain square-weave screen with m wires per unit length and wire diameter d_{wire}, we sometimes use

$$\phi \simeq 1 - \frac{\pi m d_{\text{wire}}}{4}, \tag{10.115}$$

as recommended by Organ [26], or we weigh the pile of screens, measure its overall dimensions A and Δx, and use the known density of its wire material to calculate ϕ.

Length (m) The length Δx of the heat exchanger.

rh (m) The hydraulic radius r_h of the SX. This is the ratio of the gas volume to the gas–solid contact area. For plain square-weave screen with wire diameter d_{wire}, we sometimes

use

$$r_h \simeq d_{\text{wire}} \frac{\phi}{4\,(1-\phi)}, \tag{10.116}$$

as recommended by Organ [26].

HeatIn The heat $\dot{Q}$ added to the thermoacoustic gas in the heat exchanger. Positive $\dot{Q}$ indicates heat flow from solid to gas.

FracQN If "Enable Ndot" is checked in an upstream `BEGIN`, this variable, $F_{\dot{Q}\dot{N}}$, determines what fraction of the heat $\dot{Q}$ is added to $\dot{H}_{\dot{N}} = \dot{N} m w_m$. (The remainder is added to $\dot{H}_{2,k}$.)

Solid The solid material of the wires. This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and specific heat in `SX`s.

Input variables, `PX`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}}$.

VolPor The volumetric porosity ϕ. This is the ratio of the volume of gas in the heat exchanger to the total volume of the heat exchanger.

Length (m) The length Δx of the heat exchanger.

rh (m) The hydraulic radius r_h of the heat exchanger. This is the ratio of the gas volume to the gas–solid contact area.

HeatIn The heat $\dot{Q}$ added to the thermoacoustic gas in the heat exchanger. Positive $\dot{Q}$ indicates heat flow from solid to gas.

f con The friction-factor parameter f_{con} in the equations below.

f exp The friction-factor parameter f_{exp} in the equations below.

h con The heat-transfer parameter h_{con} in the equations below.

h exp The heat-transfer parameter h_{exp} in the equations below.

FracQN If "Enable Ndot" is checked in an upstream `BEGIN`, this variable, $F_{\dot{Q}\dot{N}}$, determines what fraction of the heat $\dot{Q}$ is added to $\dot{H}_{\dot{N}} = \dot{N} m w_m$. (The remainder is added to $\dot{H}_{2,k}$.)

Solid The solid with which the `PX` is made. This is used only in the calculation of ϵ_s, which seldom differs significantly from its value for a solid with infinite conductivity and specific heat in heat exchangers.

Potential targets:

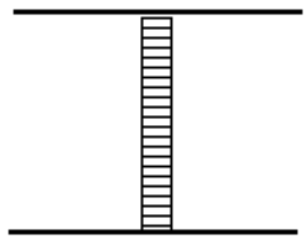
TSolid If used as a target, DELTAEC's shooting method compares T_{solid} to this value.

Master–slave links:

The length of either of these segments can be slaved to the length of another segment to keep the sum of the two lengths constant.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples:



```
SX     Hot heat exchanger
  1.029E-03 a Area      m^2  total cross sectional area
  0.690     b VolPor         volumetric porosity
  2.000E-02 c Length     m
  6.450E-05 d   rh       m   hydraulic radius
  -284.     e HeatIn     W
   300.     f Est-T      K     (t)
copper     Solid type

PX
  1.0E-4    a Area      m^2  total cross-sectional area
  0.70      b GasA/A         volumetric porosity
  0.04      c Length     m
  4.000E-05 d   rh       m   hydraulic radius = gas volume / gas-solid contact area
   900.     e HeatIn     W
  0.07      f f_con
  0.22      g f_exp
  0.035     h h_con
  0.22      i h_exp
copper     Solid type
```

Examples of `SX` occur throughout Sections 5.8 and 6.3.

Calculations:

In `SX`, dp_1/dx and $d\langle u_1\rangle/dx$ are calculated in the same way as in `STKSCREEN`, as described in [59]:

$$\frac{dp_1}{dx} = -i\omega\rho_m\left[1+\frac{(1-\phi)^2}{2(2\phi-1)}\right]\langle u_1\rangle - \frac{\mu}{r_h^2}\left(\frac{c_1(\phi)}{8}+\frac{c_2(\phi)N_{R,1}}{3\pi}\right)\langle u_1\rangle\,, \tag{10.117}$$

$$\frac{d\langle u_1\rangle}{dx} = -\frac{i\omega\gamma}{\rho_m a^2}p_1 + \frac{i\omega T_m\beta^2}{\rho_m c_p}\frac{\epsilon_s+(g_c+e^{2i\theta_p}g_v)\epsilon_h}{1+\epsilon_s+(g_c+e^{2i\theta_T}g_v)\epsilon_h}p_1, \tag{10.118}$$

using

$$c_1(\phi) = 1268 - 3545\phi + 2544\phi^2, \quad c_2(\phi) = -2.82 + 10.7\phi - 8.6\phi^2, \tag{10.119}$$

$$b(\phi) = 3.81 - 11.29\phi + 9.47\phi^2, \tag{10.120}$$

$$N_{R,1} = 4\,|\langle u_1\rangle|\,r_h\rho_m/\mu, \tag{10.121}$$

$$\epsilon_s = \phi\rho_m c_p/(1-\phi)\rho_s c_s, \quad \epsilon_h = 8ir_h^2/b(\phi)\sigma^{1/3}\delta_\kappa^2, \tag{10.122}$$

$$\delta_\kappa^2 = 2k/\omega\rho_m c_p, \tag{10.123}$$

$$\theta_p = \text{phase}(\langle u_1 \rangle) - \text{phase}\,(p_1)\,, \quad \theta_T = \text{phase}(\langle u_1 \rangle) - \text{phase}\left(\langle T \rangle_{u,1}\right), \tag{10.124}$$

$$g_c = \frac{2}{\pi}\int_0^{\pi/2} \frac{dz}{1 + N_{R,1}^{3/5}\cos^{3/5}(z)}, \quad g_v = -\frac{2}{\pi}\int_0^{\pi/2} \frac{\cos(2z)\,dz}{1 + N_{R,1}^{3/5}\cos^{3/5}(z)}. \tag{10.125}$$

The spatial average oscillatory velocity $\langle u_1 \rangle = \langle U_1 \rangle / \phi A$, where ϕ is volumetric porosity and A is regenerator cross sectional area. These expressions were derived with the assumption that the thermal and viscous penetration depths are much larger than r_h.

The `PX` segment can be used when the steady-flow friction factor and heat-transfer coefficient are known power laws in Reynolds number N_R. The derivation assumes fairly good thermal contact between gas and solid. Area A is the total cross sectional area of the heat exchanger, `VolPor` ϕ is its volumetric porosity, and Δx is its length, so that $\phi A\ \Delta x$ is the total volume of gas in the heat exchanger. The steady-state friction factor and heat-transfer coefficients must be known by the user in power-law forms:

$$f = f_{\text{con}}\,(N_R)^{-f_{\text{exp}}}\,, \tag{10.126}$$

$$N_{St}\,\sigma^{2/3} = h_{\text{con}}\,(N_R)^{-h_{\text{exp}}}\,, \tag{10.127}$$

where $N_{St} = N_{Nu}/\sigma N_R$ is the Stanton number, and the Reynolds number N_R is defined in the usual way as

$$N_R = \frac{4U r_h \rho_m}{\phi A \mu} = \frac{4\,\langle u \rangle\, r_h \rho_m}{\mu}. \tag{10.128}$$

Note: this is Fanning friction factor, the friction factor used by Kays and London, so that instantaneously

$$\frac{dp}{dx} = \frac{f}{r_h}\frac{1}{2}\rho\,\langle u \rangle^2 = \frac{\mu}{8r_h^2}\,f\,N_R\,\langle u \rangle\,. \tag{10.129}$$

The complex pressure gradient is computed using

$$\frac{dp_1}{dx} = -i\omega\rho_m\,\langle u_1 \rangle - I_f\,\frac{\mu_m}{8r_h^2}\,f_{\text{con}}\,|N_{R,1}|^{1-f_{\text{exp}}}\,\langle u_1 \rangle \tag{10.130}$$

where

$$I_f = \frac{2}{\pi}\int_0^{\pi} \sin^{3-f_{\text{exp}}}\omega t\,d(\omega t). \tag{10.131}$$

Thermal-relaxation effects due to oscillating compressibility are computed using the same equations for $d\,\langle u_1 \rangle / dx$ as for screen heat exchangers but with

$$g_c = |N_{R,1}|^{h_{\text{exp}}-1}\,\frac{2}{\pi}\int_0^{\pi/2} \cos^{h_{\text{exp}}-1}\omega t\,d(\omega t), \tag{10.132}$$

$$g_v = -\,|N_{R,1}|^{h_{\text{exp}}-1}\,\frac{2}{\pi}\int_0^{\pi/2} \cos 2\omega t\,\cos^{h_{\text{exp}}-1}\omega t\,d(\omega t), \tag{10.133}$$

$$b(\phi) = h_{\text{con}}. \tag{10.134}$$

The solid temperature is computed relative to the gas mean temperature using

$$\Delta T = I_h\,\frac{\dot{Q}}{k\,h_{\text{con}}\,|N_{R,1}|^{1-h_{\text{exp}}}}\,\frac{r_h^2}{\phi A x_{\text{eff}}}, \tag{10.135}$$

where $x_{\text{eff}} = \min\{2\,|\langle u_1\rangle|\,/\omega,\ \Delta x\}$, $N_{R,1}$ is the Reynolds number amplitude (based on the *amplitude* of the velocity), and

$$I_h = \frac{4}{\pi}\int_0^{\pi/2} \cos^{h_{\text{exp}}+1}\omega t\, d(\omega t) = \frac{h_{\text{exp}}}{h_{\text{exp}}+1}\frac{4}{\pi}\int_0^{\pi/2} \cos^{h_{\text{exp}}-1}\omega t\, d(\omega t). \tag{10.136}$$

(The second form of I_h, obtained from the first via integration by parts, expresses I_h in terms of g_c above.) To maintain DELTAEC's high speed, the trigonometric integrals are not evaluated by DELTAEC; we use simple functional fits to these integrals in DELTAEC.

The head-loss pressure gradient $dp_{2,0}/dx$ in `SX` is calculated in the same way as for `STKSCREEN`s. The nonlinear nature of the screen friction factor $f = c_1(\phi)/N_R + c_2(\phi)$ also causes the mean flow to affect dp_1/dx in `SX`, according to

$$\frac{dp_1}{dx} = -\frac{\mu U_1}{8r_h^2 A_{\text{total}}\phi}\left[c_1(\phi) + 2c_2(\phi)N_{R,2,0}\right],\quad |\varepsilon_{2,0}| \geq 1, \tag{10.137}$$

$$= -\frac{\mu U_1}{r_h^2 A_{\text{total}}\phi}\left\{\frac{c_1(\phi)}{8} + \frac{c_2(\phi)N_{R,1}}{3\pi}\left[\left(1+\frac{\varepsilon_{2,0}^2}{2}\right)\sqrt{1-\varepsilon_{2,0}^2} + \frac{3}{2}\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0}\right]\right\},\quad |\varepsilon_{2,0}| \leq 1, \tag{10.138}$$

where the acoustic Reynolds-number amplitude is

$$N_{R,1} = \frac{|U_1|\,4r_h\rho_m}{A_{\text{total}}\phi\mu}. \tag{10.139}$$

[Note $N_{R,2,0}$ in Eq. (10.137) but $N_{R,1}$ in Eq. (10.138) — *not* a typographical error.] We did not incorporate a mean flow effect on dp_1/dx in `STKSCREEN` because the energy equation restricts the magnitude of $\dot{N}$ in stacks. No such restriction exists in heat exchangers (although we're not sure why anyone would use a heat exchanger without an adjacent stack).

In `PX`, $dp_{2,0}/dx$ is calculated just as in `STKPOWERLW`. The mean flow affects dp_1/dx in `PX` according to

$$\frac{dp_1}{dx} = -i\omega\rho_m\langle u_1\rangle - \frac{\rho_m|\langle u_1\rangle|\langle u_1\rangle}{4r_h}\left[|\varepsilon_{2,0}|\,f_{M,\max} + \frac{(3-2|\varepsilon_{2,0}|)(1-2|\varepsilon_{2,0}|)}{8(1+|\varepsilon_{2,0}|)}\frac{df_M}{dN_R}N_{R,\max}\right],\quad |\varepsilon_{2,0}| \geq 1, \tag{10.140}$$

$$= -i\omega\rho_m\langle u_1\rangle - \frac{\rho_m|\langle u_1\rangle|\langle u_1\rangle}{3\pi r_h}\left\{\left[\left(1+\frac{\varepsilon_{2,0}^2}{2}\right)\sqrt{1-\varepsilon_{2,0}^2} + \frac{3}{2}\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0}\right]f_{M,\max} - \left[\left(1+\frac{\varepsilon_{2,0}^2}{2}\right)\sqrt{1-\varepsilon_{2,0}^2} + \frac{3}{2}\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0} - \frac{9\pi}{32}\frac{1+4\varepsilon_{2,0}^2}{1+|\varepsilon_{2,0}|}\right]\frac{df_M}{dN_R}N_{R,\max}\right\},\quad |\varepsilon_{2,0}| \leq 1. \tag{10.141}$$

which is a hybrid of Eq. (10.130) here and Eqs. (A13) and (A14) in Ref. [41].

For $\dot{N} = 0$, temperature and power change according to

$$\frac{dT_m}{dx} = 0, \tag{10.142}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \frac{\dot{Q}}{\Delta x}, \tag{10.143}$$

where Δx is the heat exchanger length and $\dot{Q}$ is the heat added to the heat exchanger. Positive heat $\dot{Q}$ (parameter "e") flows into the apparatus. The heat flow $\dot{Q}$ is an input for each pass of DeltaEC, and it is often used as a guess for the shooting method described in Section 2.5.

For $\dot{N} \neq 0$, temperature and power change according to

$$\frac{dT_m}{dx} = F_{\dot{Q}\dot{N}} \frac{\dot{Q}/\Delta x}{\dot{N} m c_p}, \tag{10.144}$$

$$\frac{d\dot{H}_{\text{tot}}}{dx} = \frac{\dot{Q}}{\Delta x}, \text{ so } \frac{d\dot{H}_{2,k}}{dx} = \left(1 - F_{\dot{Q}\dot{N}}\right) \frac{\dot{Q}}{\Delta x}, \tag{10.145}$$

where $F_{\dot{Q}\dot{N}}$ = FracQN is the input parameter that establishes what fraction of $\dot{Q}$ heats the steady flow. $F_{\dot{Q}\dot{N}}$ is often guessed, to give DeltaEC enough flexibility to meet nearby temperature and energy boundary conditions. It can also be set equal to zero without serious consequences in many models, or set equal to a simple expression such as $\left(1 + \rho_m \left|U_1\right| / m \left|\dot{N}\right|\right)^{-1}$ in an RPN.

In SXs, the solid temperature is computed relative to gas mean temperature using

$$\Delta T = \frac{\dot{Q}}{k} \frac{r_h^2 (g_c - g_v)}{b(\phi) \phi A x_{\text{eff}}} \tag{10.146}$$

where $x_{\text{eff}} = \min\{2\left|\langle \xi_1 \rangle\right|, \Delta x\}$. In PXs, the solid temperature is computed relative to the gas mean temperature using

$$\Delta T = I_h \frac{\dot{Q}}{k\, h_{\text{con}} N_{R,1}^{1-h_{\text{exp}}}} \frac{r_h^2}{\phi A x_{\text{eff}}}, \tag{10.147}$$

where $x_{\text{eff}} = \min\{2\left|\langle \xi_1 \rangle\right|, \Delta x\}$, $N_{R,1}$ is the Reynolds number based on the amplitude of the velocity, and I_h is defined above. Then

$$T_{\text{solid}} = \max\left[T_m(0), T_m(\Delta x)\right] + \Delta T \qquad \text{if } \dot{Q} > 0, \tag{10.148}$$

$$= \min\left[T_m(0), T_m(\Delta x)\right] + \Delta T \qquad \text{if } \dot{Q} < 0, \tag{10.149}$$

These expressions may be very inaccurate, but we believe they are more useful than nothing. The user who mistrusts them can use the gas temperature (available as an output in the adjacent stack segment, and through RPN) instead of the solid temperature for plotting or targeting, and/or use RPNs to construct customized expressions for Eqs. (10.146)–(10.149).

10.7.3. VXQ1, VXQ2, VXT1, and VXT2

General description:

For a general description of all DeltaEC heat exchangers, see the initial paragraphs of the heat-exchanger documentation, Section 10.7.

Beginning in DeltaEC version 6.0, the "variable" heat exchanger segments VXT1, VXT2, VXQ1, and VXQ2 are included, allowing gas temperature T_m and heat flux per unit length $\dot{q}$

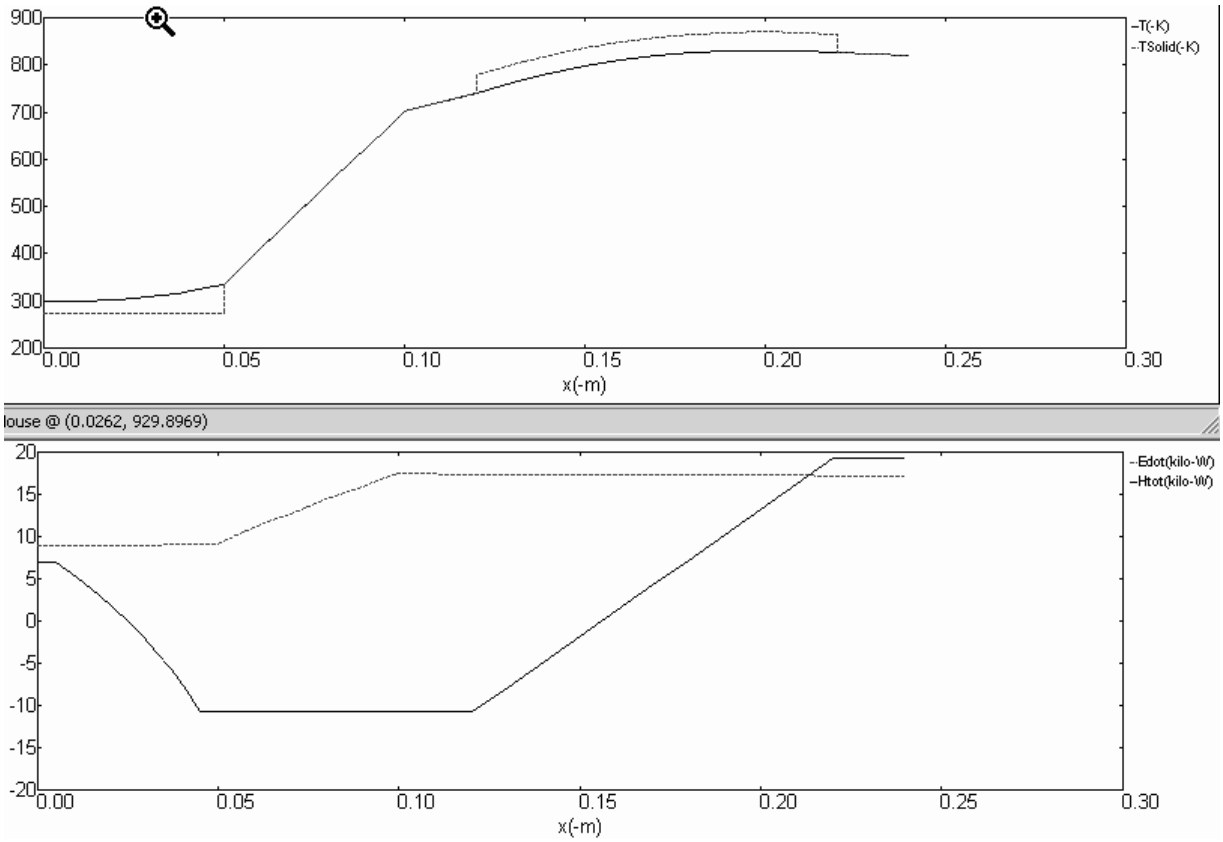


Figure 10.6: Example results from `VXT1`, `STKSCREEN`, `VXQ1` segments in series in a Stirling engine. The `VXT1` extends from $x = 0$ to $x = 0.05$ m, with tubesheets 0.005 m long at each end. The `STKSCREEN` extends from $x = 0.05$ m to $x = 0.10$ m. The `VXQ2` extends from $x = 0.10$ m to $x = 0.24$ m, with tubesheets 0.02 m long on each end. Temperatures in upper panel. The gas temperature (solid line) varies with x in all three segments. The solid temperature (dashed line) does not vary with x in the `VXT1` segment. Lower panel shows $\dot{H}_{\text{tot}}$ (solid line) and $\dot{E}$ (dashed line).

to vary with x. The two `VXQ*` segments further allow T_{solid} to depend on x. Hence, these segments combine some features of previous heat-exchanger segments with some features of previous stack segments.

The third character in the segment name, `T` or `Q`, determines which of two extreme, limiting heat-transfer conditions is assumed for the non-thermoacoustic side of the heat exchanger. In `VXT*`, T_{solid} is independent of x. Thinking of a shell-and-tube heat exchanger in which the thermoacoustic processes take place inside the tubes while a heat-transfer liquid passes through the shell, the idea is that `VXT*` should be used when the shell-side flow has such a high heat-transfer coefficient (e.g., vigorous flow of water) that the solid surfaces of the heat exchanger tubes are fixed in temperature, independent of x. In `VXQ*`, the heat flux delivered to/from the heat exchanger per unit length is independent of x. In the shell-and-tube situation, the idea is that `VXQ*` should be used when the shell-side gas is weakly coupled to the solid (e.g., combustion products at low Reynolds number) so that the temperature difference between the shell-side gas and the solid surface can be much larger than any changes in temperature along the solid surface. To decide which case is most appropriate for individual circumstances, use a standard heat-transfer book to estimate the heat-transfer coefficients on the shell side and on the thermoacoustic side. If the heat transfer is much better on the shell side, use `VXT*`. If the heat transfer is much better on the thermoacoustic side, use `VXQ*`.

The fourth character in the segment name, `1` or `2`, allows the segment to be formed of 1 or 2 separate heat-transfer pieces along x (in addition to the two "tubesheets" at either end). Thus, "`2`" is intended for heat exchangers with two "passes" on the shell side. Figure 10.7 shows some of the geometry parameters for a `VXT2` (or `VXQ2`).

To understand the philosophy behind the `VX**` calculations, consider Fig. 10.8, which shows a length dx of this segment. In contrast with the `STK**` segments, where the mean temperature of the solid is assumed to be the same as the mean temperature of the gas, here those time-mean temperatures can be different, so that large time-averaged heat flows can be driven from one to the other. Let $T_m(x)$ be the mean temperature of the gas and $T_{\text{solid}}(x)$ be the mean temperature of the solid. Both may be functions of x. Artificially, assume that T_m in the gas is independent of the coordinate perpendicular to x. This is equivalent to assuming that the heat-transfer bottleneck for the time-averaged heat transfer occurs in a thin layer adjacent to the surface.

Input variables, `VXQ1` and `VXQ2`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}} + A_{\text{shell-side}}$.

GasA/A The fraction of the total area that is available to gas, A_{gas}/A.

SolA/A The fraction of the total area occupied by solid that contributes to solid thermal conduction in the x direction, A_{solid}/A.

rh (m) The hydraulic radius r_h of the gas channels. Hydraulic radius is defined as the ratio of the gas volume to the gas–solid contact area. For the present case of an x-independent channel cross section, $r_h = A_{\text{gas}}/\Pi$ where Π is the inside perimeter of the channel.

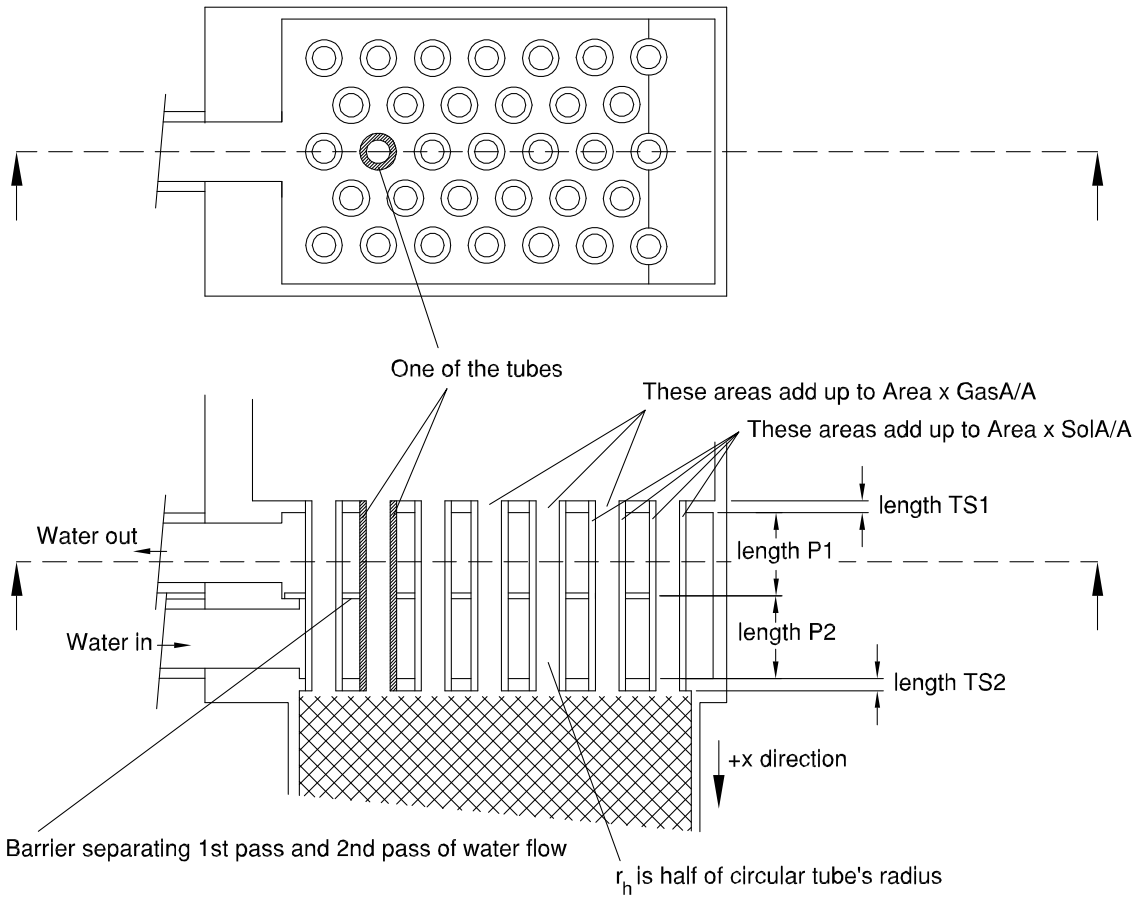


Figure 10.7: Two perpendicular cross sections through a `VXT2`, sandwiched between a duct above and a regenerator below. Thermoacoustic oscillations in the vertical direction pass through the insides of tubes, of which 7 are exposed in the lower part of this figure. Cooling water passes through the "shell" in two passes: rightward across the lower half, and then leftward across the upper half.

