Methods for Analysis of Seismic Coda Waves

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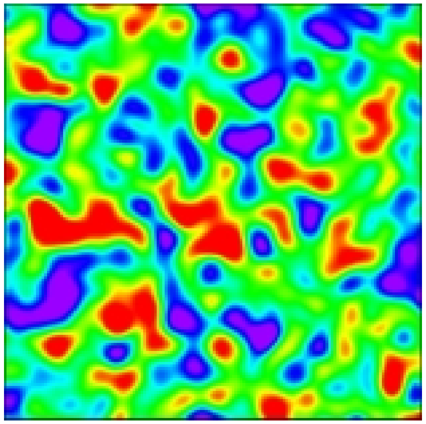
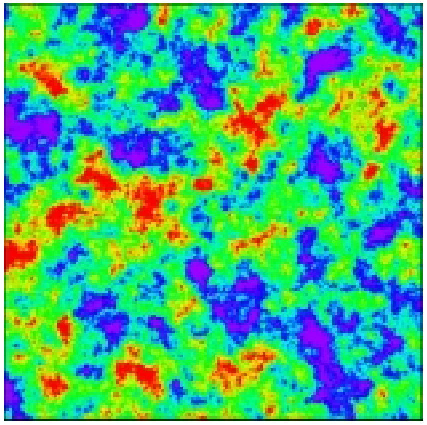


Medium Described by Gaussian Autocorrelation Function



Medium Described by Exponential Autocorrelation Function

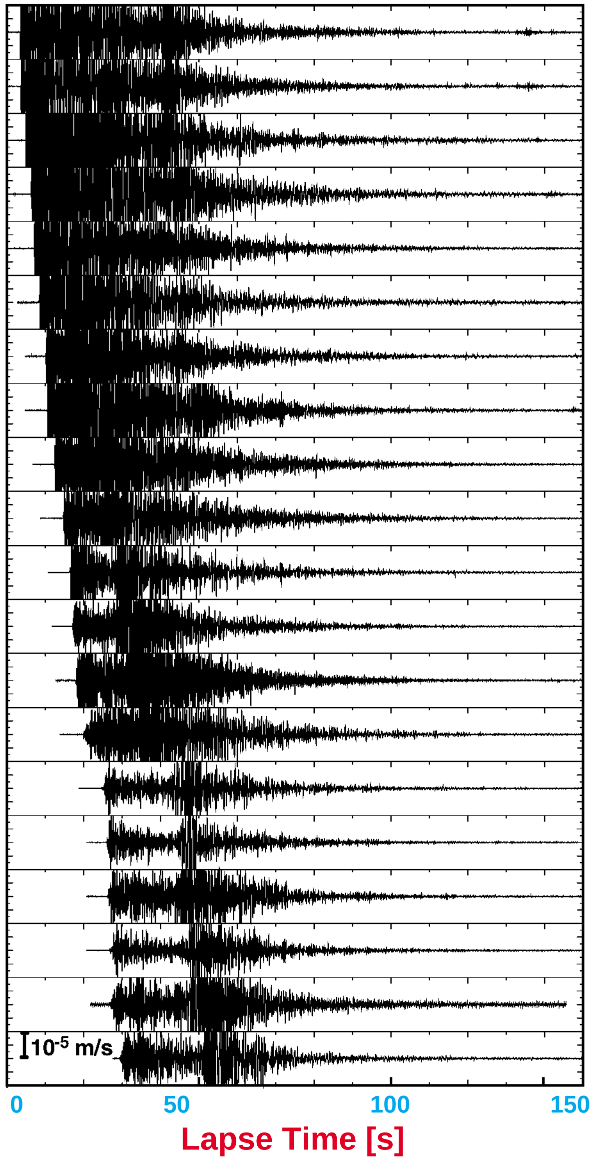
Two Topics

- Coda Normalization Method
- Radiative Transfer Theory to estimate medium parameters

Source:

Sato and Fehler, *Seismic Wave Propagation and Scattering in the Heterogeneous Earth*, AIP Press and Springer-Verlag, 1998.