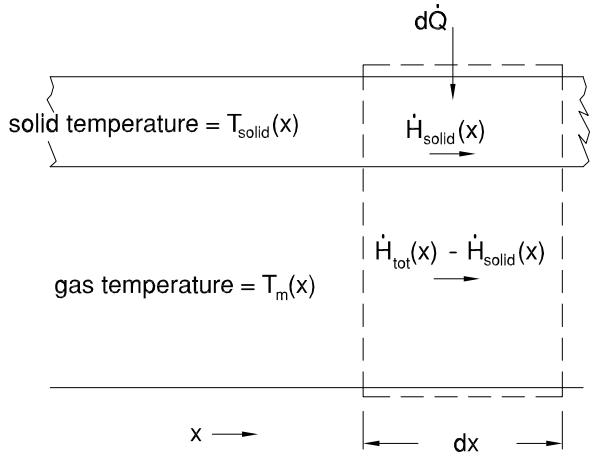


Figure 10.8: Schematic of a short section of a `VX**` segment. The solid lines show the edges of the solid and the gas. The dashed box is the short control volume, dx long, used to write the conservation laws for this situation. The space above the solid represents the external heat-transfer gas such as water or combustion gases, which transfer heat at rate $d\dot{Q}$ to the control volume.

LenTS1 (m) The length of the first tubesheet in the heat exchanger. This region experiences viscous and thermal-hysteresis loss, but not time-averaged heat transfer.

Len P1 (m) The length of the heat-transfer region in `VXQ1`; the length of the first of two heat-transfer regions in `VXQ2`.

Len P2 (m) The length of the second heat-transfer region in `VXQ2`. (Not present in `VXQ1`.)

LenTS2 (m) The length of the second tubesheet in the heat exchanger. This region experiences viscous and thermal-hysteresis loss, but not time-averaged heat transfer.

HeatP1 The heat added to the thermoacoustic gas in `VXQ1`. The heat added to the thermoacoustic gas in the first heat-transfer region in `VXQ2`. Positive heat indicates heat flow from solid to gas.

HeatP2 The heat added to the thermoacoustic gas in the second heat-transfer region in `VXQ2`. (Not present in `VXQ1`.) Positive heat indicates heat flow from solid to gas.

FracQN If "Enable Ndot" is checked in an upstream `BEGIN`, this variable, $F_{\dot{Q}\dot{N}}$, determines what fraction of heat is added to $\dot{H}_{\dot{N}} = \dot{N} m w_m$. (The remainder is added to $\dot{H}_{2,k}$.) In `VXQ2`, the same value of $F_{\dot{Q}\dot{N}}$ is applied to both passes.

Solid The solid material in the heat exchanger.

Input variables, `VXT1` and `VXT2`:

Area (m^2) The total cross-sectional area $A = A_{\text{gas}} + A_{\text{solid}} + A_{\text{shell-side}}$.

GasA/A The fraction of the total area that is available to gas, A_{gas}/A.

SolA/A The fraction of the total area occupied by solid that contributes to solid thermal conduction in the x direction, A_{solid}/A.

rh (m) The hydraulic radius r_h of the gas channels. Hydraulic radius is defined as the ratio of the gas volume to the gas–solid contact area. For the present case of an x-independent channel cross section, $r_h = A_{\text{gas}}/\Pi$ where Π is the inside perimeter of the channel.

LenTS1 (m) The length of the first tubesheet in the heat exchanger. This region experiences viscous and thermal-hysteresis loss, but not time-averaged heat transfer.

Len P1 (m) The length of the heat-transfer region in `VXT1`; the length of the first of two heat-transfer regions in `VXT2`.

Len P2 (m) The length of the second heat-transfer region in `VXT2`. (Not present in `VXT1`.)

LenTS2 (m) The length of the second tubesheet in the heat exchanger. This region experiences viscous and thermal-hysteresis loss, but not time-averaged heat transfer.

TSolP1 The solid temperature in the heat-transfer region in `VXT1`. The solid temperature in the first heat-transfer region in `VXT2`.

TSolP2 The solid temperature in the second heat-transfer region in `VXT2`. (Not present in `VXT1`.)

FracQN If "Enable Ndot" is chekced in an upstream `BEGIN`, this variable, $F_{\dot{Q}\dot{N}}$, determines what fraction of heat is added to $\dot{H}_{\dot{N}} = \dot{N} m w_m$. (The remainder is added to $\dot{H}_{2,k}$.) In `VXT2`, the same value of $F_{\dot{Q}\dot{N}}$ is applied to both passes.

Solid The solid material in the heat exchanger.

Potential targets:

HeatP1 or HeatP2 If used as a target in `VXT1` or `VXT2`, DeltaEC's shooting method compares its calculated total heat for the corresponding heat-transfer region to this value.

There are no built-in potential targets in `VXQ1` and `VXQ2`. Use `RPN` targets as needed.

Master–slave links:

None.

Format and examples:

```
VXT2       compare to segment 13
sameas  13a a Area      m^2                 1.9116  A |p|       Pa
sameas  13b b GasA/A                       -0.1565  B Ph(p)     deg
sameas  13c c SolA/A                    9.9874E-07  C |U|     m^3/s
sameas  13d d   rh       m                  90.003  D Ph(U)     deg
sameas  13e e LenTS1     m                  7.0105  E Htot      W
sameas  13f f Len P1     m              -2.6557E-09  F Edot      W
sameas  13g g Len P2     m                  1.0009  G HeatP1    W
sameas  13h h LenTS2     m                  5.0097  H HeatP2    W
  0.00      i HeatP1     W      (t)      1605.5     I qmaxP1  W/m^2
  0.00      j HeatP2     W      (t)      8034.8     J qmaxP2  W/m^2
  321.70    k TSolP1     K                  0.0000  K H2k       W
  408.00    l TSolP2     K
    1.0000  m FracQN
stainless  Solid type
```

Section 6.4 gives an example of the use of `VXQ1`.

Calculations for $\dot{N} = 0$:

The first-order continuity and momentum equations are taken to be unchanged by the time-averaged heat transfer. Hence, p_1 and U_1 evolve according to

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\text{gas}}}U_1, \tag{10.150}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\text{gas}}}{\rho_m a^2}\left(1+\frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1 + \frac{\beta(f_\kappa - f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}U_1, \tag{10.151}$$

in all four `VX**` segments, just the same as in stacks. However, the energy equation used in stacks does not suffice here: $\dot{H}_{2,k,\text{gas}}$ and $\dot{H}_{\text{solid}}$ should be considered separately instead of only considering their sum $\dot{H}_{2,k}$, and the gas mean temperature T_m and solid temperature T_{solid} might sometimes evolve separately with x. So, here there are four equations related to energy flux and energy conservation:

$$\begin{aligned}\dot{H}_{2,k} &= \frac{1}{2}\,\text{Re}\left[p_1\tilde{U}_1\left(1-\frac{T_m\beta(f_\kappa-\tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right] \\ &\quad + \frac{\rho_m c_p\left|U_1\right|^2}{2\omega A_{\text{gas}}(1-\sigma)\left|1-f_\nu\right|^2}\frac{dT_m}{dx}\,\text{Im}\left[\tilde{f}_\nu + \frac{(f_\kappa-\tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] \\ &\quad - A_{\text{gas}}k\frac{dT_m}{dx} + \dot{H}_{\text{solid}},\end{aligned} \tag{10.152}$$

$$\dot{H}_{\text{solid}} = -A_{\text{solid}}k_s\frac{dT_{\text{solid}}}{dx}, \tag{10.153}$$

$$\dot{q}(x) = \frac{d\dot{H}_{2,k}}{dx}, \tag{10.154}$$

$$\dot{q}(x) = \frac{d\dot{H}_{\text{solid}}}{dx} + \frac{A_{\text{gas}}k}{4r_h^2}N_{Nu}\left(T_{\text{solid}} - T_m\right), \tag{10.155}$$

where $\dot{q}$ is the heat per unit length externally delivered to the heat exchanger (if negative, it's the heat removed).

The first of these four equations describes the total energy transport in the x direction, and the second describes the energy transport (via heat conduction) in the x direction in the solid.

The third equation expresses conservation of energy for the dashed-line control volume in Fig. 10.8, showing that the heat input $d\dot{Q} = \dot{q}(x)\,dx$ equals the difference $(d\dot{H}_{2,k}/dx)\,dx$ between energy transport in and out along x.

The fourth equation similarly expresses conservation of energy for the solid separately within the control volume.

Equations (10.152)–(10.155) are used to obtain the evolution of the temperatures $T_m(x)$ and $T_{\rm solid}(x)$, and the energy flows $\dot{H}_2(x)$, $\dot{H}_{\rm solid}(x)$, and $\dot{q}(x)$, as functions of x, simultaneously with the use of Eqs. (10.150) and (10.151) to obtain $p_1(x)$ and $U_1(x)$. The details are different in `VXT*` segments and `VXQ*` segments:

In the `VXT*` segments, $T_{\rm solid}$ is independent of x and is a user's input. Then $\dot{H}_{\rm solid} = 0$, Eq. (10.152) is solved for dT_m/dx to yield the evolution of $T_m(x)$, Eqs. (10.154) and (10.155) together yield $d\dot{H}_{2,k}/dx$, and Eq. (10.154) yields $\dot{q}(x) = d\dot{H}_{2,k}/dx$. The heat transfer per unit length $\dot{q}(x)$ is integrated with respect to x to obtain the total heat transfer rate $\dot{Q}$. (In `VXT2`, the two "passes" can have different values of $T_{\rm solid}$.) Thus, Eqs. (10.150)–(10.155) are regarded as first-order differential equations for dp_1/dx, dU_1/dx, dT_m/dx, and $d\dot{H}_{2,k}/dx$ (and $\dot{q}(x)$ is obtained trivially from $d\dot{H}_{2,k}/dx$) :

$$\frac{dT_m}{dx} = \frac{\dot{H}_{2,k} - \frac{1}{2}\,\mathrm{Re}\left[p_1\tilde{U}_1\left(1 - \frac{T_m\beta(f_\kappa - \tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_m c_p|U_1|^2}{2\omega A_{\rm gas}(1-\sigma)|1-f_\nu|^2}\,\mathrm{Im}\left[\tilde{f}_\nu + \frac{(f_\kappa - \tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] - A_{\rm gas}k}, \tag{10.156}$$

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\rm gas}}U_1, \tag{10.157}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\rm gas}}{\rho_m a^2}\left(1 + \frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1 + \frac{\beta(f_\kappa - f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}U_1,$$

$$\text{with } \frac{dT_m}{dx} \text{ given by Eq. (10.156)}, \tag{10.158}$$

$$\frac{d\dot{H}_{2,k}}{dx} = \frac{A_{\rm gas}k}{4r_h^2}3.7\,m_k\,(T_{\rm solid} - T_m)\,. \tag{10.159}$$

The factor "3.7" in Eq. (10.159) is the laminar, isothermal-wall Nusselt number for a circular channel (Ref. [60], Table 6-1), so except for the presence of m_k this is just the simplest laminar, circular heat-transfer equation. The multiplier m_k accounts for the enhancement of the Nusselt number due to turbulence, as is described below.

Starting conditions are straightforward for `VXT*` segments: The starting p_1, U_1, T_m, and $\dot{H}_{2,k}$ are obtained directly from the end of the previous segment, and $T_{\rm solid}$ is given by the user (or as a guess, of course).

In the `VXQ*` segments, the heat exchange per unit length $\dot{q}$ is independent of x and is set by the user, as the segment's heat parameter divided by the appropriate length parameter. In `VXQ2`, the two "passes" can, of course, have different values of heat and length, and hence different values of $\dot{q}$. To make progress, it is necessary to make a couple of assumptions to simplify the starting equations. Solving Eqs. (10.152)–(10.155) for dT_m/dx, $dT_{\rm solid}/dx$,

$d\dot{H}_{2,k}/dx$, and $d\dot{H}_{\text{solid}}/dx$ and integrating them would require initial values for T_{solid} and $\dot{H}_{\text{solid}}$, in addition to the initial values of T_m and $\dot{H}_{2,k}$ that are passed into the `VXQ*` segment from the end of the previous segment. To avoid having to specify or guess the initial values of T_{solid} and $\dot{H}_{\text{solid}}$, we assume that most of $\dot{q}$ goes into the thermoacoustic gas in cases of interest to us, so that the first term on the right side of Eq. (10.155) can be neglected there. This assumption probably leads only to a slight overestimation of the temperature difference $|T_{\text{solid}} - T_m|$ in cases of interest to us. Second, we assume that $dT_{\text{solid}}/dx = dT_m/dx$. Examination of Eq. (10.155) (with the aforementioned first assumption, and with $\dot{q}$ independent of x here for a `VXQ*` segment) shows that this assumption is equivalent to $d(kN_{Nu})/dx = 0$, which seems reasonable enough (until someone figures out a better way to do all this). Then the starting equations can be rearranged as

$$\frac{dT_m}{dx} = \frac{\dot{H}_{2,k} - \frac{1}{2}\,\mathrm{Re}\left[p_1\tilde{U}_1\left(1 - \frac{T_m\beta(f_\kappa - \tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_m c_p|U_1|^2}{2\omega A_{\text{gas}}(1-\sigma)|1-f_\nu|^2}\,\mathrm{Im}\left[\tilde{f}_\nu + \frac{(f_\kappa - \tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] - A_{\text{gas}}k - A_{\text{solid}}k_s}, \tag{10.160}$$

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\text{gas}}}U_1, \tag{10.161}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\text{gas}}}{\rho_m a^2}\left(1 + \frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1 + \frac{\beta(f_\kappa - f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}U_1,$$

$$\text{with } \frac{dT_m}{dx} \text{ given by Eq. (10.160)}, \tag{10.162}$$

$$\frac{d\dot{H}_{2,k}}{dx} = \dot{q}, \tag{10.163}$$

$$T_{\text{solid}} = T_m + \frac{4r_h^2}{4.4\,m_k A_{\text{gas}}k}\,\dot{q}, \tag{10.164}$$

The factor "4.4" in Eq. (10.164) is the laminar, constant-heat-flux-density Nusselt number for a circular channel (Ref. [60], Table 6-1), so except for the presence of m_k this is just the simplest laminar, circular heat-transfer equation. The multiplier m_k accounts for the enhancement of the Nusselt number due to turbulence, as is described below.

Because of the assumptions in the previous paragraph, the starting conditions for the numerical integration are unambiguous: the four integrated variables T_m, p_1, U_1, and $\dot{H}_{2,k}$ all start with the values left over from the previous segment of the DeltaEC model.

Tubesheets are treated exactly like stacks, with $\dot{H}_{2,k}$ constant and

$$\frac{dT_m}{dx} = \frac{\dot{H}_{2,k} - \frac{1}{2}\,\mathrm{Re}\left[p_1\tilde{U}_1\left(1 - \frac{T_m\beta(f_\kappa - \tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_m c_p |U_1|^2}{2\omega A_{\mathrm{gas}}(1-\sigma)|1-f_\nu|^2}\,\mathrm{Im}\left[\tilde{f}_\nu + \frac{(f_\kappa - \tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] - A_{\mathrm{gas}}k - A_{\mathrm{solid}}k_s}, \tag{10.165}$$

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\mathrm{gas}}}U_1, \tag{10.166}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\mathrm{gas}}}{\rho_m a^2}\left(1 + \frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1 + \frac{\beta(f_\kappa - f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}U_1,$$

$$\text{with } \frac{dT_m}{dx} \text{ given by Eq. (10.165)}, \tag{10.167}$$

but with $A_{\mathrm{solid}} = A_{\mathrm{total}} - A_{\mathrm{gas}}$.

The important viscous and thermal functions f_ν and f_κ needed in Eqs. (10.150)–(10.152) and their variants are calculated as described here. For large displacement amplitudes, the effective length of the tube is its geometrical length (the *total* geometrical length, i.e., the sum of tubesheet and heat-transfer-region lengths). For small displacement amplitudes, the effective length is infinite. In between, this interpolation is used:

$$\Delta x_{\mathrm{eff}} = \frac{4\Delta x_{\mathrm{geom}}}{10\,|\xi_1|\,/\Delta x_{\mathrm{geom}} - 1} \text{ when } \frac{\Delta x_{\mathrm{geom}}}{10} < |\xi_1| < \frac{\Delta x_{\mathrm{geom}}}{2}. \tag{10.168}$$

(The choices of this functional form, and of 1/10 and 1/2 as transitions, are arbitrary and without justification, but these details have little effect on the final results.) With this effective length, shown along the x axis in Fig. 10.9, the top half of Fig. 7-1 in Kays and London is used to estimate the effect of turbulence on f_ν and f_κ in the tube. (That figure shows friction factor as a function of Reynolds number, for a few values of $\Delta x/D$.) DeltaEC's internal approximations to the Kays and London data are shown here in Fig. 10.10. Power laws

$$F = g10^{-BN_R} \tag{10.169}$$

are fit to Kays and London's high-N_R straight lines, and similar power laws with offsets are fit to their low-N_R straight lines, with interpolations for $2450 < N_R < 10,000$ using a cubic polynomial matched to the inner ends of the two straight lines in both slope and value. (Here F is the Moody friction factor, four times the Fanning friction factor that Kays and London call f.) Then, using the peak Reynolds number, a multiplier

$$m_\nu = \max\left[1,\ \frac{\delta_\nu^2 N_R}{24\pi r_h^2}\frac{F}{\mathrm{Re}\left[i/\left(1-f_\nu\right)\right]}\right] \tag{10.170}$$

gives the estimate of how turbulence (including entrance effects) affects what would otherwise be a laminar boundary layer. This expression is easily derived from the discussion about turbulence in Section 10.1.1.

Bessel-function (or, for large R, boundary-layer) expressions are used for "starting values" of f_ν and f_κ. Then m_ν as described above is used to generate modified f_ν and f_κ

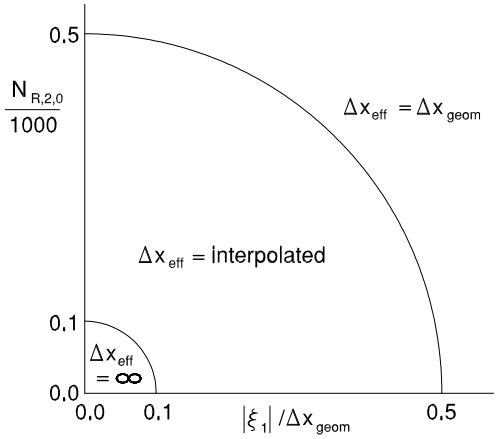


Figure 10.9: Effective length as a function of displacement amplitude $|\xi_1|$. For $\dot{N}$ disabled, only the horizontal axis is relevant. For $\dot{N}$ enabled, this plot displays Eqs. (10.182)–(10.184).

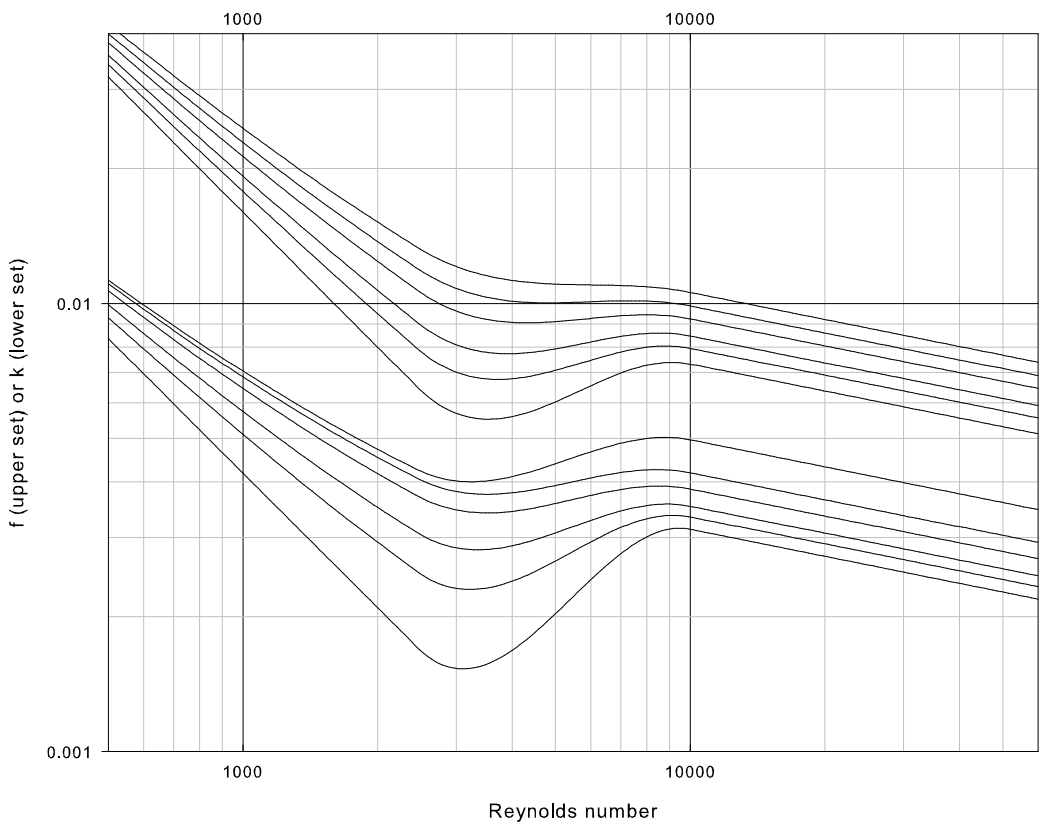


Figure 10.10: DeltaEC's fits for Kays and London's Fig. 7-1. The upper set of curves is the Fanning friction factor [Fanning = Moody $/4 = F/4$ in Eqs. (10.169) and (10.170)], and the lower set is for the heat-transfer coefficient. Within each set, the different curves are for different L/D : infinity, 100, 50, 25, 15, and 1 from bottom to top.

for the momentum and continuity equations above, exactly as described in Section 10.1.1: DeltaEC increases the resistive component of the pressure gradient, and hence the viscous power dissipation, by m_ν. It decreases the inertial pressure gradient by

$$m'_\nu = \left(\frac{1-\delta_\nu/R}{1-\delta_\nu/m_\nu R}\right)^2 \tag{10.171}$$

to correct approximately for the steeper velocity gradient at the wall, which increases the effective area open to gas contributing to inertial effects. It also multiplies the thermal penetration depth by m_ν, in an attempt to account very approximately for changes in thermal relaxation losses due to increased heat transfer. [Note that, perhaps somewhat inconsistently, m_ν enhances heat transfer in f_κ in Eqs. (10.151) and (10.152) while m_k is used in the time-averaged heat-transfer Eqs. (10.159) and (10.164).]

The value of m_k needed in Eq. (10.155) is obtained as follows. Power laws

$$\Sigma = A10^{-BN_R} \tag{10.172}$$

are fit to the low-N_R and high-N_R straight lines in the lower half of Kays and London Fig. 7-1, with interpolations for $2450 < N_R < 10,000$ using a cubic polynomial matched to the inner ends of the two straight lines in both slope and value. (That figure shows heat-transfer factor as a function of Reynolds number, for a few values of $\Delta x/D$. Here Σ is the heat-transfer factor that Ref. [60] calls $N_{St}\sigma^{2/3}$.) Figure 10.10 shows these fits. Since time-averaged heat transfer is desired, these equations are evaluated using the time-averaged Reynolds number $(2/\pi)\,|N_{R,1}|$. Finally, a multiplier

$$m_k = \frac{\Sigma}{\Sigma_{\text{laminar}}(\Delta x_{\text{eff}} = \infty)} \tag{10.173}$$

gives the turbulence correction factor in the otherwise laminar Eqs. (10.159) and (10.164).

Modifications to calculations for $\dot{N} \neq 0$:

In the presence of nonzero steady flow $\dot{N}$, the starting equations are modified and augmented as follows:

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\text{gas}}}U_1 + \frac{i\omega m\dot{N}}{\rho_m a^2 A_{\text{gas}}}\frac{1+(\gamma-1)f_\kappa/(1+\epsilon_s)}{1-f_\nu}p_1, \tag{10.174}$$

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\text{gas}}}{\rho_m a^2}\left(1+\frac{(\gamma-1)f_\kappa}{1+\epsilon_s}\right)p_1 + \frac{\beta(f_\kappa-f_\nu)}{(1-f_\nu)(1-\sigma)(1+\epsilon_s)}\frac{dT_m}{dx}U_1 + \frac{i\omega m\dot{N}}{\rho_m a^2 A_{\text{gas}}(1-f_\nu)}U_1, \tag{10.175}$$

$$
\begin{aligned}
\dot{H}_{\text{tot}} &= \frac{1}{2}\,\text{Re}\left[p_1\tilde{U}_1\left(1-\frac{T_m\beta(f_\kappa-\tilde{f}_\nu)}{(1+\epsilon_s)(1+\sigma)(1-\tilde{f}_\nu)}\right)\right] \\
&\quad +\frac{\rho_m c_p\left|U_1\right|^2}{2\omega A_{\text{gas}}(1-\sigma)\left|1-f_\nu\right|^2}\frac{dT_m}{dx}\,\text{Im}\left[\tilde{f}_\nu+\frac{(f_\kappa-\tilde{f}_\nu)(1+\epsilon_s f_\nu/f_\kappa)}{(1+\epsilon_s)(1+\sigma)}\right] \\
&\quad -A_{\text{gas}}k\frac{dT_m}{dx}+\dot{H}_{\text{solid}}+\dot{N}mw_m, \qquad (10.176)
\end{aligned}
$$

$$
\dot{H}_{\text{solid}} = -A_{\text{solid}}k_s\frac{dT_{\text{solid}}}{dx}, \qquad (10.177)
$$

$$
\dot{q}(x) = \frac{d\dot{H}_{\text{tot}}}{dx} \cong \frac{\dot{N}mc_p}{F_{\dot{Q}\dot{N}}}\frac{dT_m}{dx} \text{ in a pass, but } = 0 \text{ in a tubesheet,} \qquad (10.178)
$$

$$
\dot{q}(x) = \frac{A_{\text{gas}}k}{4r_h^2}N_{Nu}\left(T_{\text{solid}}-T_m\right), \qquad (10.179)
$$

$$
\frac{dp_{2,0,\text{HL}}}{dx} = -\text{sign}(\dot{N}_{2,0})\frac{1}{2}\rho_{\text{m}}\left(\frac{m\dot{N}_{2,0}}{\rho_{\text{m}}A_{\text{gas}}}\right)^2\frac{F}{4r_h}\sqrt{\frac{1}{2}+\left(\frac{\rho_{\text{m}}\left|U_1\right|}{m\dot{N}_{2,0}}\right)^2} \qquad (10.180)
$$

where $\dot{N}_{2,0}$ and $\dot{N}$ are related by

$$
\dot{N}_{2,0} = \dot{N}+\frac{\beta}{mc_p}\left[\dot{H}_{\text{tot}}-\dot{N}mw_m\right]-\left(\frac{1}{ma^2}+\frac{\beta}{mc_p}\right)\dot{E}_2. \qquad (10.181)
$$

Important: Equation (10.181) omits the $(A_{\text{gas}}k+A_{\text{solid}}k_s)\ dT_m/dx$ term that is present in Eq. (8.16) because we expect it to be negligible most of the time and because DELTAEC needs $\dot{N}_{2,0}$ before it knows dT_m/dx in each pass of its numerical integration. In Eqs. (10.174)–(10.180), DELTAEC evaluates m_ν and F on the basis of $\sqrt{\left|N_{R,1}\right|^2+N_{R,2,0}^2}$, and it evaluates m_κ on the basis of $\sqrt{4\left|N_{R,1}\right|^2/\pi^2+N_{R,2,0}^2}$. The square-root factor at the end of the equation for $dp_{2,0,HL}/dx$ is a rough empirical match to steady-flow `DUCT` results; this factor helps counteract the fact that high $|U_1|$ biases the friction factor F toward values that are unrealistically low for use in this equation. The friction and heat-transfer factors are based on an effective tube length Δx_{eff} plotted in Fig. 10.9 and given by

$$
\Delta x_{\text{eff}} = \infty \qquad \text{when } \left(\frac{|\xi_1|}{\Delta x_{\text{geom}}}\right)^2+\left(\frac{N_{R,2,0}}{1000}\right)^2 \le \frac{1}{100}, \qquad (10.182)
$$

$$
= \frac{4\Delta x_{\text{geom}}}{10\sqrt{\left(\frac{|\xi_1|}{\Delta x_{\text{geom}}}\right)^2+\left(\frac{N_{R,2,0}}{1000}\right)^2}-1} \qquad \text{when } \frac{1}{100}\le\left(\frac{|\xi_1|}{\Delta x_{\text{geom}}}\right)^2+\left(\frac{N_{R,2,0}}{1000}\right)^2\le\frac{1}{4}, \qquad (10.183)
$$

$$
= \Delta x_{\text{geom}} \qquad \text{when } \left(\frac{|\xi_1|}{\Delta x_{\text{geom}}}\right)^2+\left(\frac{N_{R,2,0}}{1000}\right)^2 \ge \frac{1}{4}. \qquad (10.184)
$$

10.8. Adiabatic–isothermal interface loss

10.8.1. JOIN

General description:

The JOIN segment accounts for "small" discontinuities in thermoacoustic variables at the interface between a heat exchanger or other isothermal segment and an unmixed, thermally stratified, adiabatic region such as a pulse tube. The discontinuity δT_m in mean temperatures is first order in the acoustic amplitude, the discontinuity δU_1 in volume flow rate is second order, and there is no discontinuity in pressure. See Ref. [61, 62] for δT_m, Ref. [63] for δU_1, and Ref. [12] and references therein for both δT_m and δU_1.

When it encounters a JOIN segment, DeltaEC looks both upstream and downstream (ignoring merely logistical segments such as RPN) to figure out whether the "heat exchanger" is upstream and the "pulse tube" is downstream, or vice versa. Both A and dT_m/dx are obtained from the "pulse tube" or other open duct, with $dT_m/dx = 0$ for ordinary DUCTs and CONEs, and dT_m/dx nonzero for STKDUCTs or STKCONEs. (If the relevant duct is a STKDUCT or STKCONE *downstream* of the JOIN segment, DeltaEC actually integrates ahead momentarily to evaluate dT_m/dx.)

The JOIN feature is not yet implemented for nonzero $\dot{N}$.

Input variables; Potential targets; Master–slave links:

None.

Format and examples:

```
!--------------------------------  8 --------------------------------
 SX     p.t. hot h.x.
  1.1675E-04 a Area      m^2             1.3507E+05 A |p|       Pa
     0.6470  b VolPor                     -17.664   B Ph(p)     deg
  3.0000E-03 c Length     m              2.9183E-04 C |U|     m^3/s
  1.2000E-05 d   rh       m               138.51    D Ph(U)     deg
     2.8164  e HeatIn     W      G        -13.711   E Hdot      W
   Possible targets                          -18.030   F Edot      W
 copper     Solid type                     300.00    H SolidT     K
!--------------------------------  9 --------------------------------
 JOIN       first join example
                                          1.3507E+05 A |p|       Pa
                                           -17.664   B Ph(p)     deg
                                          2.9863E-04 C |U|     m^3/s
                                           138.51    D Ph(U)     deg
                                           -13.711   E Hdot      W
                                           -18.450   F Edot      W
                                           300.00    G TBeg       K
                                           324.67    H TEnd       K
!-------------------------------- 10 --------------------------------
 STKDUCT    the pulse tube
  1.1675E-04 a Area      m^2     S=-2    1.3511E+05 A |p|       Pa
  3.8307E-02 b Perim      m    Fnc(10a)   -17.530   B Ph(p)     deg
  7.0000E-02 c Length     m              2.7164E-04 C |U|     m^3/s
  5.8678E-05 d WallA     m^2               166.83    D Ph(U)     deg
                                           -13.711   E Hdot      W
                                           -18.298   F Edot      W
                                           324.67    G TBeg       K
                                           57.932   H TEnd       K
 stainless  Solid type                       0.1518  I StkEdt     W
!-------------------------------- 11 --------------------------------
 RPN  ratio of cold end displacement to p.t. length
     0.1000  a G or T            (t)     9.2000E-02 A ChngeMe
 10C 2 / pi /  0b / 10a / 10c /
!-------------------------------- 12 --------------------------------
 JOIN       second example;  note that RPNs can go between
```

```
                                        1.3511E+05 A |p|       Pa
                                         -17.530   B Ph(p)     deg
                                        2.7854E-04 C |U|     m^3/s
                                         166.83    D Ph(U)     deg
                                         -13.711   E Hdot      W
                                         -18.763   F Edot      W
                                          57.932   G TBeg       K
                                          89.997   H TEnd       K
!--------------------------------- 13 ---------------------------------
 RPN  ratio of warm end displacement to p.t. length
     6.0000  a G or T            (t)     9.8838E-02 A ChngeMe
  8C 2 / pi /  0b / 10a / 10c /
!--------------------------------- 14 ---------------------------------
 SX     cold heat exchanger  adj rh
  1.1675E-04 a Area      m^2             1.5473E+05 A |p|       Pa
     0.6470  b VolPor                     -16.785   B Ph(p)     deg
  5.7150E-03 c Length     m              2.7985E-04 C |U|     m^3/s
  1.2000E-05 d   rh       m               169.37    D Ph(U)     deg
     7.1278  e HeatIn     W      G         -6.5828  E Hdot      W
    Possible targets                     -21.525   F Edot      W
                                           7.1278  G Heat      W
 copper     Solid type                     90.000   H SolidT    K
```

Calculations:

The discontinuity in temperature at `JOIN` is:

$$T_{m,\text{out}} - T_{m,\text{in}} = \left(\frac{T_m \beta}{\rho_m c_p} \left| p_1 \right| \sin\theta - \frac{\left| U_1 \right|_{\text{in}}}{\omega A_{\text{gas}}} \frac{dT_m}{dx} \right) F, \tag{10.185}$$

where θ is the angle by which p_1 leads U_1, dT_m/dx is evaluated on the thermally stratified, adiabatic side of the `JOIN`, and the factor F is given by

$$F = \frac{A_{\text{gas}} k \; dT_m/dx}{\dot{H}_{2,k} - \dot{E}}. \tag{10.186}$$

The factor F is a crude attempt to account for the two-dimensional nature of the `JOIN` problem within DELTAEC's inherently one-dimensional character. The derivation of the T_m discontinuity joining condition in the references cited above neglects the thermal conductivity of the wall of the tube and the boundary-layer shuttle entropy flux. If these are zero, then $F = 1$ and Eq. (10.185) represents the joining condition derived in the references cited above. At the other extreme, if the solid wall of the duct or the boundary layer is effectively a thermal short circuit compared to bulk gas conductivity, then $F = 0$ and no temperature discontinuity appears in the `JOIN` segment.

The discontinuity in $|U_1|$ at `JOIN` is

$$\begin{aligned} \left| U_1 \right|_{\text{out}} &= \left| U_1 \right|_{\text{in}} - \frac{8}{3\pi} \frac{(\gamma - 1)}{\rho_m a^2} \left| p_1 \right| \left| U_1 \right|_{\text{in}} \cos\theta \\ &= \left| U_1 \right|_{\text{in}} - \frac{16}{3\pi} \frac{\gamma - 1}{\rho_m a^2} \dot{E}. \end{aligned} \tag{10.187}$$

There is no discontinuity in p_1, in the phase of U_1, or in $\dot{H}_{2,k}$.

10.9. Mixture separation

10.9.1. `MIXBL`, `MIXSLAB`, and `MIXCIRC`

General description:

The `MIXBL`, `MIXSLAB`, and `MIXCIRC` segments calculate dp_1/dx, dU_1/dx, and dn_L/dx for sound propagation in gas mixtures, with or without steady flow, according to the theory described in Ref. [10], which builds on the theory described in Refs. [43, 64]. `MIXBL` performs the calculations in boundary-layer approximation, `MIXSLAB` in parallel-plate geometry of arbitrary gap $2y_0$, and `MIXCIRC` in circular pores of arbitrary radius R. The solid is assumed to be ideal, so it imposes a perfect isothermal boundary condition on the gas.

Input variables, `MIXBL`:

Area (m^2) Cross-sectional area A_{gas} available to the gas, based on inside dimensions.

Perim (m) Perimeter Π of the inside-cross-section described above.

Length (m) The length Δx of the segment.

Input variables, `MIXSLAB`:

Area (m^2) Total cross-sectional area A including gas and solid.

GasA/A The volumetric porosity $\phi = A_{gas}/A$. (We often fix this at 1.000 and set `Area` equal to the gas area.)

Length (m) The length Δx of the segment.

y0 (m) Half the plate spacing is y_0. Thus, $2y_0$ is the gap available to the gas.

Input variables, `MIXCIRC`:

Area (m^2) Total cross-sectional area A including gas and solid.

GasA/A The volumetric porosity $\phi = A_{gas}/A$. (We often fix this at 1.000 and set `Area` equal to the gas area.)

Length (m) The length Δx of the segment.

radius (m) The radius R of the pores.

Potential targets:

None.

Master–slave links:

The length of any of these segments can be slaved to the length of another segment to keep the sum of the two lengths constant.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples:

```
!--------------------------------  1 --------------------------------
MIXBL      boundary-layer approx
 4.0000E-04 a Area     m^2               3670.4     A |p|       Pa
 8.0000E-02 b Perim     m                  -0.2884  B Ph(p)     deg
    0.1000  c Length    m                1.4715E-03 C |U|     m^3/s
                                          -87.558   D Ph(U)     deg
                                        -2.6300E-08 E Htot      W
                                            0.1286  F Edot      W
                                            0.3406  G nL Beg
                                            0.3463  H nL End
!--------------------------------  2 --------------------------------
MIXCIRC    an array of circular pores
 2.2000E-05 a Area     m^2               5637.2     A |p|       Pa
    1.0000  b GasA/A                      162.98    B Ph(p)     deg
    0.1000  c Length    m                1.4540E-03 C |U|     m^3/s
 1.0000E-03 d radius    m                 -86.581   D Ph(U)     deg
                                        -2.6300E-08 E Htot      W
                                           -1.4313  F Edot      W
                                            0.3463  G nL Beg
                                            0.3452  H nL End
!--------------------------------  3 --------------------------------
MIXSLAB    an array of parallel plates
 1.0000E-03 a Area     m^2               5460.3     A |p|       Pa
    0.8000  b GasA/A                      165.32    B Ph(p)     deg
    0.1000  c Length    m                4.0475E-03 C |U|     m^3/s
 5.0000E-04 d  y0       m                  53.191   D Ph(U)     deg
                                        -2.6300E-08 E Htot      W
                                           -4.1618  F Edot      W
                                            0.3452  G nL Beg
                                            0.2560  H nL End
```

Section 7.3 has an example DeltaEC file using `MIXCIRC`.

Calculations:

The evolution of p_1 with x in `MIXBL`, `MIXSLAB`, and `MIXCIRC` is given by the same integration as in DeltaEC's other ducts,

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\text{gas}}}U_1, \tag{10.188}$$

because Ref. [65] showed that the first-order thermoacoustic momentum equation is unchanged by mixture-separation effects. The evolution of U_1 with x is given by Eq. (8) in Ref. [65], but they didn't write the result explicitly [10]. The explicit result, in DeltaEC's notation, is

$$\frac{dU_1}{dx} = -\frac{i\omega A_{\text{gas}}}{\rho_m a^2}\left\{1+(\gamma-1)\left[Bf_\nu + Cf_{\kappa D} + (1-B-C)\,f_{D\kappa}\right]\right\}p_1, \tag{10.189}$$

where B and C are given in Ref. [43], and the other variables carry their usual DeltaEC meanings or are defined below.

The evolution of the light fraction n_L with x is given by solving Eq. (7) in Ref. [10] for dn_H/dx, and integrating numerically for fixed $\dot{N}_L$, which is equivalent to fixed $\dot{N}_H$ and fixed $\dot{N} = \dot{N}_H + \dot{N}_L$. Solving Eq. (7) for dn_H/dx yields

$$\frac{dn_H}{dx} = \frac{\dot{N}_H - n_H\dot{N} - \frac{\delta_\kappa}{4r_h}\frac{\gamma-1}{\gamma}\frac{k_T}{R_{\rm univ}T_m}\left|p_1\right|\left|U_1\right|\left(F_{\rm trav}\cos\theta + F_{\rm stand}\sin\theta\right)}{\frac{\delta_\kappa}{4r_h}\frac{\rho_m\left|U_1\right|^2}{m_{\rm avg}\omega A_{\rm gas}}F_{\rm grad} - \frac{\rho_m}{m_{\rm avg}}A_{\rm gas}D_{12}}, \tag{10.190}$$

and then DELTAEC uses $dn_L/dx = -\,dn_H/dx$ to obtain the desired result.

In addition to the usual definitions of the viscous and thermal penetration depths $\delta_\nu = \sqrt{2\mu/\omega\rho_m}$ and $\delta_\kappa = \sqrt{2k/\omega\rho_m c_p}$, two new penetration depths are defined for mixtures:

$$\delta_{\kappa D}^2 = \frac{1}{2}\delta_\kappa^2\left[1 + (1+\varepsilon)/L + \sqrt{\left[1+(1+\varepsilon)/L\right]^2 - 4/L}\right], \tag{10.191}$$

$$\delta_{D\kappa}^2 = \frac{1}{2}\delta_\kappa^2\left[1 + (1+\varepsilon)/L - \sqrt{\left[1+(1+\varepsilon)/L\right]^2 - 4/L}\right], \tag{10.192}$$

where

$$\varepsilon = \frac{\gamma-1}{\gamma}\frac{k_T^2}{n_L\left(1-n_L\right)}, \tag{10.193}$$

k_T is the thermal diffusion ratio, $L = k/\rho_m c_p D_{12}$, and D_{12} is the binary mass-diffusion coefficient.

In `MIXBL`, DELTAEC uses the boundary-layer results of Ref. [43]. For f_ν, $f_{\kappa D}$, and $f_{D\kappa}$, the boundary layer result is

$$f_j = (1-i)\frac{\delta_j}{2r_h}, \tag{10.194}$$

where j is ν, κD, or $D\kappa$. For the three F's, the boundary-layer results are

$$F_{\rm trav} = \frac{\sigma\sqrt{\sigma L} - \sqrt{\sigma} - \sigma\sqrt{L}\left(\delta_{\kappa D}/\delta_\kappa + \delta_{D\kappa}/\delta_\kappa\right)}{\left(1+\sqrt{L}\right)\left[(1+\sigma)(1+\sigma L) + \varepsilon\sigma\right]}, \tag{10.195}$$

$$F_{\rm stand} = \frac{-\sigma\sqrt{\sigma L} + \sqrt{\sigma} - \sigma\sqrt{L}\left(\delta_{\kappa D}/\delta_\kappa + \delta_{D\kappa}/\delta_\kappa\right)}{\left(1+\sqrt{L}\right)\left[(1+\sigma)(1+\sigma L) + \varepsilon\sigma\right]}, \tag{10.196}$$

$$F_{\rm grad} = \frac{\sqrt{\sigma}L\left(1-\sigma^2\right)\left(1+\sqrt{L}\right) + \varepsilon\sqrt{\sigma}\left(\sqrt{L}-1\right) + \left[\left(\sigma^2-1\right)L + \varepsilon\sigma\sqrt{L}\right]\left(\delta_{\kappa D}/\delta_\kappa + \delta_{D\kappa}/\delta_\kappa\right)}{\left(1+\sqrt{L}\right)\left[(1+\sigma)(1+\sigma L) + \varepsilon\sigma\right]\left[(1-\sigma)(1-\sigma L) - \varepsilon\sigma\right]/\sigma}. \tag{10.197}$$

In MIXSLAB and MIXCIRC, DELTAEC uses the following expressions:

$$F_{\text{trav}} = -\frac{2r_h}{\delta_\kappa}\text{Re}\left[\frac{G}{\left(1-\tilde{f}_\nu\right)}\right], \tag{10.198}$$

$$F_{\text{stand}} = \frac{2r_h}{\delta_\kappa}\text{Im}\left[\frac{G}{\left(1-\tilde{f}_\nu\right)}\right], \tag{10.199}$$

$$\begin{aligned} F_{\text{grad}} = & -\frac{2r_h}{\delta_\kappa}\frac{1}{|1-f_\nu|^2}\frac{\sigma}{(1-\sigma)(1-\sigma L)-\varepsilon\sigma} \\ & \cdot \text{Im}\left[\frac{(1-\sigma)}{\sigma S}f_\nu\left(\frac{\delta_\kappa^2-\delta_{\kappa D}^2}{\delta_\nu^2+\delta_{\kappa D}^2}f_{\kappa D}-\frac{\delta_\kappa^2-\delta_{D\kappa}^2}{\delta_\nu^2+\delta_{D\kappa}^2}f_{D\kappa}+\frac{(1+\sigma)LQ}{M}\tilde{f}_\nu+S\right)+\varepsilon G\right], \end{aligned} \tag{10.200}$$

where r_h is the hydraulic radius and

$$S = \left(\frac{\delta_\kappa^2}{\delta_{D\kappa}^2}-1\right)f_{D\kappa}-\left(\frac{\delta_\kappa^2}{\delta_{\kappa D}^2}-1\right)f_{\kappa D}, \tag{10.201}$$

$$Q = \frac{\delta_{\kappa D}^2-\delta_{D\kappa}^2}{\delta_\kappa^2}, \tag{10.202}$$

$$M = (1+\sigma)(1+\sigma L)+\varepsilon\sigma, \tag{10.203}$$

$$G = \frac{\sigma LQ}{MS}f_{\kappa D}f_{D\kappa}+\frac{\tilde{f}_\nu}{S}\left(\frac{f_{\kappa D}}{1+\delta_\nu^2/\delta_{D\kappa}^2}-\frac{f_{D\kappa}}{1+\delta_\nu^2/\delta_{\kappa D}^2}\right). \tag{10.204}$$

These expressions appear in Ref. [10] in the context of a circular tube.[2] In MIXCIRC the three f's are given by

$$f_j = \frac{2J_1[(i-1)R/\delta_j]}{(i-1)(R/\delta_j)J_0[(i-1)R/\delta_j]}, \tag{10.205}$$

where R is the circle radius, $r_h = R/2$, and j is ν, κD, or $D\kappa$. In MIXSLAB, the three f's are given by

$$f_j = \frac{\tanh[(1+i)y_0/\delta_j]}{(1+i)y_0/\delta_j}, \tag{10.206}$$

where the plate separation is $2y_0 = 2r_h$ and j is ν, κD, or $D\kappa$.

The head-loss pressure gradient $dp_{2,0,HL}/dx$ is calculated on the basis of laminar flow.

10.9.2. MIXTBL, MIXTSLAB, and MIXTCIRC

General description:

The MIXTBL, MIXTSLAB, and MIXTCIRC segments calculate dp_1/dx, dU_1/dx, dn_L/dx, and dT_m/dx for sound propagation in gas mixtures, with or without steady flow, according to

[2] There was a typographical error in Eq. (10.200) in the Version 6.0 Users Guide. The same typographical error appears in Eq. (A12), Ref. [10], for which an erratum has been published in J. Acoust. Soc. Am. in 2008.

the theory described in Ref. [66], which builds on the theory described in Refs. [43, 64, 10]. `MIXTBL` performs the calculations in boundary-layer approximation, `MIXTSLAB` in parallel-plate geometry of arbitrary gap $2y_0$, and `MIXTCIRC` in circular pores of arbitrary radius R.

The solid is assumed to be laterally ideal, so it imposes a perfect isothermal boundary condition on the oscillating temperature of the gas. This assumption would correspond to $\epsilon_s = 0$ in a `STK**` segment. However, the solid's finite thermal conductivity in the x direction is included in the calculation of dT_m/dx in `MIXT**` segments, in the same way as in `STK**` segments.

Input variables, `MIXTBL`:

Area (m^2) Cross-sectional area A_{gas} available to the gas, based on inside dimensions.

Perim (m) Perimeter Π of the inside-cross-section described above.

Length (m) The length Δx of the segment.

WallA (m^2) The cross-sectional area A_{solid} of the wall material.

Solid The material comprising the walls.

Input variables, `MIXTSLAB`:

Area (m^2) Total cross-sectional area A including gas and solid.

GasA/A The volumetric porosity $\phi = A_{\mathrm{gas}}/A$. (We often set this to 1.000 and set `Area` equal to the gas area.)

Length (m) The length Δx of the segment.

y0 (m) Half the plate spacing is y_0. Thus, $2y_0$ is the gap available to the gas between the plates.

Solid The material comprising the walls.

Input variables, `MIXTCIRC`:

Area (m^2) Total cross-sectional area A including gas and solid.

GasA/A The volumetric porosity $\phi = A_{\mathrm{gas}}/A$. (We often set this to 1.000 and set `Area` equal to the gas area.)

Length (m) The length Δx of the segment.

radius (m) The radius R of the pores.

Solid The material comprising the walls.

Potential targets:

None.

Master–slave links:

The length of any of these segments can be slaved to the length of another segment to keep the sum of the two lengths constant.

(For an introduction to master–slave links, and a complete listing of linking options for all segments, see Section 2.7.2.)

Format and examples:

```
!--------------------------------  1 --------------------------------
MIXTCIRC    an array of circular pores
 2.2000E-05 a Area     m^2                5637.2     A |p|       Pa
    0.9100  b GasA/A                      162.98    B Ph(p)     deg
    0.1000  c Length    m                1.4540E-03 C |U|     m^3/s
 1.0000E-03 d radius    m                 -86.581   D Ph(U)     deg
                                        -2.6300E-08 E Htot      W
                                           -1.4313  F Edot      W
                                              0.3463  G nL Beg
                                              0.3452  H nL End
                                              310.00  I TBeg
 stainless    Solid type                   451.23  J TEnd
```

Calculations:

In `MIXTBL`, `MIXTSLAB`, and `MIXTCIRC`, the integration variables evolve according to

$$\frac{dT_m}{dx} = \frac{\dot{H}_{\text{tot}} - \dot{N} m w_m - \frac{1}{2}\text{Re}\left[p_1 \tilde{U}_1 \left(1 - \frac{f_\kappa - \tilde{f}_\nu}{(1+\sigma)(1-\tilde{f}_\nu)}\right)\right]}{\frac{\rho_m c_p |U_1|^2}{2\omega A_{\text{gas}}(1-\sigma)|1-f_\nu|^2}\text{Im}\left[\tilde{f}_\nu + \frac{(f_\kappa - \tilde{f}_\nu)}{(1+\sigma)}\right] - A_{\text{gas}} k - A_{\text{solid}} k_s}, \tag{10.207}$$

$$\frac{dp_1}{dx} = -\frac{i\omega\rho_m}{(1-f_\nu)A_{\text{gas}}} U_1, \tag{10.208}$$

$$\frac{dn_H}{dx} = \frac{\dot{N}_H - n_H \dot{N} - \frac{\delta_\kappa}{4r_h}\frac{(\gamma-1)k_T}{\gamma R_{\text{univ}} T_m}|p_1||U_1|\left(F_{\text{trav}}\cos\theta + F_{\text{stand}}\sin\theta\right) + \frac{\rho_m}{m}\frac{k_T}{T_m}\frac{dT_m}{dx}\left(A_{\text{gas}} D_{12} - \frac{\delta_\kappa}{4r_h}\frac{|U_1|^2 F_{\nabla T}}{|1-f_\nu|^2 \omega A_{\text{gas}}}\right)}{\frac{\rho_m}{m}\left(\frac{\delta_\kappa}{4r_h}|U_1|^2 \frac{F_{\text{grad}}}{\omega A_{\text{gas}}} - A_{\text{gas}} D_{12}\right)}, \tag{10.209}$$

$$\frac{dU_1}{dx} = -i\omega A_{\text{gas}}\left(\frac{p_1}{p_m} - \frac{\langle T_1\rangle}{T_m} + \frac{m_H - m_L}{m_{\text{avg}}}\langle n_{H,1}\rangle\right) + \frac{1}{T_m}\frac{dT_m}{dx}U_1 - \frac{m_H - m_L}{m_{\text{avg}}}\frac{dn_H}{dx}U_1, \tag{10.210}$$

where

$$\frac{\langle T_1\rangle}{T_m} = \frac{\gamma-1}{\gamma}\left[1-Bf_\nu - Cf_{\kappa D} - (1-B-C)\,f_{D\kappa}\right]\frac{p_1}{p_m} - \frac{(1-f_{D\kappa})}{i\omega A_{\rm gas}\,(1-f_\nu)}\frac{1}{T_m}\frac{dT_m}{dx}U_1, \tag{10.211}$$

$$\begin{aligned}\langle n_{H,1}\rangle &= -\frac{\gamma-1}{\gamma}\frac{m_L}{m_{\rm avg}}\frac{k_T}{\varepsilon}\left[B\left(1-\frac{1}{\sigma}\right)f_\nu + C\left(1-\frac{\delta_\kappa^2}{\delta_{\kappa D}^2}\right)f_{\kappa D} + (1-B-C)\left(1-\frac{\delta_\kappa^2}{\delta_{D\kappa}^2}\right)f_{D\kappa}\right]\frac{p_1}{p_m}\\ &\quad -\frac{1}{i\omega A_{\rm gas}}\frac{dn_H}{dx}U_1 + \frac{m_L}{m_{\rm avg}}\frac{k_T}{\varepsilon}\frac{1}{i\omega A_{\rm gas}\,(1-f_\nu)}\left[\left(1-\frac{\delta_\kappa^2}{\delta_{D\kappa}^2}\right)f_{D\kappa} - f_\nu\right]\frac{1}{T_m}\frac{dT_m}{dx}U_1,\end{aligned} \tag{10.212}$$

and

$$B = \frac{ie^{-i\theta}}{1-f_\nu}\frac{\sigma}{(1-\sigma)\,(1-\sigma L)-\varepsilon\sigma}\left[(\sigma L-1)\frac{dT_m/dx}{\nabla T_{\rm crit}} + \varepsilon\frac{dn_H/dx}{\nabla n_{\rm crit}}\right], \tag{10.213}$$

$$\begin{aligned}C &= \frac{\sqrt{L}\delta_{\kappa D}-\delta_{D\kappa}}{\left(1+\sqrt{L}\right)(\delta_{\kappa D}-\delta_{D\kappa})}\left[1-B\left(1-\frac{1-\sigma}{\sqrt{\sigma}}\frac{\delta_\kappa}{\sqrt{L}\delta_{\kappa D}-\delta_{D\kappa}}\right)+\right.\\ &\quad \left.+\frac{ie^{-i\theta}}{1-f_\nu}\left(1+\sqrt{\sigma}\frac{\delta_\kappa}{\sqrt{L}\delta_{\kappa D}-\delta_{D\kappa}}\right)\frac{dT_m/dx}{\nabla T_{\rm crit}}\right],\end{aligned} \tag{10.214}$$

$$\nabla T_{\rm crit} = \frac{\gamma-1}{\gamma}\frac{|p_1|}{p_m}\frac{\omega A_{\rm gas}}{|U_1|}T_m, \tag{10.215}$$

$$\nabla n_{\rm crit} = \frac{\gamma-1}{\gamma}\frac{|p_1|}{p_m}\frac{\omega A_{\rm gas}}{|U_1|}k_T. \tag{10.216}$$

In addition to the usual definitions of the viscous and thermal penetration depths $\delta_\nu = \sqrt{2\mu/\omega\rho_m}$ and $\delta_\kappa = \sqrt{2k/\omega\rho_m c_p}$, two new penetration depths are defined for mixtures:

$$\delta_{\kappa D}^2 = \frac{1}{2}\delta_\kappa^2\left[1+(1+\varepsilon)/L + \sqrt{\left[1+(1+\varepsilon)/L\right]^2 - 4/L}\right], \tag{10.217}$$

$$\delta_{D\kappa}^2 = \frac{1}{2}\delta_\kappa^2\left[1+(1+\varepsilon)/L - \sqrt{\left[1+(1+\varepsilon)/L\right]^2 - 4/L}\right], \tag{10.218}$$

where

$$\varepsilon = \frac{\gamma-1}{\gamma}\frac{k_T^2}{n_L\,(1-n_L)}, \tag{10.219}$$

k_T is the thermal diffusion ratio, $L = k/\rho_m c_p D_{12}$, and D_{12} is the binary mass-diffusion coefficient.

For `MIXTBL`,

$$f_j = (1-i)\frac{\delta_j}{2r_h}, \tag{10.220}$$

where j is ν, κD, or $D\kappa$ and r_h is the hydraulic radius, and the four F's are given by

$$F_{\text{trav}} = \frac{\sigma\sqrt{\sigma L} - \sqrt{\sigma} - \sigma\sqrt{L}\left(\delta_{\kappa D}/\delta_\kappa + \delta_{D\kappa}/\delta_\kappa\right)}{\left(1+\sqrt{L}\right)\left[(1+\sigma)(1+\sigma L)+\varepsilon\sigma\right]}, \tag{10.221}$$

$$F_{\text{stand}} = \frac{-\sigma\sqrt{\sigma L} + \sqrt{\sigma} - \sigma\sqrt{L}\left(\delta_{\kappa D}/\delta_\kappa + \delta_{D\kappa}/\delta_\kappa\right)}{\left(1+\sqrt{L}\right)\left[(1+\sigma)(1+\sigma L)+\varepsilon\sigma\right]}, \tag{10.222}$$

$$F_{\text{grad}} = \frac{\sqrt{\sigma}L\left(1-\sigma^2\right)\left(1+\sqrt{L}\right) + \varepsilon\sqrt{\sigma}\left(\sqrt{L}-1\right) + \left[\left(\sigma^2-1\right)L + \varepsilon\sigma\sqrt{L}\right]\left(\delta_{\kappa D}/\delta_\kappa + \delta_{D\kappa}/\delta_\kappa\right)}{\left(1+\sqrt{L}\right)\left[(1+\sigma)(1+\sigma L)+\varepsilon\sigma\right]\left[(1-\sigma)(1-\sigma L)-\varepsilon\sigma\right]/\sigma}, \tag{10.223}$$

$$-F_{\nabla T} = \frac{\sqrt{\sigma L}\left(1+\sqrt{L}\right)\left(1+\sigma^2\sqrt{L}\right) + \sigma^{\frac{3}{2}}L + \varepsilon\sqrt{\sigma} - \frac{1}{\sqrt{\sigma}} + \left[\sqrt{L}\left(1-\sigma L-\varepsilon\sigma\right) - \sigma L(1+\sigma)\right]\left(\frac{\delta_{\kappa D}}{\delta_\kappa} + \frac{\delta_{D\kappa}}{\delta_\kappa}\right)}{\left(1+\sqrt{L}\right)\left[(1+\sigma)(1+\sigma L)+\varepsilon\sigma\right]\left[(1-\sigma)(1-\sigma L)-\varepsilon\sigma\right]/\sigma}. \tag{10.224}$$

In `MIXTSLAB` and `MIXTCIRC`, DeltaEC uses the following expressions:

$$F_{\text{trav}} = -\frac{2r_h}{\delta_\kappa}\text{Re}\left[\frac{G}{\left(1-\tilde{f}_\nu\right)}\right], \tag{10.225}$$

$$F_{\text{stand}} = \frac{2r_h}{\delta_\kappa}\text{Im}\left[\frac{G}{\left(1-\tilde{f}_\nu\right)}\right], \tag{10.226}$$

$$\begin{aligned} F_{\text{grad}} = & -\frac{2r_h}{\delta_\kappa}\frac{1}{\left|1-f_\nu\right|^2}\frac{\sigma}{(1-\sigma)(1-\sigma L)-\varepsilon\sigma} \\ & \cdot \text{Im}\left[\frac{(1-\sigma)}{\sigma S}f_\nu\left(\frac{\delta_\kappa^2-\delta_{\kappa D}^2}{\delta_\nu^2+\delta_{\kappa D}^2}f_{\kappa D} - \frac{\delta_\kappa^2-\delta_{D\kappa}^2}{\delta_\nu^2+\delta_{D\kappa}^2}f_{D\kappa} + \frac{(1+\sigma)LQ}{M}\tilde{f}_\nu + S\right) + \varepsilon G\right], \end{aligned} \tag{10.227}$$

$$\begin{aligned} F_{\nabla T} = & \frac{2r_h}{\delta_\kappa}\frac{1}{\left|1-f_\nu\right|^2}\frac{\sigma}{(1-\sigma)(1-\sigma L)-\varepsilon\sigma} \\ & \cdot \text{Im}\left[\frac{f_\nu}{S}\left(\frac{\delta_\kappa^2-\delta_{\kappa D}^2}{\delta_\nu^2+\delta_{\kappa D}^2}f_{\kappa D} - \frac{\delta_\kappa^2-\delta_{D\kappa}^2}{\delta_\nu^2+\delta_{D\kappa}^2}f_{D\kappa} + \frac{(1+\sigma)LQ}{M}\tilde{f}_\nu + S\right) + \frac{1-\sigma L-\varepsilon\sigma}{\sigma}G\right], \end{aligned} \tag{10.228}$$

where r_h is the hydraulic radius and

$$S = \left(\frac{\delta_\kappa^2}{\delta_{D\kappa}^2} - 1\right) f_{D\kappa} - \left(\frac{\delta_\kappa^2}{\delta_{\kappa D}^2} - 1\right) f_{\kappa D}, \tag{10.229}$$

$$Q = \frac{\delta_{\kappa D}^2 - \delta_{D\kappa}^2}{\delta_\kappa^2}, \tag{10.230}$$

$$M = (1+\sigma)(1+\sigma L) + \varepsilon\sigma, \tag{10.231}$$

$$G = \frac{\sigma L Q}{MS} f_{\kappa D} f_{D\kappa} + \frac{\tilde{f}_\nu}{S}\left(\frac{f_{\kappa D}}{1+\delta_\nu^2/\delta_{D\kappa}^2} - \frac{f_{D\kappa}}{1+\delta_\nu^2/\delta_{\kappa D}^2}\right). \tag{10.232}$$

These expressions appear in Refs. [10, 66] in the context of a circular tube.

In `MIXTCIRC` the three f's are given by

$$f_j = \frac{2J_1[(i-1)R/\delta_j]}{(i-1)(R/\delta_j)J_0[(i-1)R/\delta_j]}, \tag{10.233}$$

where R is the circle radius, $r_h = R/2$, and j is ν, κD, or $D\kappa$.

In `MIXTSLAB`, the three f's are given by

$$f_j = \frac{\tanh[(1+i)y_0/\delta_j]}{(1+i)y_0/\delta_j}, \tag{10.234}$$

where the plate separation is $2y_0 = 2r_h$ and j is ν, κD, or $D\kappa$.

The head-loss pressure gradient $dp_{2,0,HL}/dx$ is calculated on the basis of laminar flow in `MIXT**` segments.

11. Logistical segments

Details about DeltaEC's physically realistic thermoacoustics segments were described in Chapter 10. Here, we detail the logistical segments, which perform only trivial computations or no computations but are used to coordinate other segments, impose boundary conditions, and display non-standard results.

The behavior of some of these segments depends on whether the DeltaEC model using the segments is pure acoustics or thermoacoustics or includes nonzero $\dot{N}$ or includes separation effects in gas mixtures. The discussion of $\dot{N}_L$ and n_L only applies when `MIX**` segments and gas mixtures are in use. The discussion of $\dot{N}$ and $p_{2,0,HL}$ only applies when nonzero time-averaged flow is involved. Users who are interested only in acoustics can focus entirely on p_1 and U_1 here, and can safely ignore all discussion of $\dot{H}$, T_m, $\dot{N}$, $p_{2,0,HL}$, $\dot{N}_L$, and n_L.

An alphabetical listing of segments is in the Segment Index at the end of the Users Guide.

11.1. Starting and ending

11.1.1. `TITLE`

General description:

`TITLE` has no numerical inputs. It must come at the very beginning of a DeltaEC file, and its heading and (optional) notes should be descriptive, to remind the user what the file is about.

Format and examples:

```
TITLE   July 2006 engine with Pete's modifications.
```

11.1.2. `BEGIN`

General description:

After the `TITLE` segment, `BEGIN` is counted as the zeroth segment of the file. It is used to initialize "global" variables that are shared among subsequent segments (e.g., frequency and mean pressure) and the integration variables required by each pass of DeltaEC to get started (e.g., real and imaginary parts of p_1 and U_1).

`BEGIN` segments can be used anywhere in a DeltaEC file, in order to set any or all of DeltaEC's integration variables or global variables to new values. This can be useful for packing two or more related DeltaEC models into a single file or for changing from one gas to another in the middle of a model. (Do not use such a `BEGIN` within a teebranch; instead,

use RPN "change" assignments to set or reset variables in a teebranch.) When more than one BEGIN is used in a model, each BEGIN starts a new leg. (Legs are described in Section 11.2.1.)

Starting with DeltaEC version 6.0, BEGIN causes subsequent DUCTs, CONEs, SURFACEs, and IMPEDANCEs to set $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}}$, so the user should imagine such segments to be thermally insulated. ANCHOR, described in Section 11.3.1, can be used to override this deafault, and the user should imagine segments downstream of ANCHOR to be attached to a heat sink at the local temperature T_m. (ANCHOR is useful for very low power systems and simple systems where total energy flow is irrelevant, and of course for systems with DUCTs and other components that actually are well-attached to a heat sink.)

Input variables:

Mean P (Pa) The mean pressure of the gas, p_m.

Freq (Hz) The frequency f of the sinusoidal oscillations in subsequent segments. Most equations in the Users Guide use $\omega = 2\pi f$.

TBeg (K) The initial temperature T_m of the gas.

|p| (Pa) The initial value of $|p_1|$.

Ph(p) (deg) The initial phase of p_1.

|U| (m^3/s) The initial value of $|U_1|$.

Ph(U) (deg) The initial phase of U_1.

Gas The gas type, e.g., helium or air.

Default Htot / Other Htot (W) With this choice, the user can let DeltaEC assign the default value to the total energy flow $\dot{H}_{\text{tot}}$ or can assign a numerical value or let it be a guess.

Enable Ndot / Disable Ndot This check box controls some calculations and results in many subsequent segments.

Ndot (mol/s) If "Enable Ndot" is checked, the user can specify the steady molar flow rate $\dot{N}$.

nL If the gas type is a gas mixture, this parameter initializes the light-component mole fraction n_L.

Bulk NLdot / Other NLdot (mol/s) If the gas type is a gas mixture, this check box controls whether DeltaEC assigns the default, bulk value to the light component's steady molar flow rate $\dot{N}_L$ or lets the user assign a different value.

Potential targets; Master–slave links:

None.

Format and examples:

```
BEGIN This example is the minimal format, as saved in an out file.
1.0e6 Pa    a Mean P
500. Hz     b Freq
300. K      c TBeg
3.0e4 Pa    d |p|
0.00 deg    e Ph(p)
0.00 m3/s   f |U|
0.00 deg    g Ph(U)
helium      Gas type
```

```
BEGIN This example is the most complicated possible format, as saved in an out file.
1.0e6 Pa    a Mean P
500. Hz     b Freq
300. K      c TBeg
3.0e4 Pa    d |p|
0.00 deg    e Ph(p)
0.00 m3/s   f |U|
0.00 deg    g Ph(U)
2000.       h Htot
1.00E-8     i Ndot
0.800       j nL
1.00E-9     k NLdot
HeAr      Gas type
```

```
TITLE     Argon cover on top of sodium-potassium column, illustrating two BEGINs in one model
!->2beg.out
!Created@10:20:31 29-Mar-07 with DeltaE Vers. 6.0g7 for the IBM/PC-Compatible
!--------------------------------  0 --------------------------------
 BEGIN      Initialize in the argon
  1.0000E+05 a Mean P    Pa               3.9676E-05 A  |U|   G(  0f)     P
   100.00    b Freq      Hz                 87.799   B Ph(U)  G(  0g)     P
   400.00    c TBeg      K
  1000.0     d  |p|      Pa
     0.0000  e Ph(p)     deg
  3.9676E-05 f  |U|     m^3/s    G
    87.799   g Ph(U)     deg     G
     0.0000  h Htot       W
     0.0000  j nL
 HeAr       Gas type
!--------------------------------  1 --------------------------------
 DUCT       the argon on top
  1.0000E-04 a Area      m^2     S= -2    1015.7     A |p|       Pa
  3.5449E-02 b Perim      m    Fn(  1a)  -7.9065E-02 B Ph(p)     deg
     0.1000  c Length     m               1.9924E-08 C |U|     m^3/s
                                            89.527   D Ph(U)     deg
                                             0.0000  E Htot      W
 ideal      Solid type                    6.9504E-08 F Edot      W
!--------------------------------  2 --------------------------------
 BEGIN      transition from argon to Na-K
 sameas   0a a Mean P    Pa
 sameas   0b b Freq      Hz
 sameas   0c c TBeg      K
 sameas   1A d  |p|      Pa
 sameas   1B e Ph(p)     deg
 sameas   1C f  |U|     m^3/s
 sameas   1D g Ph(U)     deg
 sameas   1E h Htot       W
 NaK-78     Gas type
!--------------------------------  3 --------------------------------
 DUCT       The sodium-potassium column
 sameas   1a a Area      m^2              1069.7     A |p|       Pa
 sameas   1b b Perim      m                 -0.1210  B Ph(p)     deg
     1.0000  c Length     m               3.7365E-13 C |U|     m^3/s
                                             -0.1156  D Ph(U)     deg
                                              0.0000  E Htot      W
 ideal      Solid type                    1.9984E-10 F Edot      W
!--------------------------------  4 --------------------------------
 SURFACE    Bottom end
 sameas   1a a Area      m^2              1069.7     A |p|       Pa
                                             -0.1210  B Ph(p)     deg
                                          3.5753E-17 C |U|     m^3/s
                                             83.367   D Ph(U)     deg
                                              0.0000  E Htot      W
```

```
 ideal      Solid type                    2.1686E-15 F Edot      W
!---------------------------------  5 ---------------------------------
 HARDEND   terminate in the liquid metal
     0.0000  a R(1/z)          =  5G?     1069.7     A |p|       Pa
     0.0000  b I(1/z)          =  5H?       -0.1210  B Ph(p)     deg
                                          3.5753E-17 C |U|     m^3/s
                                            83.367   D Ph(U)     deg
                                             0.0000  E Htot      W
                                          2.1686E-15 F Edot      W
                                          6.3787E-11 G R(1/z)
                                          5.5883E-10 H I(1/z)
```

Calculations:

`BEGIN` establishes initial values of p_m, f, T_m, p_1, U_1, $\dot{H}_{2,k}$, $\dot{N}$, and (if applicable) n_L and $\dot{N}_L$. It establishes the gas type. Unless subsequently overridden by a `ANCHOR` segment, it establishes "insulate" mode, effectively putting thermal insulation around all subsequent segments except heat exchangers.

If `Htot` is omitted from the <`.out`> file, or if the "Typical Htot" box is checked in the user interface, `BEGIN` sets an initial value of

$$\dot{H}_{\text{tot}} = \dot{E} + \dot{N} m w_m, \tag{11.1}$$

which implies that $\dot{H}_{2,k} = \dot{E}$. Otherwise, `Htot` is an ordinary input parameter, which can be set by the user or guessed.

If the "Ignore steady flow" box is checked or the `Ndot` input parameter is missing from the <`.out`> file, DELTAEC assigns $\dot{N} = 0$, skips all subsequent $p_{2,0,HL}$ calculations, and ignores any possible effect of nonzero $\dot{N}_{2,0}$ on dp_1/dx. If the "Enable Ndot" box is checked, $p_{2,0,HL}$ is initialized to zero and its dependence on x is calculated in all subsequent segments. This also allows input parameter $\dot{N}$ to appear in `BEGIN`, and allows other input and output parameters that are relevant to steady flow to appear in subsequent segments. When this box is checked, nonnegligible $dp_{2,0,HL}/dx$ and nonlinear effects on dp_1/dx can occur in subsequent segments even if $\dot{N} \simeq 0$, because of the difference between $\dot{N}$ and $\dot{N}_{2,0}$ described in Eq. (8.16).

If the gas type is a binary mixture, the initial mole fraction of the light component, n_L, appears as an input parameter, which can only change in `MIX**` segments downstream.

Other, less-commonly-used variables are invisibly initialized in `BEGIN`:

`vol` $= 0$, described in Section 11.4.1.

$F_1 = 0$, described in Section 11.4.1.

`Tzero` $= 300$ Kelvin, described above Eq. (6.3) in the context of $\dot{H}$ and below Eq. (8.9) in the context of $\dot{X}$.

`nLzero` $= 0.5$, described below Eq. (8.9) in the context of $\dot{X}$.

$\dot{N}_L = n_L \dot{N}$, if the gas is a binary gas mixture and the "Bulk NLdot" box is checked.

$T_{\text{solid}} = T_m$.

11.1.3. `HARDEND` and `SOFTEND`

General description:

Often, the final segment (except possibly `RPN` segments) will be either `HARDEND` or `SOFTEND`. These contain three or four potential targets.

HARDEND targets are appropriate when the user wants the complex volume flow rate somewhere to be zero. This is the usual case at the end of a closed thermoacoustic system.

SOFTEND targets are appropriate when the user wants the complex pressure amplitude somewhere to be zero. This is often useful for symmetrical systems, where SOFTEND indicates that the remainder of the apparatus is a mirror image of what is in the <.out> file and forces a complex-pressure node. Also use the SOFTEND segment, but without using its targets, to signal the end of a TBRANCHed sequence of segments that will be reattached elsewhere at a UNION. The third default target in these segments, $\dot{H}_{\text{tot}}$, can be targeted to zero to represent thermal insulation at a HARDEND or the energy aspect of a mirror-image symmetry location in SOFTEND.

In both **ENDs, the complex specific impedances are made dimensionless by dividing by $\rho_m a$ evaluated at the local temperature.

Disabling the targets in **ENDs is often useful in debugging a new model that doesn't readily converge. Set these targets nonzero to model a known, nonzero end impedance—or use BRANCH (followed by a zero-targeted HARDEND) to work with Z instead of $z/\rho a$.

Input variables:

None.

Potential targets, HARDEND:

R(1/z) If targeted, this number is compared with DeltaEC's calculation of the real part of the inverse of the normalized specific impedance, $\text{Re}[1/z_n]$, at the HARDEND.

I(1/z) If targeted, this number is compared with DeltaEC's calculation of the imaginary part of the inverse of the normalized specific impedance, $\text{Im}[1/z_n]$, at the HARDEND.

Htot (W) Total energy flow $\dot{H}_{\text{tot}}$. This potential target appears in insulated mode. If targeted, this number is compared with DeltaEC's calculation of $\dot{H}_{\text{tot}}$ at the HARDEND.

Ndot (mol/s) Steady molar flow rate $\dot{N}$. This potential target appears when "Enable Ndot" is checked in an upstream BEGIN. If targeted, this number is compared with DeltaEC's calculation of $\dot{N}$ at the HARDEND.

Potential targets, SOFTEND:

R(z) If targeted, this number is compared with DeltaEC's calculation of the real part of the normalized specific impedance, $\text{Re}[z_n]$, at the SOFTEND.

I(z) If targeted, this number is compared with DeltaEC's calculation of the imaginary part of the normalized specific impedance, $\text{Im}[z_n]$, at the SOFTEND.

Htot (W) Total energy flow $\dot{H}_{\text{tot}}$. This potential target appears in insulated mode. If targeted, this number is compared with DeltaEC's calculation of $\dot{H}_{\text{tot}}$ at the SOFTEND.

Master–slave links:

None.

Format and examples:

```
HARDEND   looks like this if steady-flow calculations are turned off
0.000     R(1/z)
0.000     I(1/z)
0.000     Htot

SOFTEND
0.000  R(z)
0.000  I(z)
```

Calculations:

The normalized impedance in `HARDEND` and `SOFTEND` is calculated according to

$$z_n = \frac{Ap_1}{\rho_m a U_1}, \tag{11.2}$$

with A from the most recent segment having an area (or $A = 10^{-6}$ m^2 if no such segment exists), and with ρ_m and a evaluated at the local T_m.

11.2. Structured branches and unions

Figure 11.1 shows how `TBRANCH` and `UNION` can be used to model complicated branched and multiply connected thermoacoustic equipment.

11.2.1. `TBRANCH`

General description:

Use `TBRANCH` for branched systems that are too complicated for `BRANCH` or `OPNBRANCH`.

When it encounters a `TBRANCH`, DeltaEC treats subsequent segments as the sequential members of a side branch (which we call a "teebranch" in the Users Guide), until it reaches a `HARDEND` or `SOFTEND`. Then it "returns to the trunk," treating the rest of the segments as trunk members. The `HARDEND` termination of a teebranch is appropriate for a closed-end leg. The `SOFTEND` termination of a teebranch is appropriate for reconnection to the trunk at a `UNION`.

To manage the plotter display of results from models with one or more `TBRANCH`es, the <`.sp`> file's initial column, "leg number," is incremented each time a `TBRANCH` or `BEGIN` is encountered, and decremented when a `SOFTEND` or `HARDEND` is encountered. The user can display plots of one or more selected legs, suppressing results from uninteresting legs.

The complex impedance in the `TBRANCH` segment tells DeltaEC how to split up the volume flow rate between the teebranch and the trunk. Usually this complex impedance is used as two guesses to hit two downstream targets, often either complex $1/z_n$ in a `HARDEND` at the end of a dead-ended teebranch or complex p_1 at a subsequent `UNION` for a multiply

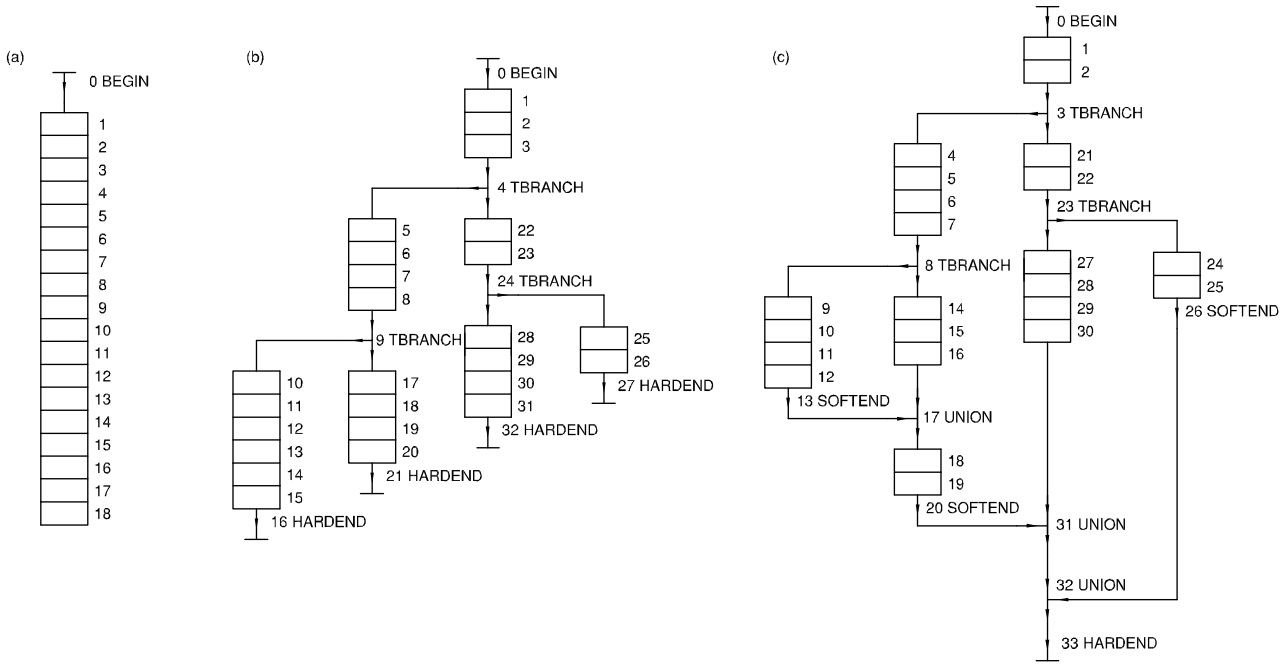


Figure 11.1: (a) Example of an 18-segment DeltaEC model of a linear array of thermoacoustic components. (b) Example of a model of a multiply branched apparatus. (c) Example of a model of a multiply connected apparatus. Line `a` of `UNION` 17 must point to `SOFTEND` 13, line `a` of `UNION` 31 must point to `SOFTEND` 20, and line `a` of `UNION` 32 must point to `SOFTEND` 26.

connected teebranch. Unless `ANCHOR` appears upstream, `HtotBr` tells DeltaEC how to split up the total power $\dot{H}_{\rm tot}$ between the branch and the trunk. This is often used as a guess to hit a downstream temperature target, such as at a `UNION`. The parameter `NdotBr` tells DeltaEC how to split any nonzero time-averaged molar flow $\dot{N}$ between the branch and the trunk. Similarly, `NLdotB` specifies the portion of the time-averaged mole flux of the lighter component of a mixture, $\dot{N}_L$, that goes into the branch.

For duct networks in which temperature is constant and p_1 and U_1 are the only integration variables of interest, use `ANCHOR` after the `BEGIN` segment, so that `TBRANCH` will not worry about how to split $\dot{H}_{\rm tot}$ and `UNION` will not think of temperature as a potential default target.

Nested teebranches are allowed, as shown in Figs. 11.1(b) and (c). However, never use a `BEGIN` segment within a teebranch to re-initialize variables.

The incoming values of the `RPN` variables `F1` and `vol` are retained for the trunk, and these variables are initialized to zero for the teebranch.

Input variables:

Re(Zb) (Pa·s/m^3) The real part of the branch impedance $Z_{\rm br}$.

Im(Zb) (Pa·s/m^3) The imaginary part of the branch impedance $Z_{\rm br}$.

HtotBr (W) This optional input variable, $\dot{H}_{\rm br}$, can be used in insulated mode. It specifies

how much total power flows into the teebranch. The default value is simply equal to $\dot{E}_{\rm br}$.

NdotBr (mol/s) This input variable, $\dot{N}_{\rm br}$, appears when "Enable Ndot" is checked in an upstream `BEGIN`. It specifies how much steady flow goes into the teebranch.

NLdotB (mol/s) This input variable, $\dot{N}_{L,\rm br}$, appears when "Enable Ndot" is checked in an upstream `BEGIN` and when a gas mixture and `MIX**` segments are in use. It specifies how much steady flow of the light component of the mixture goes into the teebranch. The default value represents bulk flow into the branch, $\dot{N}_{L,\rm br} = n_L \dot{N}_{\rm br}$.

Potential targets:

None.

Master–slave links:

The imaginary part of the branching impedance can be linked to the real part in two possible ways: to preserve either the magnitude or the phase while the real part is changed.

(For an introduction to master–slave links, and a complete listing of these link options for all segments, see Section 2.7.2.)

Format and examples:

```
TBRANCH      the fork in the road, in the default "insulated" mode
 4.412E+07 a Re(Zb) Pa-s/m^3  G
-3.528E+06 b Im(Zb) Pa-s/m^3  G
 0.49      c  HtotBr          G

TBRANCH      the fork in the road, if "ANCHOR" segment appears somewhere upstream
 4.412E+07 a Re(Zb) Pa-s/m^3  G
-3.528E+06 b Im(Zb) Pa-s/m^3  G
```

Calculations:

The `TBRANCH` segment leaves T_m, p_1, and p_{20HL} unchanged. Other variables change according to

$$U_{1,\rm br} = p_1/Z_{\rm br}, \tag{11.3}$$
$$U_{1,\rm trunk} = U_{1,\rm in} - U_{1,\rm br}, \tag{11.4}$$
$$\dot{H}_{\rm br} = \dot{H}_{\rm br}, \tag{11.5}$$
$$\dot{H}_{\rm trunk} = \dot{H}_{\rm in} - \dot{H}_{\rm br}, \tag{11.6}$$
$$\dot{N}_{\rm br} = \dot{N}_{\rm br}, \tag{11.7}$$
$$\dot{N}_{\rm trunk} = \dot{N}_{\rm in} - \dot{N}_{\rm br}, \tag{11.8}$$
$$\dot{N}_{L,\rm br} = \dot{N}_{L,\rm br}, \tag{11.9}$$
$$\dot{N}_{L,\rm trunk} = \dot{N}_{L,\rm in} - \dot{N}_{L,\rm br}, \tag{11.10}$$

$$F_{1,\mathrm{br}} = 0, \tag{11.11}$$
$$F_{1,\mathrm{trunk}} = F_{1,\mathrm{in}}, \tag{11.12}$$
$$\mathrm{vol}_{\mathrm{br}} = 0, \tag{11.13}$$
$$\mathrm{vol}_{\mathrm{trunk}} = \mathrm{vol}_{\mathrm{in}}. \tag{11.14}$$

`TBRANCH` has an extra output, `EdotTr`, to display the acoustic power flowing past the branch in the trunk.

11.2.2. `UNION`

General description:

A `UNION` is used in concert with a `TBRANCH` and a `SOFTEND` somewhere upstream, to create loops of segments as illustrated in Fig. 11.1(c). In this arrangement, when it encounters a `TBRANCH`, DeltaEC treats subsequent segments as the sequential members of a branch until it reaches a `SOFTEND`; then it "returns to the trunk," treating the rest of the segments as trunk members. Subsequently, a `UNION` segment in the trunk tells DeltaEC where to connect the teebranch's `SOFTEND` back to the trunk.

When reconnected with a `UNION`, the teebranch's `SOFTEND` impedance targets should not be used; instead, enable the `UNION`'s targets to ensure that p_1 (complex) at the `UNION` is equal to p_1 (complex) at the `SOFTEND` of the teebranch. Often, guessing the teebranch impedance determines how the (complex) volume flow rate splits up at the `TBRANCH` and allows the complex p_1 target to be met.

In all but the simplest acoustic duct network, additional potential targets can be used at a `UNION` to properly tie the teebranch's `SOFTEND` to the trunk at the `UNION` with continuity of relevant variables. These include T_m, $p_{2,0,HL}$, and n_L.

`UNION` targets are unusual, because their target values are dynamically rewritten by DeltaEC during each iteration, so they always reflect the current result at the referenced `SOFTEND`. (When the user enables a `UNION`'s target, the target input parameters, e.g., magnitude and phase of pressure, can be initialized to any values, because DeltaEC will overwrite them during each integration pass with the current magnitude and phase of pressure at the referenced `SOFTEND`.)

Oscillating volume flow rates are added at the `UNION`, as are time-averaged energy flows and mole fluxes. The `RPN` variables `F1` and `vol` are also added.

When `TBRANCH` and `UNION` are used for duct networks, where temperature is constant and hence p_1 and U_1 are the only variables of interest, an `ANCHOR` segment upstream of the `TBRANCH` will eliminate confusion that might arise from the display of uninteresting, irrelevant variables.

Input variables:

SegNum The segment number of a `SOFTEND`, upstream of the `UNION`, which terminates a teebranch and which should be connected to the trunk here. See Fig. 11.1(c).

Potential targets:

|p|Sft (Pa) If used as a target, DELTAEC's shooting method compares its calculation of $|p_1|$ here in the trunk with its calculation of $|p_1|$ at the `SOFTEND` identified by `SegNum`.

Ph(p)S (deg) If used as a target, DELTAEC's shooting method compares its calculation of the phase of p_1 here in the trunk with its calculation of the phase of p_1 at the `SOFTEND` identified by `SegNum`.

TSoft (K) This potential target appears in insulated mode. If used as a target, DELTAEC's shooting method compares its calculation of T_m here in the trunk with its calculation of T_m at the `SOFTEND` identified by `SegNum`.

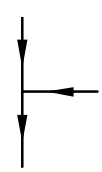
p20HLS (Pa) This potential target appears when "Enable Ndot" is checked in an upstream `BEGIN`. If used as a target, DELTAEC's shooting method compares its calculation of $p_{2,0,HL}$ here in the trunk with its calculation of $p_{2,0,HL}$ at the `SOFTEND` identified by `SegNum`.

nLSoft This potential target appears when a gas mixture is in use and any `MIX**` segment appears in the model. If used as a target, DELTAEC's shooting method compares its calculation of n_L here in the trunk with its calculation of n_L at the `SOFTEND` identified by `SegNum`.

Master–slave links:

None.

Format and examples:



```
!--------------------------------- 13 ---------------------------------
 UNION      reconnect the "final" end of the loop, segment 8, here
     8.000   a SegNum                    2.5420E+05 A |p|       Pa
  2.5420E+05 b |p|Sft    Pa    = 13A?       1.9444  B Ph(p)     deg
     1.9444  c Ph(p)S    deg   = 13B?     3.0515E-02 C |U|     m^3/s
   781.66    d TSoft     K       (t)      -44.624   D Ph(U)     deg
  1603.6     e p20HLS    Pa    = 13H?     2674.7     E Htot      W
                                          2666.4     F Edot      W
                                           702.66    G T         K
                                          1603.6     H p20HL     Pa

UNION    connect two paths together, matching complex pressure without worrying about any other variables
   4.000   a segment number of SOFTEND of the TBRANCH somewhere upstream
    3.e4   b |p|Sft  Pa
   1.234   c ph(p)S  deg
```

Calculations:

Calculations at `UNION` are simple arithmetic:

$$T_{m,\text{out}} = T_{m,\text{in}}, \tag{11.15}$$
$$p_{1,\text{out}} = p_{1,\text{in}}, \tag{11.16}$$
$$U_{1,\text{out}} = U_{1,\text{in}} + U_{1,\text{end}}, \tag{11.17}$$
$$\dot{H}_{\text{tot,out}} = \dot{H}_{\text{tot,in}} + \dot{H}_{\text{tot,end}}, \tag{11.18}$$
$$\dot{N}_{\text{out}} = \dot{N}_{\text{in}} + \dot{N}_{\text{end}}, \tag{11.19}$$
$$\dot{N}_{L,\text{out}} = \dot{N}_{L,\text{in}} + \dot{N}_{L,\text{end}}, \tag{11.20}$$
$$F_{1,\text{out}} = F_{1,\text{in}} + F_{1,\text{end}}, \tag{11.21}$$
$$\text{vol}_{\text{out}} = \text{vol}_{\text{in}} + \text{vol}_{\text{end}}, \tag{11.22}$$

where the subscript "in" refers to the segment immediately preceding the `UNION` and the subscript "end" refers to the `SOFTEND` that is identified by `SegNum` in the first input parameter of the `UNION`, i.e., the `SOFTEND` that terminates the teebranch that is being joined to the trunk at the `UNION`. Note that Eq. (11.18) implies $\dot{H}_{2,k,\text{out}} = \dot{H}_{2,k,\text{in}} + \dot{H}_{2,k,\text{end}}$ only if T_m is successfully targeted.

In thermally anchored mode, Eq. (11.18) is replaced by $\dot{H}_{\text{tot,out}} = \dot{E}$.

11.3. Insulation

11.3.1. `ANCHOR` and `INSULATE`

General description:

By default, DeltaEC assumes that the only places total power can flow into or out of a model are

- heat exchangers, where heat can be added to the total power (or, if heat is negative, subtracted from the total power),
- electroacoustic transducers, where electrical power can be added to or subtracted from the total power,
- `BRANCH`, `OPNBRANCH`, and `PISTBRANCH`,
- `BEGIN`, unless $\dot{H}_{\text{tot}}$ is explicitly set equal to zero (or is set equal to $\dot{E} + mw_m\dot{N}$ which in turn happens to equal zero),
- `SOFTEND` and `HARDEND`, unless $\dot{H}_{\text{tot}}$ is targeted to zero.

Other segments—stacks and regenerators, ducts and cones, and compliances, surfaces, and impedances—are assumed to have insulation wrapped around their side-wall boundaries by default, so whatever total power $\dot{H}_{\text{tot}}$ flows into them from the previous segment must flow out of them to the subsequent segment.

However, sometimes it is useful to change the behavior of ducts, cones, compliances, surfaces, and impedances so that the total power flowing out of them is equal to the acoustic power out of them. In a series of such segments, acoustic power dissipation in each segment "disappears" from the model, as it might if the segment were thermally anchored to a heat sink at the same temperature as the local gas temperature T_m. This behavior can be started by an `ANCHOR` segment before the segments that are supposed to be thermally anchored. Thereafter, such segments obey $\dot{H}_{2,k,\mathrm{out}} = \dot{E}_{\mathrm{out}}$. The complementary `INSULATE` segment, or a `BEGIN` segment, can be used to return the computation to the default, thermally insulated mode in which $\dot{H}_{2,k,\mathrm{out}} = \dot{H}_{2,k,\mathrm{in}}$. `ANCHOR` is most useful for very low power systems and simple systems where total energy flow is irrelevant, and of course for components that actually are immersed in a heat sink.

Stacks and regenerators remain insulated when `ANCHOR` is in effect.

Some users who are interested only in acoustics, not thermoacoustics, habitually put `ANCHOR` immediately after `BEGIN`, to reduce irrelevant clutter.

Use the default, insulated mode for ducts, compliances, or impedances that come between stacks and heat exchangers, and for modeling insulated acoustic networks attached to thermoacoustic devices.

Format and examples:

```
ANCHOR   this segment has no inputs or outputs
INSULATE  this segment also has no inputs or outputs
```

11.4. Math segments

Use this class of segments to access or display variables that DeltaEC would not otherwise display, to perform simple arithmetic operations, and to create targets other than DeltaEC default targets. See Chapter 4 for many examples.

11.4.1. `RPN`

General description:

Results of the `RPN` segment are computed by interpreting the formula (line "`b`") in Reverse Polish Notation, as described in [16, 17] and especially in all of Chapter 4. The formula is a sequence of numbers, addresses, internal DeltaEC variables, and algebraic operators. Variables such as `Htot` can be typed as lower case; DeltaEC will automatically convert to its preferred case. The formula in an `RPN` must not be longer than 80 characters. (Note that DeltaEC stores the line so that each segment number occupies 3 characters. The practical typed length is therefore a little shorter than 80 characters.) As with RPN calculators, when operators do not consume all numbers on the "stack," more than one output is generated—a feature that can be exploited to display multiple results. The stack, whose members may be complex numbers, grows downward, from `A`–`J`. Valid operators and inputs are summarized in Tables 11.1–11.6. A few of the variables and operations are unusual and are not described

Table 11.1: List of RPN number-entry methods. Each number is piled on top of the stack.

Item	Description	Comment or example
<Constant>	Real or complex number	8.3; 6.02e23; (-0.01, 1.3)
pi	3.14159265	
i	$\sqrt{-1}$	complex
<Input address>	Segment number and l.c. letter	3c
<Output address>	Segment number and cap letter	3F
<filename:addr>	Filename:segment number and letter (linking two or more files together)	bottle:3F
<%:addr>	% = shortcut for previous filename	bottle:3A %:3a * %:3C /
inp	Parameter "a" of this RPN segment	
rcl	Recall from Storage Register	see also sto
#	Put a copy of the stack's top number on top of the stack.	
lstx	Recall what was on top of the stack before the most recent math operation, and put it on top of the stack.	

elsewhere in the Users Guide or in published books or articles; these are discussed in detail below.

Format and examples of RPN:

```
!---------------------6-------------------------
RPN   magU1 over 2 pi freq Area (meters)
!  gas displacement amplitude after duct in segment five
0.00                                      0.0153    A ChngeMe
5C 2 / pi / 0b / 5a /

! -----------------23 ---------------------
RPN efficiency divided by Carnot's efficiency
! electrical power out of alternator is 19G
! heat into engine is 6e
! hot temperature is 6f
! ambient temperature is 8f
1.00                                       0.2438   A ChngeMe
19G 6e / 1 8f 6f / - /

! ----------- 21 -----------------
RPN   B=diam.  A= Reynolds number amplitude here
! 20a is the area in segment 20
0.0000                                    2838.5    A NR1
                                           0.123    B D(m)
20a 4 * pi / sqrt # U1 mag * rho * 20a / mu /


! --------- 128 ------------------
RPN some gas properties here
    0.0000  a G or T                        1.6667  A gamma
                                          519.22    B rho c
                                           0.6625   C Prandtl
 mu cp * k / ; rho a * ; gamma
```

Potential targets in RPN:

Input line a is the potential target to which output line A can be compared. If so used, A should be real.

Table 11.2: `RPN` variable-entry methods. Each number is piled on top of the stack.

Item	Description	Comment or units
Thermophysical properties		
`a`	a; sound speed	m/s
`beta`	β; expansion coefficent	1/K
`cp`	c_p; heat capacity	J/kg·K
`cs`	c_s; solid heat capacity	J/kg·K
`dk`	δ_κ; thermal penetration depth	m
`dn`	δ_ν; viscous penetration depth	m
`ds`	δ_s; solid thermal penetration depth	m
`D12`	D_{12}; binary mass diffusion coefficient	m^2/s, for gas mixtures
`enth`	w_m; enthalpy per unit mass	J/kg
`gamma`	$\gamma = c_p/c_v$	
`ks`	k_s; solid thermal conductivity	W/m·K
`kT`	k_T; thermal diffusion ratio	for gas mixtures
`k`	k; thermal conductivity	W/m·K
`m`	m_{avg}; molar mass (avg. mass of mixture)	kg/mol
`mu`	μ; viscosity	kg/m·s
`mL`	m_L; molar mass of light comp. of mixture	kg/mol
`mH`	m_H; molar mass of heavy comp. of mixture	kg/mol
`nLzero`	$n_{L,0}$; reference mole fraction for $\dot{X}$	
`rho`	ρ_m; density	kg/m^3
`rhos`	ρ_s; solid density	kg/m^3
`Tzero`	T_0; reference temperature for w_m and $\dot{X}$	K
State variables		
`Edot`	$\dot{E}$; acoustic power flow	W
`f`	f; frequency	Hz
`F1`	F_1; cumulative force phasor	N; complex; **see text**
`H2k`	$\dot{H}_{2,k}$; thermoacoustic plus conduction power flow	W
`Htot`	$\dot{H}_{tot}$; total power flow, incl. steady flow	W
`Ndot`	$\dot{N}$; time-averaged molar flow	mol/s
`nL`	molar fraction of light gas in mixture	
`NLdot`	$\dot{N}_L$; time-avg. molar flow, light comp. of mixt.	mol/s
`pm`	p_m; mean pressure	Pa
`p1`	p_1; oscillatory pressure	Pa; complex
`p20HL`	$p_{2,0,HL}$; time-averaged head-loss pressure	Pa
`p20tot`	$p_{2,0,tot}$; time-averaged pressure [Eq. (8.11)]	Pa
`Tm`	T_m; mean temperature	K
`U1`	U_1; oscillatory volume flow rate	m^3/s; complex
`vol`	cumulative gas volume	m^3; **see text**
`w`	ω; radian frequency	rad/s
`x`	integration coordinate	m
`Xdot`	$\dot{X}$; exergy flow	W

Table 11.3: List of `RPN` unary functions. Each function removes a number from the top of the stack, acts on it, and puts the result on top of the stack. These functions accept and return complex numbers, unless otherwise noted. Whether phase angles are returned as radians or degrees is noted.

Item	Description	Comment or explanation
Algebra		
`sqrt`	$x = \sqrt{x}$	
`sqrd`	$x = x * x$	
$\sim$	$x = -x$	change sign
`abs`	$x = \|x\|$	absolute value
`inv`	$x = 1/x$	
`real`	$x =$real(x)	
`imag`	$x =$imag(x)	
`mag`	$x = \|x\|$	magnitude of complex x
`conj`	$x = \tilde{x}$	complex conjugate of x
`arg`		phase of complex x (degrees)
`argr`		phase of complex x (radians)
Trigonometry in degrees		
`sin`; `asin`	$x = \sin(x)$;$x = \sin^{-1}(x)$	These six functions
`cos`; `acos`	$x = \cos(x)$;$x = \cos^{-1}(x)$	require real arguments
`tan`; `atan`	$x = \tan(x)$;$x = \tan^{-1}(x)$	
Trigonometry in radians		
`sinr`	$x = \sin(x)$	
`asinr`	$x = \sin^{-1}(x)$	real argument only
`cosr`	$x = \cos(x)$	
`acosr`	$x = \cos^{-1}(x)$	real argument only
`tanr`	$x = \tan(x)$	
`atanr`	$x = \tan^{-1}(x)$	real argument only
Hyperbolic functions		
`sinh`	$x = \sinh(x)$	
`cosh`	$x = \cosh(x)$	
`tanh`	$x = \tanh(x)$	
Bessel functions		
`J0`	$x = J_0(x)$	Bessel function of zero order
`J1`	$x = J_1(x)$	Bessel function of first order
`Y0`	$x = Y_0(x)$	Neumann function of zero order
`Y1`	$x = Y_1(x)$	Neumann function of first order
Logarithms and exponentials		
`log`	$x = \log_e(x)$	
`exp`	$x = e^x$	
`log10`	$x = \log_{10}(x)$	real argument only
`tenx`	$x = 10^x$	

Table 11.4: Ordinary `RPN` binary operations and functions. Each operation or function removes two numbers from the top of the stack, combines them, and puts the result on top of the stack. In this table, "x" refers to the number on top of the stack, and "y" refers to the number second from the top of the stack.

Item	Description	Comment or example
+	$x = y + x$	
-	$x = y - x$	
*	$x = y * x$	
/	$x = y/x$	
^	$x = y^x$	
`min`; `max`	$x = \min[x, y]$; $x = \max[x, y]$	real arguments only
`cmplx`	$x = x(\cos y + i \sin y)$	y in degrees
`cmplxr`	$x = x(\cos y + i \sin y)$	y in radians
`atan2`	$x = \tan^{-1}(x/y)$; 2-argument, 4-quadrant arctangent	degrees
`atan2r`	$x = \tan^{-1}(x/y)$; 2-argument, 4-quadrant arctangent	radians

Table 11.5: `RPN` actions that have no effect on the RPN stack, but have an effect elsewhere.

Item	Description	Comment
`sto`	$S = x$	Store the number that's on top of the stack in the Storage Register. (See also `rcl`)
`=f`	$f = x$	Set f equal to the number that's on top of the stack.
`=F1`	$F_1 = x$	Set the accumulated-force phasor equal to the number that's on top of the stack.
`=H2k`	$\dot{H}_{2,k} = x$	Set $\dot{H}_{2,k}$ equal to the number that's on top of the stack.
`=Ndot`	$\dot{N} = x$	Set $\dot{N}$ equal to the number that's on top of the stack.
`=nL`	$n_L = x$	Set n_L equal to the number that's on top of the stack.
`=NLdot`	$\dot{N}_L = x$	Set $\dot{N}_L$ equal to the number that's on top of the stack.
`=NLzro`	$n_{L,0} = x$	Set $n_{L,0}$ equal to the number that's on top of the stack.
`=pm`	$p_m = x$	Set p_m equal to the number that's on top of the stack.
`=p1`	$p_1 = x$	Set p_1 phasor equal to the number that's on top of the stack.
`=p20HL`	$p_{2,0,HL} = x$	Set $p_{2,0,HL}$ equal to the number that's on top of the stack.
`=Tm`	$T_m = x$	Set T_m equal to the number that's on top of the stack.
`=Tzero`	$T_0 = x$	Set T_0 equal to the number that's on top of the stack.
`=U1`	$U_1 = x$	Set U_1 phasor equal to the number that's on top of the stack.
`=vol`	`vol` $= x$	Set the accumulated volume equal to the number that's on top of stack.

Table 11.6: RPN actions that affect the top two numbers on the RPN stack.

Item	Description	Comment or example
a<>b	$x = y$; $y = x$	Interchange top two numbers on the stack.

Table 11.7: RPN tokens that have no effect.

Item	Description	Comment or example
;	No effect	Punctuation mark for human readability.

Details of F1 calculation in RPN:

Using the F1 variable in RPN segments, DeltaEC can calculate the net oscillating force exerted by the oscillating gas (and oscillating parts of speakers) on the surrounding pressure vessel, which is assumed to be rigid. As far as we know, the approach is not published anywhere, so the theory behind it is derived in Chapter 14. Here, we describe the implementation in DeltaEC, and we give two examples of its use.

DeltaEC always keeps track of the integral on the right-hand side of

$$F_1 = -i\omega \int (\rho_m u_1)\ dV = -i\omega \int \rho_m(x) U_1(x)\ dx \tag{11.23}$$

during the course of its other integrations. The derivation in Chapter 14 shows that this integral gives the total force exerted by the gas on the pressure vessel. The result is accessible to the user at any location by putting "F1" in line b of an RPN segment, which displays the complex number F_1 and makes it available for further RPN calculations. (As usual, lowercase typing is accepted.)

DeltaEC initializes F1 to zero at BEGIN and at the beginning of the teebranch of every TBRANCH. The trunk's F1 and the referenced-SOFTEND's F1 are added in UNION. Any other segment adds its own F1 contribution [according to Eq. (11.23)] to whatever total F1 has accumulated before it.

These defaults can be overridden via RPN segments that include the change command "=F1". The defaults must often be overridden for thermoacoustic systems with elbows or side branches sticking out at different angles, to resolve the components of the force vector in different directions and to take care of minus signs in toroidal acoustic paths.

For a 90° miter elbow, split the elbow into two segments. Displaying and then rezeroing the force variable between those segments (with "F1" and "=F1", respectively) is a useful approach. For a smoothly radiused 90 degree elbow, strictly speaking we ought to do an integral of a vector velocity around the corner to obtain a vector force. This is beyond DeltaEC's current capability. However to the extent that U_1 does not vary from one end of such an elbow to the other, that integral yields simply $2/\pi$ times DeltaEC's full-elbow DUCT integral for the vertical direction and $2/\pi$ times DeltaEC's full-elbow DUCT integral for the horizontal direction. The user who requires F_1 in such an elbow can use RPN calculations to implement the factor of $2/\pi$, as shown in the second example below.

For IESPEAKER and VESPEAKER segments, DELTAEC evaluates $\int \rho_m U_1 \, dx$ in Eq. (11.23) as simply the speaker's moving mass times its velocity phasor.

Example 1

To find the oscillating velocity of a freely suspended, 14-kg pressure vessel enclosing a half-wavelength resonance driven by a noncompliant transducer at the far end, use this series of segments.

```
!--------------------------------- 0 ---------------------------------
BEGIN no transducer at this end
2.000E+06  a Mean P Pa
173.53     b Freq   Hz
300.00     c TBeg   K
1.0000E+05 d |p|    Pa
0.0000     e Ph(p)  deg
0.0000     f |U|    m/s
0.0000     g Ph(U)  deg
!--------------------------------- 1 ---------------------------------
SURFACE first end
sameas 2a  a Area   m^2
!--------------------------------- 2 ---------------------------------
DUCT
1.0000E-02 a Area   m^2
0.3545     b Perim  m
1.0000     c Length m
!--------------------------------- 3 ---------------------------------
SURFACE noncompliant transducer at this end
sameas 2a  a Area   m^2
!--------------------------------- 4 ---------------------------------
HARDEND
0.0000     a R(1/z) (t)
0.0000     b I(1/z) = 4H?
!--------------------------------- 5 ---------------------------------
RPN [A]= velocity phasor of pressure vessel (m/s). (a)=mass of pressure vessel (kg)
14.000     a G or T (t)           (-8.8680E-05, 0.1309 ) A
 F1 5a / i / w /
!--------------------------------- 6 ---------------------------------
RPN viscous drag force phasor (newtons)
0.0000     a G or T (t)           ( 1.3536 , -1.3536 ) A
 F1 p1 3a * 0d 1a * - -
guessz 0b
targs 4b
```

We arbitrarily defined zero phase to be that of the pressure at segments 0 and 1. Segment 5 shows that the phase of the velocity is almost 90°, which is consistent with the phases of the pressure on the two ends of the half-wavelength resonator at segments 1 and 3. Segment 6 subtracts the normal end-cap forces from the total force to display what's left over, which must be the integrated viscous shear force on the side walls.

Example 2

To find oscillating forces and torques on the half-wavelength refrigerator shown in Fig. 11.2, we use a DELTAEC model like this:

```
   TITLE symmetric speaker-driven refrigerator, example2.out
00 BEGIN in the motor enclosure. Guess temperature and complex p1.
01 INSULATE
02 SURFACE top lid
03 DUCT backside of piston = motor enclosure
04 SURFACE extra surface area for motor parts
05 IESPEAKER motor-driven piston. Guess phase of current.
06 RPN annular surface area around piston
07 SURFACE annular surface area around piston
08 DUCT short duct
09 HX ambient heat exchanger. Guess heat rejected and target temperature.
10 STKSLAB the stack
11 HX cold heat exchanger
12 CONE adapter to small-diam resonator
13 RPN capture F1 here, and reset it to zero
   0.0000 a G or T (t)                    0.0000 A ChngeMe
                            ( 9.2386 , -113.62 ) B ChngeMe
   F1 0 =F1
14 DUCT 90 degree elbow
15 RPN vertical and horizontal forces on elbow; then set F1=0
   0.0000 a G or T (t)                    0.0000 A ChngeMe
                         ( 3.0179E-03, -3.6644 ) B ChngeMe
```

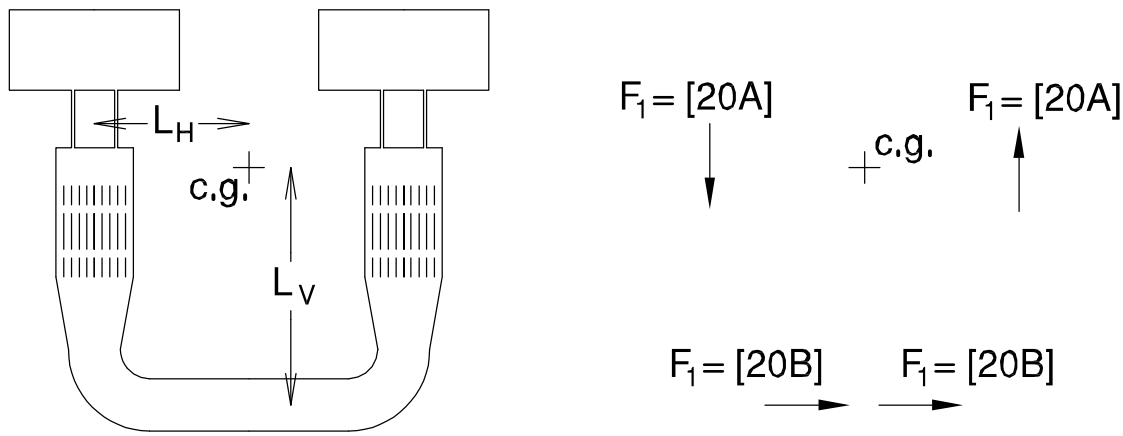


Figure 11.2: Example 2. Left, a sketch of a half-wavelength, two-speaker, standing-wave refrigerator. Right, force diagram, in which the directions of the arrows indicate the "positive" direction of calculated forces. In both parts of the figure, "c.g." indicates the center of gravity.

```
                         ( 3.0179E-03, -3.6644 ) C ChngeMe
   F1 2 * pi / # 0 =F1
16 DUCT horizontal run of pipe, to mirror-image location
17 SOFTEND mirror-image plane. Target zero complex impedance and zero Hdot.
18 RPN for aesthetics, enforce phase(U1)=0 here. Target this.
   0.0000 a G or T = 18A?              -2.5791E-14 A ChngeMe
   U1 arg
19 RPN force due to horizontal segment
   0.0000 a G or T (t)     ( 6.9004E-04, -6.0857 ) A ChngeMe
   F1
20 RPN [A]=net vertical force; [B]=net horizontal force (newtons)
   0.0000 a G or T (t)        ( 9.2416 , -117.29 ) A ChngeMe
                         ( 3.7079E-03, -9.7501 ) B ChngeMe
   15C 19A + 13B 15B +
```

Most of the input and output have been eliminated from the listing for clarity, leaving only the relevant `RPN` segments fully displayed. Segment 13 captures F_1 for the vertical components, segment 15 captures F_1 for the elbow using the approximation that U_1 is uniform in the elbow, and segment 19 captures F_1 for the horizontal part of the resonator. Then segment 20 adds up the previous results to yield the net horizontal and vertical F_1s for this half of the full refrigerator.

Considering the symmetry of the full refrigerator as shown in the figure (only half of which is modeled above with DeltaEC) to get the minus signs and factors of two correct, we see that the total horizontal force exerted on the pressure vessel is $2\times$ [20B], the total vertical force is zero, and the total counter-clockwise torque around the center of gravity "c.g." is $2 \times ([20\text{A}]L_H + [20\text{B}]L_V)$ where L_H and L_V are distances between the center of gravity and the axes of the parts of the refrigerator as shown in the figure. Both the total horizontal force and the torque have time phases of about $-90°$, relative to the time phase of the velocity at the mirror-image plane, which is set to $0°$ with segment 18.

Details of `vol` calculation in `RPN`:

Beginning with version 6, DeltaEC keeps track of the cumulative gas volume in a series of segments while performing its other integrations. The result is accessible to the user at any location by putting "`vol`" in line `b` of an `RPN` segment. (A similar result is accessible

through the VOLUME segment, described in Section 11.4.2, which calculates the volume in a range of segments, including both gas volume and the volume of fins, plates, and struts in STK**s and heat exchangers.)

By default, DeltaEC initializes "vol" to zero at BEGIN and at the beginning of a TBRANCH's teebranch, and adds the branch "vol" and trunk "vol" at UNIONs. Any other segment adds its own contribution to whatever total "vol" has accumulated before it. These defaults can be overridden via RPN segments that include the change command "=vol".

Example

```
   TITLE symmetric speaker-driven refrigerator, example2v.out
00 BEGIN in the motor enclosure. Guess temperature and complex p1.
01 INSULATE
02 SURFACE top lid
03 DUCT backside of piston = motor enclosure
04 SURFACE extra surface area for motor parts
05 IESPEAKER motor-driven piston. Guess phase of current.
06 SURFACE annular surface area around piston
07 DUCT short duct
08 HX ambient heat exchanger. Guess heat rejected and target temperature.
09 STKSLAB the stack
10 HX cold heat exchanger
11 CONE adapter to small-diam resonator
12 DUCT 90 degree elbow
13 DUCT horizontal run of pipe, to mirror-image location
14 SOFTEND mirror-image plane. Target zero complex impedance and zero Hdot.
15 RPN for aesthetics, enforce phase(U1)=0 here. Target this.
16 RPN volume of gas (cubic meters) in segments 01 through 14
   0.0000 a G or T (t)                      0.6587 A ChngeMe
   vol
```

The vol operator in segment 16 causes the cumulative volume to be displayed: The total volume of gas in the DUCTs, HXs, STKSLAB, and CONE. (The other segments in this example have no gas volume.)

11.4.2. VOLUME

General description:

Result A of a VOLUME segment is the sum of the volumes in all duct, cone, stack, compliance, and heat exchanger segments beginning with BegAddr and ending with EndAddr (parameter letters are inconsequential). Porosity is ignored in calculating this volume, so the result is the volume of the gas plus the volume of internal fins, plates, etc. in heat exchangers and stacks. (In contrast, the vol variable, accessible in RPN segments, gives the cumulative gas volume, not including internal fins and plates.) This segment is intended to give an indication of the overall size of an apparatus for doing trade-off analysis during design.

Input variables:

BegSeg The first of the segments whose volumes are to be summed.

EndSeg The last of the segments whose volumes are to be summed.

Potential targets:

Line "a" is the target at which VOLUME output line "A" can be aimed.

Master–slave links:

None.

Format and examples:

```
VOLUME
0.50    a targeted volume (cubic meters)
1       b BegSeg
10      c EndSeg
```

11.4.3. CONSTANTS

General description:

The CONSTANTS segment allows the user to input a dozen real numbers in one segment. This is useful when a subsequent RPN segment depends on a large number of nonstandard parameters that the user must control or that DeltaEC must guess. (Without CONSTANTS, a dozen RPN segments would be required.) This segment performs no calculations and has no potential targets.

Format and examples:

```
CONSTANTS      test of CONSTANTS
1.55     a
2.00     b
3.22     c
4.00     d
5.00     e
6.00     f
7.00     g
8.00     h
9.0E6    i
-0.048   j
11.01    k
12.02    l
```

11.5. Interfaces to external programs and files

11.5.1. BLKDATA

General description:

BLKDATA is a specialized segment designed for users who have a quantity of tabular data that they want DeltaEC to follow. The data may represent experimental results, the results from other computational methods, or simply an irregular parameter space to be explored that is not well served by DeltaEC's incremental-plot capability (e.g., irregular step size or parametric plotting). The data file is in text format, delimited with spaces or tabs, and it may contain up to 14 columns and any number of lines, all with the same number of entries. The first one or two lines can optionally contain descriptive strings that are read into the parameters' description and unit fields, respectively. Only one BLKDATA segment per model is allowed. The data file must have the suffix <.blk> and its root name is supplied in the heading of the BLKDATA segment. When the file is opened, the number of lines and columns are counted, and initial column headings are read in, if present. The number of input parameters for the segment is equal to the number of columns in the <.blk> file.

During a run, each numeric line of the <.blk> file is read sequentially, and DeltaEC runs the model once for each line. Each line of the file is inserted into the input parameters of the BLKDATA segment, one line per data point. The use of sameas provides a direct means to tie the values of DeltaEC *input* parameters to the tabulated data.

Enforcing values that normally appear as DeltaEC *outputs* is a little more subtle. This requires an RPN target to be sameas'ed to an input parameter in the BLKDATA segment, so that the RPN's calculated output value can be forced to hit the targeted value. This target must be used, of course, and an appropriately related input parameter must be added to the guess list. Segment 5 in the example below illustrates such a target.

Input variables:

The heading field, which is to the right of "BLKDATA," must contain the root name of the <.blk> file. For example, if the user's data is in <mydata.blk>, the BLKDATA segment looks like the example below.

Potential targets; Master–slave links:

None.

Format and examples:

If the BLKDATA segment looks like this:

```
 BLKDATA    mydata
```

and the file <mydata.blk> looks like this:

```
   Freq        |p|@0       SkrAmp
     Hz          Pa          A
    511.4       6653.       0.031
    511.0      14401.       0.076
    511.9       9872.       0.055
```

then DeltaEC will immediately transform the BLKDATA segment to this:

```
!--------------------------------  1 --------------------------------
 BLKDATA    test
   511.40    a Freq   Hz
  6653.0     b |p|@0  Pa
  3.0000E-02 c SkrAmp  A
```

and will read subsequent lines of <mydata.blk> into place in subsequent iterations of the run. If the file <mydata.blk> is a log of experimental data, we can ask DeltaEC to emulate the experimental conditions by inserting appropriate sameas statements, as in the following fragment that matches the experimental applied frequencies, pressure amplitudes, and speaker currents:

```
!--------------------------------  0 --------------------------------
 BEGIN      the setup
  7.8000E+04 a Mean P    Pa
 sameas  1a  b Freq      Hz
   300.00    c TBeg      K
 sameas  1b  d |p|@0     Pa
     0.0000  e Ph(p)0    deg
     0.0000  f |U|@0    m^3/s
     0.0000  g Ph(U)0    deg
 air        Gas type
```

```
 ideal      Solid type
!---------------------------------  1 ---------------------------------
 BLKDATA    test
   511.40    a Freq   Hz
  6653.0     b |p|@0  Pa
  3.0000E-02 c SkrAmp  A
+  2 SURFACE     cavity back surface
+  3 DUCT    back duct
!---------------------------------  4 ---------------------------------
 VESPEAKER  ...using an IESPEAKER would be simpler,
 !             but would not illustrate BLKDATA plus target.
  1.5000E-04 a Area      m^2               68.286   A |p|    Hz
     4.6000  b   R      ohms              -91.445   B Ph(p)     deg
  5.5000E-05 c L         H               5.2965E-06 C |U|     m^3/s
     3.3000  d B x L    T-m               -90.455   D Ph(U)     deg
  4.0000E-04 e   M      kg               1.8081E-04 E Hdot      W
  8560.0     f   K      N/m              1.8081E-04 F Edot      W
     0.8000  g   Rm   N-s/m              1.8597E-03 G EdotIn    W       P
     0.1945  h  |V|     V                   0.1945  H Volts     V
   243.00    i Ph(V)   deg               2.2615E-02 I Amps      V
                                           32.263   J Ph(Ze)   deg
 sameas  0  Gas type                      121.99    K  |Px|    Pa
 ideal      Solid type                   -145.97    L Ph(Px)   deg
!---------------------------------  5 ---------------------------------
 RPN   use to match speaker current output to measured value
 sameas  1c  a                = 5A?      2.2615E-02 A
 4I
```

11.5.2. SYSEXEC

General description:

The SYSEXEC segment is not yet working in DeltaEC, as of version 6.2. We retain the documentation here, hoping that future versions will have this feature.

The SYSEXEC segment allows DeltaEC to send data to an external program via the computer's file-handling system, execute that external program, and bring any results of that external program back into DeltaEC. The use of sameas allows any input or output numbers in DeltaEC to be brought into the SYSEXEC segment's input parameters so they can be sent to the external program, and subsequently allows any DeltaEC input parameters to be set equal to results of the external program.

The first action taken by SYSEXEC is to write all of its 14 real input parameters to a text file called <sysin.dat> in single-column format. SYSEXEC then calls an external program using the entire command string specified in the heading field of the segment. The external program may or may not make use of the numbers in sysin.dat. If the external program's results need to be brought back in to DeltaEC, the external program should generate those results in a file named <sysout.dat> in the same ASCII, single-column format. (If more than 14 values are required, the leftovers can be read in through a BLKDATA segment.) When the external program has finished running, control returns to DeltaEC, which will then read up to 14 parameters from the file <sysout.dat> if it exists, put those numbers in the output list of the SYSEXEC segment, and resume its numerical integrations.

Format and examples:

The following is a trivial example showing how a set of integers assigned to SYSEXEC input parameters can be sorted by the MS-DOS sort command. The SYSEXEC segment is initially prepared like this:

```
!-----------------------------------1---------------------------------
RPN  a silly example
87.0     a                                        13.000 A
```

```
5  8  +
!---------------------------------  2 ---------------------------------
SYSEXEC    sort sysin.dat > sysout.dat
     3.0000  a In  1
     4.0000  b In  2
     6.0000  c In  3
     7.0000  d In  4
     2.0000  e In  5
 sameas 1a   f In  6
     2.0000  g In  7
     1.0000  h In  8
    10.000   i In  9
 sameas 1A   j In 10
     5.0000  k In 11
     4.0000  l In 12
    11.000   m In 13
     9.0000  n In 14
```

When the `SYSEXEC` segment is encountered by DELTAEC during a run, DELTAEC creates the file <`sysin.dat`> containing the parameters `a` through `n`, like this:

```
     3.0000
     4.0000
     6.0000
     7.0000
     2.0000
    87.000
     2.0000
     1.0000
    10.000
    13.000
     5.0000
     4.0000
    11.000
     9.0000
```

and passes the segment's heading field, `sort sysin.dat > sysout.dat`, to the operating system as a command string. Next, the operating system pipes the newly created file <`sysin.dat`> to the `sort` function. The MS-DOS `sort` command sorts the numbers into ascending order. The results of the sort are piped into the <`sysout.dat`> file using the 'greater than' character, i.e., the '>' character, in the command string. Finally, the `SYSEXEC` segment reads the <`sysout.dat`> file and puts the results in the output column of the segment, so the segment looks like this:

```
! Sort input parameters into output parameters (DOS version)
!----------------------------------1---------------------------------
RPN  a silly example
87.0     a                                          13.000 A
5  8  +
!---------------------------------  2 ---------------------------------
 SYSEXEC    sort sysin.dat > sysout.dat
     3.0000  a In  1                        1.0000  A Out  1
     4.0000  b In  2                        2.0000  B Out  2
     6.0000  c In  3                        2.0000  C Out  3
     7.0000  d In  4                        3.0000  D Out  4
     2.0000  e In  5                        4.0000  E Out  5
sameas 1a   f In  6                        4.0000  F Out  6
     2.0000  g In  7                        5.0000  G Out  7
     1.0000  h In  8                        6.0000  H Out  8
    10.000   i In  9                        7.0000  I Out  9
sameas 1A   j In 10                        9.0000  J Out 10
     5.0000  k In 11                       10.000   K Out 11
     4.0000  l In 12                       11.000   L Out 12
    11.000   m In 13                       13.000   M Out 13
     9.0000  n In 14                       87.000   N Out 14
```

Now an `RPN` or `sameas` in a subsequent segment that points to output `2N` will access the largest number in the original list, which is 87.000 in this example.

12. Gases (and liquids)

We provide an artificial temperature floor of 10 Kelvin to prevent DeltaEC from trying to use negative temperatures when it is really lost. Consequently, no temperature below 10 Kelvin can be used. (Most of the equations for the gases are very inaccurate when this limit is reached anyway.) This floor can be modified within Options in the Edit pulldown menu.

In what follows, `ta` is temperature in Kelvin, `t1` is temperature in Celsius.

DeltaEC looks for a 10-character field to determine gas type. If you are modifying a model with a text editor, be sure to use plenty of trailing spaces after short gas names like "air" to get character strings like "gas-type" out of the field.

A "fluid" is a gas or a liquid. Gases are the most commonly used fluids in thermoacoustics and DeltaEC. However, liquid sodium and eutectic liquid sodium-potassium are included in DeltaEC because they are thermodynamically powerful, with non-negligible thermal expansion coefficients and non-negligible $(\partial s/\partial p)_T$, when close enough to their critical points. For further details on the use of these liquids in thermoacoustics, see Ref. [13]. Perhaps we should refer to the "gases" used in DeltaEC as "fluids," but for simplicity we always call them gases despite the fact that some liquids are included.

Equations used here accurately cover ranges of T and p that have been of interest to the Los Alamos thermoacoustics team. No warning is issued by DeltaEC if it goes outside the accurate range. It is up to the user to study the equations here, or examine the results of these equations via the properties tokens in an `RPN` segment or the thermophysical-properties dialog, and decide if these equations are accurate enough or if a user-defined gas is called for.

12.1. Helium (`helium`)

Ideal gas approximation for equation of state (including sound speed and expansion coefficient) and specific heat. Transport from Ref. [67].

```
k0=0.0025672*ta**0.716
mu=0.412e-6*ta**0.68014
```

12.2. Helium–argon mixtures (`HeAr`)

Number in `0j` is helium fraction. Ideal gas approximation for equation of state and specific heat. Transport from Refs. [68, 69].

```
k0he=0.0025672*ta**0.716
amuhe=0.412e-6*ta**0.68014
k0ar=(1.39e-4*ta**0.852-1.5e-8*(ta-300.)*(ta-300.))*(1.+2.e-8*pm)
amuar=(1.77e-7*ta**0.852-25.e-12*(ta-300.)*(ta-300.))*(1.+2.e-8*pm)
k0=x1*k0ar+x2*k0he-(k0ar+k0he)*x1*x2**1.5
```

```
mu=x1*amuar+x2*amuhe+0.2*(amuar+amuhe)*x1*x2
diff12= 7.49d0*(1.d0 - 0.05d0*x2)*(ta/295.d0)**1.66d0/pm
ktherm= 0.38d0 * x2**1.2d0 * x1**0.8d0
```

12.3. Helium–xenon mixtures (HeXe)

Number in `0j` is helium fraction. Ideal gas approximation for equation of state and specific heat. Transport from Refs. [68, 69]. Our fits to the published transport data are only accurate for `frxe` < 0.5 or for `frxe` $= 1.000$.

```
k0he=0.0025672*ta**0.716
amuhe=0.412e-6*ta**0.68014
k0xe=4.75e-5*ta**0.84*(1.+1.e-7*pm)
amuxe=0.187e-6*ta**0.85*(1.+25.e-9*pm)
frxe=1.-fhe
k0=k0he*fhe+k0xe*frxe-2.*(k0he+k0xe)*frxe*fhe*fhe
mu=amuhe*fhe+amuxe*frxe+(amuhe+amuxe)*frxe*fhe*fhe*(0.8+3.7*fhe*fhe*(0.25-frxe))
diff12=5.9d0*(1.d0-0.074d0*litefrac)*(ta/300.d0)**1.72d0/0.96d0/pm
ktherm= 0.4d0 * litefrac**1.3d0 * frxe**0.7d0
```

12.4. Neon (neon)

Ideal gas approximation for equation of state and specific heat. Transport from Ref. [67].

```
k0=0.001149*ta**0.65907
mu=0.735e-6*ta**0.66065
```

12.5. Air (air)

Ideal gas approximation for equation of state and specific heat. Transport from Ref. [56].

```
parameter (tps=110.4,tpa=245.4,tpb=27.6,tp0=300.,tpexp=223.8306)
k0=2.624e-2*(ta/tp0)**1.5*(tp0+tpexp)/(ta+tpa*exp (-tpb/ta))
mu=1.846e-5*(ta/tp0)**1.5*(tp0+tps)/(ta+tps)
```

12.6. Humid air and fog (HumidAir)

The number in `0j` is the water mole fraction in a mixture of air and water. As with other gas mixtures, the water mole fraction can be modified or used as an independent plot variable. However, unlike other gas mixtures, the `MIX**` segments do not act on humid air. We worry that future improvements to understanding will bring dramatic change to this part of DeltaEC. Nevertheless, for the present:

In DeltaEC, we think of humid and wet air as a sort of single gas, having two or three interpenetrating components: dry air, water vapor, and sometimes liquid water.

The molar volume v and molar enthalpy w are DeltaEC's primary dependent variables, as functions of three independent variables: mean pressure p_m, mean temperature T_m, and mole fraction of water $n_{\rm wat}$. To determine whether a given mixture is wet or merely humid, DeltaEC compares $n_{\rm wat}$ with $p_{\rm sat}/p_m$, where $p_{\rm sat}(T_m)$ is the saturated vapor pressure at temperature T_m.

For humid air, DeltaEC uses the ideal-gas equation of state for v and a slightly non-linear temperature dependence for w (due to the T dependence of c_p for water vapor in

the ASHRAE tables). Also in accordance with ASHRAE recommendations, we set μ and k equal to their dry-air values.

In wet air, in accordance with Hiller's measurements [70], the *oscillating* thermodynamics is that of humid air at saturation, while the *mean flow* thermodynamics includes the enthalpy of the liquid when $\dot{N} \neq 0$. In other words, if there is no mean flow, the calculation proceeds exactly as for an ideal-gas mixture and ignores the condensate, but if mean flow is nonzero DeltaEC performs stack integrations with ideal-gas-mixture properties in the momentum and continuity equations but with the heat of condensation/evaporation (plus the small enthalpy of the condensate itself) included in the energy equation as it is integrated to find dT_m/dx. In all cases, we ignore any dynamic effects of oscillating diffusion of the water vapor through the air.

Although the enthalpy of wet air calculated in DeltaEC's gas-properties algorithm includes the latent heat of freezing and melting as the condensate passes through 0°C, we have not yet incorporated this latent heat, of the liquid-to-solid phase transition, into the numerical integrations in `STK**` segments, so integrating through $T_m = 0$°C in wet air is of dubious value. (Anyway, we have not thought of any good reason to use wet air in DeltaEC below 0°C, because we believe the stack would simply plug up with ice.)

The saturated vapor pressure is accurate from −100°C to 370°C; other properties are reasonably accurate from −50°C to 150°C.

12.7. Nitrogen (`nitrogen`)

Ideal gas approximation for equation of state and specific heat. Transport from Ref. [67].

```
k0=0.0003609*ta**0.7512
mu=0.3577e-6*ta**0.6885
```

12.8. Hydrogen (`hydrogen`)

Ideal gas approximation for equation of state and specific heat. Transport from Ref. [67].

```
k0=0.002627*ta**0.744
mu=0.19361e-6*ta**0.6723
```

12.9. Deuterium (`deuterium`)

Ideal gas approximation for equation of state and specific heat. Transport from Ref. [67].

```
k0=0.002795*ta**0.686
mu=0.2726e-6*ta**0.6721
```

12.10. Carbon dioxide (`CO2`)

Ideal gas approximation for equation of state and specific heat. Transport from Ref. [67].

```
k10=2.8646E-5*ta**1.1318
k20=3.692E-5*ta**1.0940
k0=k10+(pm-1.01e6)/(1.01e6)*(k20-k10)
u10=1.4187E-7*ta**.8216
u20=1.5416E-7*ta**.8094
mu=u10+(pm-1.01e6)/(1.01e6)*(u20-u10)
```

12.11. Neon–xenon mixtures (NeXe)

Number in Oj is neon fraction. Ideal gas approximation for equation of state and specific heat. Transport from Refs. [68, 69]. (Thermal conductivity not very accurate for high xenon concentrations.)

```
k0he=0.001149*ta**0.65907
amuhe=0.735e-6*ta**0.66065
k0xe=4.75e-5*ta**0.84*(1.+1.e-7*pm)
amuxe=0.187e-6*ta**0.85*(1.+25.e-9*pm)
frxe=1.-fhe(ns)
k0=k0he*fhe(ns)+k0xe*frxe-1.3*(k0he+k0xe)*frxe*fhe(ns)**2.5
mu=amuhe*fhe(ns)+amuxe*frxe+0.12*(amuhe+amuxe)*frxe*fhe(ns)**4
diff12=2.21d0*(1.d0+0.0326d0*frxe/(1.d0+0.531d0*frxe))*(ta/300.d0)**1.71d0/pm
ktherm= 0.258d0 * litefrac**1.31d0 * frxe**0.69d0
```

12.12. Natural-gas combustion products (NGCbProd)

Natural Gas combustion products with 5% excess air. Use around 1 atm only. Our 1993 notes say "Data supplied by British Gas (private communication from R. J. Tucker to Bill Ward) for typical North-Sea gas. Molar weight is a fit to data in Ref. [71] between 288 K and 4000 K."

```
      gamma=1.4
      cp=gasprop(ta,1392.02d0,39.3769d0,-3.89819d0,-0.0317961d0,
     &    0.0327554d0,-1.44149d-3)
      if (ta.gt.2000.) then
        mass=27.9495-7.81175*dexp(-((ta-4151.85)/1047.42)**2)
      else
        mass=27.84
      endif
      r=8314.
      a=dsqrt(gamma*r*ta/mass)
      rho=pm*mass/(r*ta)
      beta=1./ta
      k0=gasprop(ta,0.0997279d0,0.0125516d0,6.73728d-5,4.22761d-4,
     &    1.43198d-4,1.35508d-5)
      mu=gasprop(ta,50.2973d0,4.68523d0,-0.12061d0,0.0140082d0,
     &    -0.001488951d0,4.97968d-5)*1.d-6
      goto 900
      real*8 function gasprop(ta,a,b,c,d,e,f)
      real*8 z,ta,a,b,c,d,e,f
      z=(ta-1400)/200
      gasprop=a+z*(b+ z*(c + z*(d +z*(e+f*z))))
      return
      end
```

12.13. Liquid sodium (sodium)

Data for sodium from Ref. [72].

```
a0=2578.
at1=-.52
ap=6.1e-7
r0=950.1
rt1=-2.2976e-1
rt2=-1.46e-5
rt3=5.638e-9
c0=1.4361e3
ct1=-5.8024e-1
ct2=4.6208e-4
k0=.918e2-4.9e-2*t1
if(t1.le.500.) then
 e1=.697
 e2=1.235e-5
else
 e1=1.04
 e2=8.51e-6
endif
a=a0+at1*t1
```

```
rho=r0+rt1*t1+rt2*t1**2+rt3*t1**3
beta=(-rt1-2.*rt2*t1-3.*rt3*t1**2)/rho
bt=beta**2-(2.*rt2+6.*rt3*t1)/rho
cp=c0+ct1*t1+ct2*t1**2
rp=1./a/a+ta*beta**2/cp
bp=-beta/(rho*a**2)+2.*at1/(rho*a**3)-beta**2/rho/cp
bp=bp-2.*ta*beta*bt/rho/cp-ta*beta**3/rho/cp
bp=bp+ta*beta**2*(ct1+2.*ct2*t1)/rho/cp/cp
cpp=-ta*(beta**2+bt)/rho
c So far, everything is evaluated at p=0.
a=a+ap*pm
rho=rho+rp*pm
beta=beta+bp*pm
cp=cp+cpp*pm
gamma=1.+ta*beta**2*a**2/cp
mu=e2*rho**(1./3.)*exp (e1*rho/ta)
```

12.14. Liquid sodium–potassium eutectic (NaK-78)

This is for eutectic sodium-potassium; data from Ref. [72].

```
a0=2051.
at1=-.53
ap=0.
r0=876.4
rt1=-2.183e-1
rt2=-2.982e-5
rt3=0.
c0=970.69
ct1=-.36903
ct2=3.4309e-4
k0=21.4+2.07e-2*t1-2.2e-5*t1**2
if(t1.le.400.) then
 e1=.688
 e2=1.16e-5
else
 e1=.979
 e2=8.2e-6
endif
a=a0+at1*t1
rho=r0+rt1*t1+rt2*t1**2+rt3*t1**3
beta=(-rt1-2.*rt2*t1-3.*rt3*t1**2)/rho
bt=beta**2-(2.*rt2+6.*rt3*t1)/rho
cp=c0+ct1*t1+ct2*t1**2
rp=1./a/a+ta*beta**2/cp
bp=-beta/(rho*a**2)+2.*at1/(rho*a**3)-beta**2/rho/cp
bp=bp-2.*ta*beta*bt/rho/cp-ta*beta**3/rho/cp
bp=bp+ta*beta**2*(ct1+2.*ct2*t1)/rho/cp/cp
cpp=-ta*(beta**2+bt)/rho
c So far, everything is evaluated at p=0.
a=a+ap*pm
rho=rho+rp*pm
beta=beta+bp*pm
cp=cp+cpp*pm
gamma=1.+ta*beta**2*a**2/cp
mu=e2*rho**(1./3.)*exp (e1*rho/ta)
```

12.15. User-defined gases

12.15.1. User-defined pure gases

Files that define non-standard gases can have any name valid under the operating system under which DeltaEC is running, and should end with the extension <`.tpf`>. If the root filename is the same as any pre-defined gases, DeltaEC replaces its internal calculations for that gas with those given in the user's file. The <`.tpf`> file should be in the same directory or folder as the model file. The name of the gas is set to the root filename of the user-defined gas file. Up to five distinct user-defined gases can be active at one time. Please limit the root of the file name to eight characters.

Each gas property is specified by a line containing 1–10 real coefficients to be read in as C_0, C_1, ... C_9, where trailing unspecified parameters are taken to be zero. The order of the property lines is ρ_m, c_p, k, a^2, and μ. Comment lines can be inserted anywhere in the <`.tpf`> file with an initial '!', and blank lines are not allowed.

It is critical that the properties be arranged in this order:

line 1: ρ_m
line 2: c_p
line 3: k
line 4: a^2
line 5: μ

Each of the five properties is derived from its 10 coefficients using the following equation:

$$\text{gas property} = C_0 + C_1 \frac{p_m}{T_m + p_m C_2} + C_3 T_m + C_4 T_m^2 + C_5 T_m^{C_6} + C_7 p_m^2 T_m^{C_8} + C_9 p_m, \quad (12.1)$$

where T_m and p_m are the absolute temperature (K) and mean pressure (Pa). An example <`.tpf`> file is shown in Section 2.6.2.

Equation (12.1) is a compromise between simplicity and flexibility; it is intended for use with a variety of simple expressions for gases and liquids and has a uniform syntax for specifying all five properties. There is only a limited mean pressure dependence, suitable for nearly ideal gases; for more complicated mean pressure dependence, multiple <`.tpf`> files should be written for each mean pressure range used.

DeltaEC also needs the ratio of specific heats, γ, and the expansion coefficient β, but these are calculated internally from

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T} \text{ and } \gamma = 1 + \frac{T\beta^2 a^2}{c_p}. \quad (12.2)$$

To employ a user-defined gas, select its filename as the gas type in a `BEGIN` segment.

12.15.2. User-defined gas mixtures

The user can define the properties of his/her own binary ideal-gas mixture that is not a member of DeltaEC's internal library of mixtures, by using a file <`filename.tpm`> in the folder in which the DeltaEC model is running. If `filename` is one of DeltaEC's known gases, DeltaEC will replace its own properties with the user's properties. Please limit `filename` to eight characters.

The file format, shown below, is similar to that of <`.tpf`> files for pure gases. Blank lines are not allowed, and extra lines beginning with "!" are regarded as comments, which are very useful for keeping track of which line is which.

Properties are functions of mean pressure p_m (Pa), absolute temperature T_m (K), and mole fraction n_L. Each function is specified by a line containing a sequence of real coefficients (separated by blanks) which are read sequentially. Unused trailing parameters are set to zero. The properties must be arranged in this order:

line 1: m_L m_H γ_L γ_H
line 2: $k_{\text{pure } L}$
line 3: $k_{\text{pure } H}$
line 4: $\mu_{\text{pure } L}$
line 5: $\mu_{\text{pure } H}$
line 6: k_{mix}
line 7: μ_{mix}
line 8: D_{12}
line 9: k_T

The four numbers in line 1, which are m_L, m_H, γ_L, and γ_H, are used to calculate state properties based on ideal-gas behavior as follows:

$$m_{\text{avg}} = m_L n_L + m_H (1 - n_L), \tag{12.3}$$

$$\rho_m = \frac{p_m m_{\text{avg}}}{R_{\text{univ}} T_m}, \tag{12.4}$$

$$\frac{1}{\gamma - 1} = \frac{n_L}{\gamma_L - 1} + \frac{1 - n_L}{\gamma_H - 1}, \tag{12.5}$$

$$a^2 = \frac{\gamma R_{\text{univ}} T_m}{m_{\text{avg}}}, \tag{12.6}$$

$$c_p = \frac{\gamma}{\gamma - 1} \frac{R_{\text{univ}}}{m_{\text{avg}}}. \tag{12.7}$$

The pure-gas transport properties, $k_{\text{pure } L}$, $k_{\text{pure } H}$, $\mu_{\text{pure } L}$, and $\mu_{\text{pure } H}$, are calculated from numbers C_0, C_1, ... C_9 in each of lines 2 through 5 using the following equation:

$$\text{property} = C_0 + C_1 \frac{p_m}{T + p_m C_2} + C_3 T + C_4 T^2 + C_5 T^{C_6} + C_7 p_m^2 T^{C_8} + p_m C_9. \tag{12.8}$$

Equation (12.8) is a compromise between simplicity and flexibility. It is intended for use with most coefficients set to zero in a variety of simple expressions. Finally, the numbers in lines 6 through 9 are used in these equations:

$$\text{line 6:} \quad k_{\text{mix}} = n_L k_{\text{pure } L} + (1 - n_L) k_{\text{pure } H} + \left(C_0 + C_1 T_m^{C_2}\right) n_L^{C_3} (1 - n_L)^{C_4}, \tag{12.9}$$

$$\text{line 7:} \quad \mu_{\text{mix}} = n_L \mu_{\text{pure } L} + (1 - n_L) \mu_{\text{pure } H} + \left(C_0 + C_1 T_m^{C_2}\right) n_L^{C_3} (1 - n_L)^{C_4}, \tag{12.10}$$

$$\text{line 8:} \quad D_{12} = \frac{C_0}{\left(p_m/[10^5 \text{ Pa}]\right)} (1 - C_1 n_L) \left(\frac{T_m}{[300 \text{ K}]}\right)^{C_2} + C_3, \tag{12.11}$$

$$\text{line 9:} \quad k_T = C_0 n_L^{C_1} (1 - n_L)^{C_2} \left(\frac{T_m}{[300 \text{ K}]}\right)^{C_3} + C_4. \tag{12.12}$$

As an example of the use of these coefficients, consider the example below for neon, a mixture of two isotopes.

```
! user-defined mixture: neon, a mixture of two isotopes, 20 and 22.
! Natural abundance is nL = 0.91
! mL(kg/mole)  mH(kg/mole)  gammaL  gammaH:
  0.01999  0.02199  1.6667  1.6667
! k pure L (W/m-K):
  0.  0.  0.  0.  0.  0.00122  0.65
! k pure H (W/m-K):
```

```
  0.  0.  0.  0.  0.  0.00116  0.65
! mu pure L (kg/m-s):
  0.  0.  0.  0.  0.  7.41E-7  0.66
! mu pure H (kg/m-s):
  0.  0.  0.  0.  0.  7.70E-7  0.66
! k mixture (W/m-K):
  0.
! mu mixture (kg/m-s):
  0.
! D12 (m2/s):
  0.53E-4  0.  1.72
! kT:
  0.0267 1.0 1.0
```

To employ a user-defined mixture, select its filename as the gas type in a `BEGIN` segment.

13. Solids

We provide an artificial temperature floor of 10 Kelvin to prevent DeltaEC from trying to use negative temperatures when it is really lost. Consequently no temperature below 10 Kelvin can be used.

In what follows, `ta` is temperature in Kelvin, `t1` is temperature in Celsius.

DeltaEC looks for a 10-character field to determine solid type. When editing a <`.out`> file with a text editor, be sure to use plenty of trailing spaces after short solid names like "`mylar`" to get character strings like "`solid-type`" out of the field.

Equations used here accurately cover ranges of T and p that have been of interest to the Los Alamos thermoacoustics team. No warning is issued by DeltaEC if it goes outside the accurate range. It is up to the user to study the equations here, or examine the results of these equations via the properties tokens in an `RPN` segment or the thermophysical-properties dialog, and decide if these equations are accurate enough or if a user-defined solid is necessary.

13.1. Ideal solid (`ideal`)

`ks, rhos`, and `cs` are effectively infinite, so $\epsilon_s = 0$.

```
ks=1.d8
rhos=1.d8
cs=1.d8
```

13.2. Copper (`copper`)

```
ks=398.-.0567*(ta-300.)
rhos=9000.
cs=420.
```

13.3. Nickel (`nickel`)

```
if (ta.lt.631) then
 ks=63.8+.08066*(631.-ta)
else
 ks=63.8+.02156*(ta-631.)
endif
rhos=8700.
cs=530.
```

13.4. Stainless steel (`stainless`)

```
rhos=8274.55 -1055.23 *dexp(-((T1-2171.05)/2058.08)**2)
ks=(266800*ta**(-5.2)+0.21416*ta**(-1.6))**(-0.25)
cs=(1.7054e-6*ta**(-0.88962)+23324/ta**6)**(-1/3) + 15/ta
```

Prior to version 3.5b2, DeltaE's stainless steel properties were very inaccurate at cryogenic temperatures.

13.5. Molybdenum (molybdenum)

```
rhos= 10868.6 -2637.52 * exp (-((T1-11383.7)/9701.36)**2)
cs= 253.791 +0.0583812 *T1-2.73919e-06*T1**2
ks= (33.9616 -0.00947953 *T1-4.12809e-08*T1**2)*4.186
```

13.6. Tungsten (tungsten)

```
cs=.13576e3*(1.-4805./ta**2)+.0091159*ta+2.31341e-9*ta**3
ks=135.5+1.05e4/ta-.023*ta
rhos=19254*(1.-3.*(-8.69e-5+3.83e-6*t1+7.92e-10*t1**2))
```

13.7. Kapton (kapton)

```
ks=0.2*(1.-exp(-ta/100.))
rhos=1445.-0.085*ta
cs=3.64*ta
```

13.8. Mylar (mylar)

```
ks=0.11+1.7e-4*ta
rhos=1400.-0.175*ta
cs=3.7*ta
```

13.9. Celcor (celcor)

Properties from Steve Garrett, Penn State University (private communication).

```
ks=2.5
rhos=2510.
cs=262.5 + 1.864*ta - 0.001011*ta**2
```

13.10. User-defined solids

User-defined solids, like user-defined gases, are derived from coefficients in user-written text files that must be named <`filename.tpf`>. Please limit `filename` to eight characters. Up to five user-defined solids can be active at once. Each property is specified by a line containing 1–10 real coefficients to be read in as C_0, C_1, C_2, ... C_9, where unspecified parameters are taken to be zero. The order of the property lines is

line 1: ρ_s
line 2: c_s
line 3: k_s

Comment lines can be added with an initial '!', and blank lines are not allowed.

Each of the three properties is derived from its 10 coefficients using the following equation:

$$\text{property} = C_0 + C_1 \exp(-TC_2) + C_3T + C_4T^2 + C_5T^{C_6} + C_7p_m^2T^{C_8} + p_mC_9. \qquad (13.1)$$

To request a user-defined solid, simply use the root file name as the solid type. The <`.tpf`> file should be in the same directory or folder as the model file. If the name matches any pre-defined solid name, the (constant) user-defined properties will replace DeltaEC's internal calculations.

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Back Matter

14. Derivations

Here we present our derivations of a few equations that are used in DeltaEC but are not derived in the archival thermoacoustics literature.

Force F_1 on pressure vessel

Consider a rigid, uniaxial pressure vessel containing a thermoacoustic gas oscillation at angular frequency ω. Newton's laws of motion show that the force exerted *by* the gas (and by moving parts of transducers, if present) *on* the rigid pressure vessel equals the force exerted *on* the gas (and optional transducers) *by* the rigid vessel, and this force equals the rate of change of momentum $\rho\mathbf{v}$ of the gas (and optional transducers). Consider only forces and motion at ω. Then the total first-order oscillating force exerted on the rigid pressure vessel by the oscillations inside it, in the x direction, is

$$F_1 = -i\omega \int (\rho_m u_1)\ dV = -i\omega \int \rho_m(x) U_1(x)\ dx. \tag{14.1}$$

The right-hand side of this equation is what DeltaEC's RPN force operator, `F1`, calculates; this operator is described in Section 11.4.1. If $F_1 = 0$, the vessel does not vibrate. Otherwise, if the vessel is freely suspended and we want to learn its acceleration amplitude, we simply divide by its mass M; if we want its velocity amplitude, we divide by $i\omega M$; its displacement amplitude, we divide by $-\omega^2 M$.

Equation (14.1) includes both the normal force exerted by gas pressure on the vessel and the shear force exerted by viscosity and gas velocity on the vessel. Newton's laws say that this must be true, but it can also be verified directly, as follows. Calculus shows that

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) = \rho\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\frac{\partial \rho}{\partial t}. \tag{14.2}$$

Switching to "sum over repeated indices" notation (as in Ref. [35]), the continuity equation is

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x_k}(\rho v_k) \tag{14.3}$$

and the momentum equation is

$$\rho\frac{\partial v_i}{\partial t} = -\rho\left(v_k\frac{\partial v_i}{\partial x_k}\right) - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_k}\sigma'_{ik}, \tag{14.4}$$

where σ'_{ik} is the viscous stress tensor. Substituting Eqs. (14.3) and (14.4) into (14.2) and doing a little algebra to combine terms yields

$$\frac{\partial}{\partial t}(\rho v_i) = -\frac{\partial}{\partial x_k}\left(\rho v_i v_k + p\delta_{ik} - \sigma'_{ik}\right). \tag{14.5}$$

Now integrate over the whole volume of gas, and use Green's formula to convert the volume integral of the divergence on the right side into a surface integral:

$$\frac{\partial}{\partial t}\int (\rho v_i)\ dV = -\oint (\rho v_i v_k + p\delta_{ik} - \sigma'_{ik})\ dS_k, \tag{14.6}$$

where dV is a differential element of the volume and dS_k is a vector normal to an element of surface with area dS. The velocity is zero on the unmoving surface, so

$$\frac{\partial}{\partial t}\int (\rho v_i)\ dV = -\oint (p\delta_{ik} - \sigma'_{ik})\ dS_k. \tag{14.7}$$

The right side is the total force exerted on the gas by the solid vessel bounding the gas. It is the sum of the pressure forces and the viscous forces. The x component of the left side is Eq. (14.1), which is what DeltaEC's `F1` operator calculates.

Third-order p_1 dependence in $dp_{2,0,HL}/dx$

Derivations of $dp_{2,0,HL}/dx$ in publications of the Los Alamos thermoacoustics team through 2006, such as Refs. [41, 12], kept terms only through second order. In 2006, Scott Backhaus found one example (a stacked-screen regenerator with a small $u_{2,0}$) in which a third-order term, proportional to $|\rho_1|\,|u_1|^2$, contributed significantly to $dp_{2,0,HL}/dx$. This led us to include that term in DeltaEC's `STKSCREEN` calculations, and for consistency we have included similar third-order contributions in other segments, including `MINOR`, `DUCT`, and `CONE`. This section outlines the derivations of these expressions for $dp_{2,0,HL}/dx$, which include these small terms proportional to p_1. Scott Backhaus's contributions to these derivations are gratefully acknowledged.

`MINOR`

The time-averaged pressure drop across a minor-loss component is, by definition, given by

$$\overline{\Delta p} = -\frac{1}{2}\overline{K\rho u\,|u|}, \tag{14.8}$$

where u is the spatially averaged velocity, ρ is the density, K is the minor-loss coefficient, and all three of these variables can be time dependent. In previous derivations, such as in Ref. [12], we have taken ρ to be independent of time, because we were satisfied with results to second order. Here we retain the fundamental component of ρ's time dependence.

Write Eq. (14.8) as

$$\overline{\Delta p_{\text{minor}}} = -\frac{1}{2}\overline{K(t)\left[\rho_m + |\rho_1|\cos(\omega t+\phi)\right](|u_1|\cos\omega t + u_{2,0})\,||u_1|\cos\omega t + u_{2,0}|} \tag{14.9}$$

$$= -\frac{1}{4\pi}\int_0^{2\pi} K(t)\left[\rho_m + |\rho_1|\cos(\omega t+\phi)\right](|u_1|\cos\omega t + u_{2,0})\,||u_1|\cos\omega t + u_{2,0}|\ d(\omega t). \tag{14.10}$$

In the interval $0 < \omega t < 2\pi$, the argument of the absolute value changes sign at $\omega t = \text{Arccos}(-u_{2,0}/|u_1|) \equiv \tau_0$ and $\omega t = 2\pi - \text{Arccos}(-u_{2,0}/|u_1|) = 2\pi - \tau_0$ if $|u_{2,0}/u_1| < 1$; at these zero crossings, the value of K changes from K_+ to K_- and back again. Thus

$$\overline{\Delta p_{\text{minor}}} = -\frac{1}{4\pi}\left(K_+ \int_{-\tau_0}^{\tau_0} - K_- \int_{\tau_0}^{2\pi-\tau_0}\right)[\rho_m + |\rho_1|\cos(\omega t + \phi)]\left(|u_1|\cos\omega t + u_{2,0}\right)^2 \; d(\omega t) \tag{14.11}$$

if $u_{2,0}/|u_1| < 1$. Evaluating this (e.g., with the symbolic-mathematics software MATHEMATICA) yields

$$\begin{aligned}\overline{\Delta p_{\text{minor}}} &= \frac{\rho_m |u_1|^2}{4\pi}\left\{(K_- - K_+)\frac{\pi}{2}\left(1 + 2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0}\frac{|\rho_1|}{\rho_m}\cos\phi\right)\right. \\ &\quad - (K_+ + K_-)\left[\left(1 + 2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0}\frac{|\rho_1|}{\rho_m}\cos\phi\right)\sin^{-1}\varepsilon_{2,0}\right. \\ &\quad \left.\left. + \left(3\varepsilon_{2,0} + \frac{4}{3}\left[1 + \varepsilon_{2,0}^2/2\right]\frac{|\rho_1|}{\rho_m}\cos\phi\right)\sqrt{1 - \varepsilon_{2,0}^2}\right]\right\}\end{aligned} \tag{14.12}$$

if $|\varepsilon_{2,0}| \le 1$, where $\varepsilon_{2,0} = u_{2,0}/|u_1|$.

If $|\varepsilon_{2,0}| \ge 1$, the integral is less complicated because the argument of the absolute value never crosses zero; the results are

$$\overline{\Delta p_{\text{minor}}} = -\frac{\rho_m |u_1|^2}{4}K_+\left(1 + 2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0}\frac{|\rho_1|}{\rho_m}\cos\phi\right) \text{ for } \varepsilon_{2,0} \ge 1, \tag{14.13}$$

$$= \frac{\rho_m |u_1|^2}{4}K_-\left(1 + 2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0}\frac{|\rho_1|}{\rho_m}\cos\phi\right) \text{ for } \varepsilon_{2,0} \le -1. \tag{14.14}$$

We take the density oscillations to be adiabatic, so $\rho_1/\rho_m = p_1/\gamma p_m$. These results appear in Section 10.2.4.

STKSCREEN and SX

The momentum equation in a screen bed is [59, 12]

$$\frac{dp}{dx} = -\frac{c_1}{8r_h^2}\mu u - \frac{c_2}{2r_h}\rho u|u|, \tag{14.15}$$

where u is the spatially averaged velocity, ρ is the density, μ is the viscosity, and r_h is the hydraulic radius. We take

$$u = |u_1|\cos\omega t + u_{2,0}, \tag{14.16}$$

$$\rho = \rho_m + |\rho_1|\cos(\omega t + \phi). \tag{14.17}$$

We might also write $\mu = \mu_m + |\mu_1|\cos(\omega t + \theta)$, but the oscillations in a regenerator are nearly isothermal and viscosity is independent of pressure, so we neglect μ_1 in the first term in Eq. (14.15).

The time average of the left-hand side, to second order, is $dp_{2,0,HL}/dx$.

The time average of the first term on the right-hand side, to second order, is

$$-\frac{c_1}{8r_h^2}\mu_m u_{2,0}. \tag{14.18}$$

Taking the time average of the second term involves the integral

$$\begin{aligned} I_{\text{screen}} &= \overline{\left[\rho_m + |\rho_1|\cos(\omega t+\phi)\right]\left(|u_1|\cos\omega t + u_{2,0}\right)\left||u_1|\cos\omega t + u_{2,0}\right|} && (14.19)\\ &= \frac{1}{2\pi}\int_0^{2\pi}\left[\rho_m + |\rho_1|\cos(\omega t+\phi)\right]\left(|u_1|\cos\omega t + u_{2,0}\right)\left||u_1|\cos\omega t + u_{2,0}\right|\ d(\omega t). && (14.20)\end{aligned}$$

In the interval $0 < \omega t < 2\pi$, the argument of the absolute value near the end of the equation changes sign at $\omega t = \text{ Arccos}(-u_{2,0}/|u_1|) \equiv \tau_0$ and $\omega t = 2\pi - \text{ Arccos}(-u_{2,0}/|u_1|) = 2\pi - \tau_0$ if $|u_{2,0}/u_1| < 1$, so we can write

$$I_{\text{screen}} = \frac{1}{2\pi}\left(\int_{-\tau_0}^{\tau_0} - \int_{\tau_0}^{2\pi-\tau_0}\right)\left[\rho_m + |\rho_1|\cos(\omega t+\phi)\right]\left(|u_1|\cos\omega t + u_{2,0}\right)^2\ d(\omega t). \tag{14.21}$$

Evaluating this with the symbolic-mathematics software MATHEMATICA yields

$$\begin{aligned} I_{\text{screen}} &= \frac{\rho_m |u_1|^2}{\pi}\left\{\left[\left(1+2\varepsilon_{2,0}^2\right)\sin^{-1}\varepsilon_{2,0} + 3\varepsilon_{2,0}\sqrt{1-\varepsilon_{2,0}^2}\right]\right.\\ &\quad \left. +2\frac{|\rho_1|}{\rho_m}\cos\phi\left[\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0} + \frac{2+\varepsilon_{2,0}^2}{3}\sqrt{1-\varepsilon_{2,0}^2}\right]\right\}. && (14.22)\end{aligned}$$

where $\varepsilon_{2,0} = u_{2,0}/|u_1|$.

Combining all of these intermediate results and taking $\rho_1/\rho_m = p_1/p_m$ for approximately isothermal oscillations yields

$$\begin{aligned} \frac{dp_{2,0,HL}}{dx} &= -\frac{c_1}{8r_h^2}\mu_m u_{2,0} - \frac{c_2}{2\pi r_h}\rho_m |u_1|^2\left\{\left(1+2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0}\frac{|p_1|}{p_m}\cos\phi\right)\sin^{-1}\varepsilon_{2,0}\right.\\ &\quad \left. + \left[3\varepsilon_{2,0} + \frac{2}{3}\left(2+\varepsilon_{2,0}^2\right)\frac{|p_1|}{p_m}\cos\phi\right]\sqrt{1-\varepsilon_{2,0}^2}\right\} && (14.23)\end{aligned}$$

for $|\varepsilon_{2,0}| \le 1$.

In SX, $|\varepsilon_{2,0}| \ge 1$ may sometimes be of interest. In that case,

$$\frac{dp_{2,0,HL}}{dx} = -\frac{c_1}{8r_h^2}\mu_m u_{2,0} - \frac{c_2}{4r_h}\rho_m |u_1|^2\ \text{sign}(\varepsilon_{2,0})\left(1+2\varepsilon_{2,0}^2 + 2\varepsilon_{2,0}\frac{|p_1|}{p_m}\cos\phi\right). \tag{14.24}$$

These results appear in Sections 10.5.2 and 10.7.2.

DUCT and CONE

We follow the outline in the appendix of Ref. [41], but we change the time dependence from sine to cosine, to be consistent with other results in this section, and we keep some dependence on ρ_1 by writing Eq. (A6) in Ref. [41] as

$$\frac{dp}{dx} = -f_M(t)\frac{\rho_m + |\rho_1|\cos(\omega t + \phi)}{2D}u(t)\,|u(t)|\,, \tag{14.25}$$

where $u = U/S$ in Ref. [41]. Using Eq. (A4) in Ref. [41] for f_M yields

$$\begin{aligned}\frac{dp_{2,0,HL}}{dx} &= -\frac{\rho_m\,|u_1|^2}{2D}\frac{1}{2\pi}\left(\int_{-\tau_0}^{\tau_0} - \int_{\tau_0}^{2\pi-\tau_0}\right)\left[f_{M,\max} + \frac{df_M}{dN_R}N_{R,\max}\left(\frac{|\cos\omega t + \varepsilon_{2,0}|}{1+|\varepsilon_{2,0}|} - 1\right)\right] \\ &\quad \times\left[1 + \frac{|\rho_1|}{\rho_m}\cos(\omega t + \phi)\right](\cos\omega t + \varepsilon_{2,0})^2\;\; d(\omega t),\end{aligned} \tag{14.26}$$

where $\varepsilon_{2,0} = u_{2,0}/\,|u_1|$ and $\tau_0 = \mathrm{Arccos}(-u_{2,0}/\,|u_1|)$ as for the STKSCREEN derivation above. We neglect $|u_1|^2\,N_{R,\max}\,|\rho_1|$ because it is fourth order. Then

$$\begin{aligned}\frac{dp_{2,0,HL}}{dx} &= -\frac{\rho_m\,|u_1|^2}{2D}f_{M,\max}\frac{1}{2\pi}\left(\int_{-\tau_0}^{\tau_0} - \int_{\tau_0}^{2\pi-\tau_0}\right)\left[1 + \frac{|\rho_1|}{\rho_m}\cos(\omega t + \phi)\right](\cos\omega t + \varepsilon_{2,0})^2\;\; d(\omega t) \\ &\quad -\frac{\rho_m\,|u_1|^2}{2D}\frac{df_M}{dN_R}N_{R,\max}\frac{1}{2\pi}\left(\int_{-\tau_0}^{\tau_0} - \int_{\tau_0}^{2\pi-\tau_0}\right)\left(\frac{|\cos\omega t + \varepsilon_{2,0}|}{1+|\varepsilon_{2,0}|} - 1\right)(\cos\omega t + \varepsilon_{2,0})^2\;\; d(\omega t).\end{aligned} \tag{14.27}$$

The integral in the first line was evaluated above, in Eqs. (14.21)–(14.22). The integral in the second line was evaluated to obtain the bottom line in Eq. (A10) in Ref. [41]. Pulling all the pieces together, and using $\rho_1/\rho_m = p_1/\gamma p_m$ for adiabatic oscillations, yields

$$\begin{aligned}\frac{dp_{2,0}}{dx} &= -\frac{\rho_m\,|u_1|^2}{2\pi D}\Bigg\{\Bigg[\left(1+2\varepsilon_{2,0}^2\right)\sin^{-1}\varepsilon_{2,0} + 3\varepsilon_{2,0}\sqrt{1-\varepsilon_{2,0}^2}\;f_{M,\max} \\ &\qquad + \frac{2\,|p_1|}{\gamma p_m}\cos\phi\left(\varepsilon_{2,0}\sin^{-1}\varepsilon_{2,0} + \frac{2+\varepsilon_{2,0}^2}{3}\sqrt{1-\varepsilon_{2,0}^2}\right)\Bigg] \\ &\quad + \left[\pi\varepsilon_{2,0}\frac{3/2+\varepsilon_{2,0}^2}{1+|\varepsilon_{2,0}|} - 3\varepsilon_{2,0}\sqrt{1-\varepsilon_{2,0}^2} - \left(1+2\varepsilon_{2,0}^2\right)\sin^{-1}\varepsilon_{2,0}\right]\frac{df_M}{dN_R}\,N_{R,\max}\Bigg\}\,, \\ &\qquad\qquad |\varepsilon_{2,0}| \le 1,\end{aligned} \tag{14.28}$$

$$\begin{aligned}\frac{dp_{2,0}}{dx} &= -\,\mathrm{sign}(u_{2,0})\frac{\rho_m u_{2,0}^2}{2D}\left[\left(1 + \frac{1}{2\varepsilon_{2,0}^2} + \frac{|p_1|}{\gamma p_m}\frac{\cos\phi}{\varepsilon_{2,0}}\right)f_{M,\max} - \frac{2\varepsilon_{2,0}^2 - 2\,|\varepsilon_{2,0}| + 1}{2\varepsilon_{2,0}^2(1+|\varepsilon_{2,0}|)}\frac{df_M}{dN_R}\,N_{R,\max}\right]\,, \\ &\qquad\qquad |\varepsilon_{2,0}| \ge 1,\end{aligned} \tag{14.29}$$

These results appear in Section 10.1.1 for DUCT and 10.1.2 for CONE.

STKPOWERLW and PX

The result for $dp_{2,0,HL}/dx$ in STKPOWERLW and PX is the same as for DUCT and CONE above, except that we replace γp_m with p_m to model isothermal oscillations.

Pole-tip flux in linear motor or alternator, **SPEAKER

The pole-tip magnetic-flux phasor is the sum of the flux phasor due to current and the flux phasor due to motion:

$$\Phi_{pt,1} = \Phi_{I,1} + \Phi_{bEMF,1} \tag{14.30}$$

where the subscript "1" indicates a complex phasor as usual. Obviously

$$\Phi_{I,1} = -LI_1. \tag{14.31}$$

We use "−" instead of "+" here because we always write $|V| = |L\,dI/dt|$ and basic-physics books write $V = -d\Phi/dt$ and $\oint E \cdot dl = -L\,dI/dt$ with minus signs expressing Lenz's law (induced EMF *opposes* changes in current). Similarly, we know that the unmeasurable internal voltage $V_{bEMF,1} = -d\Phi_{bEMF,1}/dt = -i\omega\ \Phi_{bEMF,1}$, and in the linear operating regime we express this as $V_{bEMF,1} = -(Bl)v_1 = -(Bl)U_1/A$ in DELTAEC's language of "Bl" product and velocity. (Note DELTAEC's electromechanical sign convention, described in Sections 10.3.1 and 10.4.2, differs from some others' conventions, e.g., Ref. [11], via $V_1 \to -V_1$, $I_1 \to -I_1$, and $Bl \to -Bl$. Basically, this amounts to a rotation of all electrical and magnetic phasors by 180° relative to all mechanical and acoustic phasors.) Hence

$$\Phi_{bEMF,1} = -i(Bl)U_1/\omega A. \tag{14.32}$$

Combining these three equations gives us

$$\Phi_{pt,1} = -LI_1 - i(Bl)U_1/\omega A. \tag{14.33}$$

Using the law of cosines for triangles, we obtain the magnitude of the pole-tip flux

$$|\Phi_{pt,1}| = \sqrt{[L\,|I_1|]^2 + [(Bl)\,|U_1|\,/\omega A]^2 - 2\,[L\,|I_1|]\,[(Bl)\,|U_1|\,/\omega A]\sin\alpha} \tag{14.34}$$

where α is the phase angle by which U_1 leads I_1. (Note: $\cos(\alpha - 90°) = \sin\alpha$.)

(Thanks to John Corey for help with this section.)

Example

Here's a DELTAEC fragment giving $|\Phi_{pt,1}|$ as the result of RPN segment 17, while segment 16 displays $|\Phi_{I,1}| = L\,|I_1|$ and $|\Phi_{bEMF,1}| = (Bl)\,|U_1|\,/\omega A$.

```
!---------------------------- 15 ----------------------
IESPEAKER  the driver 7.5 diam
2.8500E-02 a Area  m^2          9.7598E+04 A |p| Pa
0.5200     b R     ohms             154.96 B Ph(p) deg
3.2200E-02 c L     H            7.3587E-02 C |U| m^3/s
84.700     d B x L T-m             -115.16 D Ph(U) deg
15.000     e M     kg               7.4617 E Hdot W
7.3300E+04 f K     N/m              7.4617 F Work W
105.00     g Rm    N-s/m            6602.7 G WorkIn W
59.466     h |I|   A     G          654.43 H Volts V
93.911     i Ph(I) deg   G          59.466 I Amps A
                                    70.164 J Ph(Ze) deg
                                4.3376E+05 K |Px| Pa
                                    174.36 L Ph(Px) deg
ideal      Solid type              -1273.9 M HeatIn W
!------------------------------------ 16 ---------------
RPN A = IL flux [weber];  B = bEMF flux [weber];  C = alpha [degree]
0.0000 a Target (t)                 1.9148 A ChngeMe
                                    0.5801 B ChngeMe
                                    150.93 C ChngeMe
15D 15i - 15d 15C * 2 / pi / 0b / 15a / 15c 15h *
!----------------------------------- 17 ---------------
RPN peak pole-tip flux. [weber = tesla.m^2]
0.0000 a Target (t)                 1.7099 A ChngeMe
16A sqrd 16B sqrd + 2 16A * 16B * 16C sin * - sqrt
```

Dividing by pole-tip effective area would turn 17A into the magnetic field, in tesla.

15. Symbols

Variables not found here are intermediate factors, constants, or functions defined in the text close to where they are used.

English letters

A	area, m^2
a	sound speed, m/s
a	half of rectangular pore's width, m
B	magnetic field, tesla
B	first virial coefficient, m^3/mol
b	a constant or function
b	half of rectangular pore's breadth, m
C	compliance, m^3/Pa
C	a constant or function
c	a constant or function
c	heat capacity per unit mass, J/kg·K
c_p	isobaric heat capacity per unit mass, J/kg·K
COP	coefficient of performance, $\dot{Q}/\dot{W}$ or $\dot{Q}/\dot{E}$
D_{12}	binary mass diffusion coefficient, m^2/s
D	diameter, m
d	diameter, m
d	differential
$\mathcal{E}$	energy, J
$\dot{E}$	acoustic power, W
e	2.71828...
F	force, N
F	a factor, fraction, friction factor, constant, or function
$\mathcal{F}$	a function
f	frequency, Hz
f	spatially averaged diffusion function
f	friction factor
f_M	Moody friction factor
$\dot{H}$	rate at which energy flows, W
h	heat transfer coefficient per unit area, W/m^2·K

I	electric current, amp
i	$\sqrt{-1}$
J	Bessel function
K	spring constant, N
K	minor-loss coefficient
`ksFrac`	fudge factor by which $(1-\phi)k_s$ is adjusted in `STKSCREEN`
k	thermal conductivity, W/m·K
k_T	thermal diffusion ratio
k	real or complex wave number, m^{-1}
L	$= k/\rho_m c_p D_{12}$
L	electric inductance, henry
L	length, m
l	length, m
l	half of plate thickness, m
M	mass, kg
m	molar mass, kg/mol (e.g., 0.004002 kg/mol for helium)
m	screen mesh number, wires/m
m	multiplier for turbulent effects
m'	turbulence multiplier for inertial pressure gradient
N_{Nu}	Nusselt number
N_R	Reynolds number
N_{St}	Stanton number, $= N_{Nu}/\sigma N_R$
$\dot{N}$	molar flow rate, mol/s (One mole $= 6.022 \times 10^{23}$ molecules)
n	mole fraction
p	pressure, Pa
Q	heat, J
$\dot{Q}$	rate at which heat is transferred (thermal power), W
$\dot{q}$	rate at which heat is transferred per unit length, W/m
R	acoustic resistance, Pa·s/m^3
R	electric resistance, ohm
R	mechanical resistance, kg/s
R	radius, m
R_{univ}	universal gas constant, J/mol·K
r	radius, m
r	radial coordinate, m
S	entropy, J/K
S	surface area, m^2
$\dot{S}$	rate of entropy generation, J/K·s
s	entropy per unit mass, J/kg·K
T	temperature, K
t	time, s
U	volume flow rate, m^3/s
u	x component of velocity, m/s
V	volume, m^3
V	voltage, volt

v molar volume, m^3/mol
$\mathbf{v}$ vector velocity, m/s
W work, J
$\dot{W}$ rate at which work is done (mechanical or electrical power), W
w enthalpy per unit mass, J/kg
X exergy, J
$\dot{X}$ rate at which exergy flows, W
x coordinate along sound-propagation direction, m
Y Neumann function
y coordinate perpendicular to x direction, m
Z acoustic impedance, $Pa{\cdot}s/m^3$
Z electric impedance, ohm
z coordinate perpendicular to x and y, m
z specific acoustic impedance, $kg/m^2{\cdot}s = Pa{\cdot}s/m$
z_n $= z/\rho a$

Greek letters

β $= -(1/\rho)(\partial\rho/\partial T)_p$, thermal expansion coefficient, K^{-1}
γ ratio of isobaric to isochoric specific heats
Δ big difference
δ penetration depth, m
δ small difference
δ_{ik} the unit tensor, equal to unity for $i = k$ and zero for $i \neq k$
ε surface roughness
ε in mixtures, function defined by Eq. (10.219)
ε_{20} $= m\dot{N}_{20}/\rho_m\,|U_1|$
ε $= m\dot{N}/\rho_m\,|U_1|$
ϵ_s correction factor for finite solid heat capacity
η efficiency, $\dot{W}/\dot{Q}_H$ or $\dot{E}/\dot{Q}_H$
θ phase angle
κ thermal diffusivity, m^2/s
μ dynamic viscosity, kg/m·s
ν kinematic viscosity, m^2/s
ξ displacement of gas in x direction, m
Π perimeter, m
π 3.14159...
ρ density, kg/m^3
σ Prandtl number
σ_{ik} nine-component viscous stress tensor, Pa
τ transduction coefficient, N/amp = volt/(m/s)
τ ωt
ϕ volumetric porosity
ω $= 2\pi f$, radian frequency, s^{-1}

Subscripts

$bEMF$	back EMF
br	branch
C	cold
e	electric
eff	effective
ext	external
fric	friction
gen	generated
geom	geometrical
grad	gradient
H	hot
H	heavy component of mixture
h	hydraulic (i.e., r_h is hydraulic radius)
HL	head loss
i	inner
imp	impedance
ini	initial
k	thermal
M	Moody
M	Moody
mix sep	mixture separation
m	mean (or mechanical)
max	maximum
$\dot{N}$	steady flow
Nu	Nusselt(i.e., N_{Nu} is Nusselt number))
o	outer
p	pressure
$\dot{Q}$	heat
R	Reynolds (i.e., N_R is Reynolds number)
rad	radiation
rev	reversible
s	series
s	solid
sat	saturation
stand	standing
T	temperature
tot	total
trav	traveling
wat	water
$D\kappa$	coupled mass and thermal diffusion
κD	coupled mass and thermal diffusion

κ	thermal
ν	viscous
0	"environment" or "ambient" or independent of time
1	first order, usually a complex amplitude
2	second order
$2, k$	sum of second-order enthalpy and ordinary conduction
$+$	positive-x direction
$-$	negative-x direction
univ	universal (i.e., $R_{\text{univ}} \simeq 8.314$ J/mol·K is the molar gas constant)

Special symbols

Im[ ]	Imaginary part of
Re[ ]	Real part of,
$\langle\,\rangle$	spatial average perpendicular to x
$\lvert\,\rvert$	magnitude of complex number
overbar	time average
overdot	time derivative, time rate
tilde	complex conjugate

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