Seismograms Vs. Distance from an Earthquake in Japan



Continuous wavetrains between P and S arrivals and those following S arrival are not easily explained by deterministic models

Fundamental Characteristics

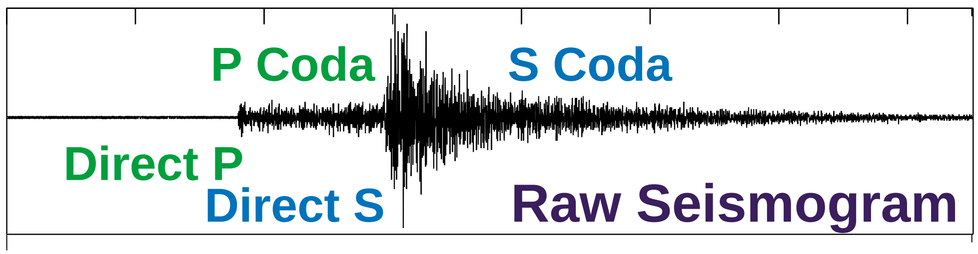
Regional Distances (< ~100km)

- Spectral content of later portions of S-coda are similar at all stations ($t > 2t_s$)
- Coda Duration is reliable measurement of magnitude
- Temporal shape of (narrow-band) coda is independent of epicentral distance
- Temporal shape of (narrow-band) coda is independent of magnitue for M<~6
- S-coda amplitude depends on local geology of recording site

Coda concept powerful

- Some analysis doesn’ t require calibrated stations
- Can gain quantitative information from data that have clipped direct arrivals
- Some analysis can be done with few computational resources
- Yield valuable information
- Spectral information obtained from larger portion of waveform than first arrival

Aki (1969) discussed continuous wave trains in tail portion of seismograms of local earthquakes

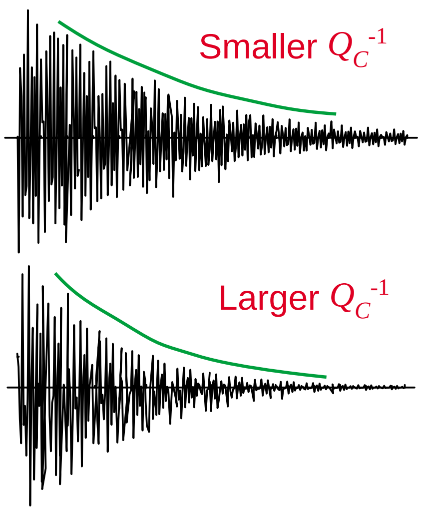


- Seismic Coda
  - Superposition of incoherent waves scattered by heterogeneities in the Earth
- Proposed analysis approach to investigate shape of coda envelope and obtain frequency-dependent information about Earth's heterogeneity

Initial Analysis Approach:

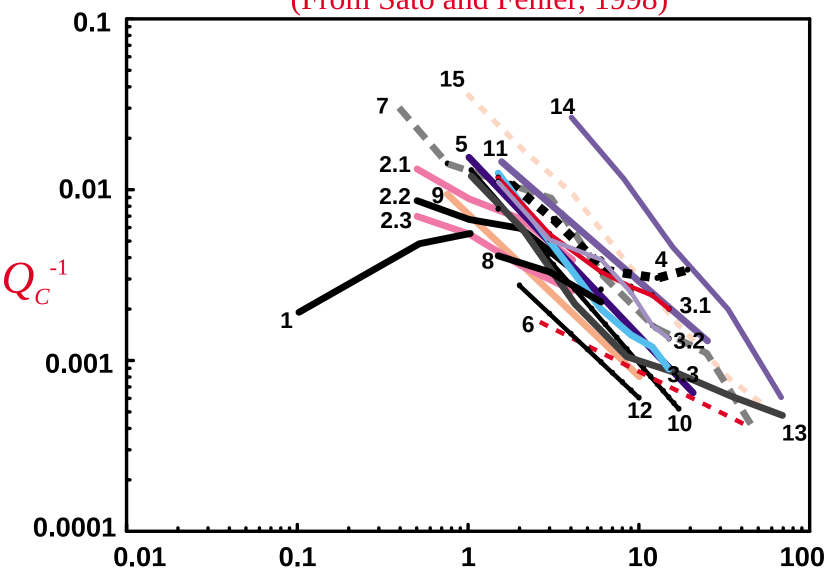
$$A \sim A_0 \frac{1}{t} e^{-\omega t / 2Q_c} \qquad Q_c \text{ is Coda } Q$$

Q_C^{-1} and Seismogram Envelope Shape



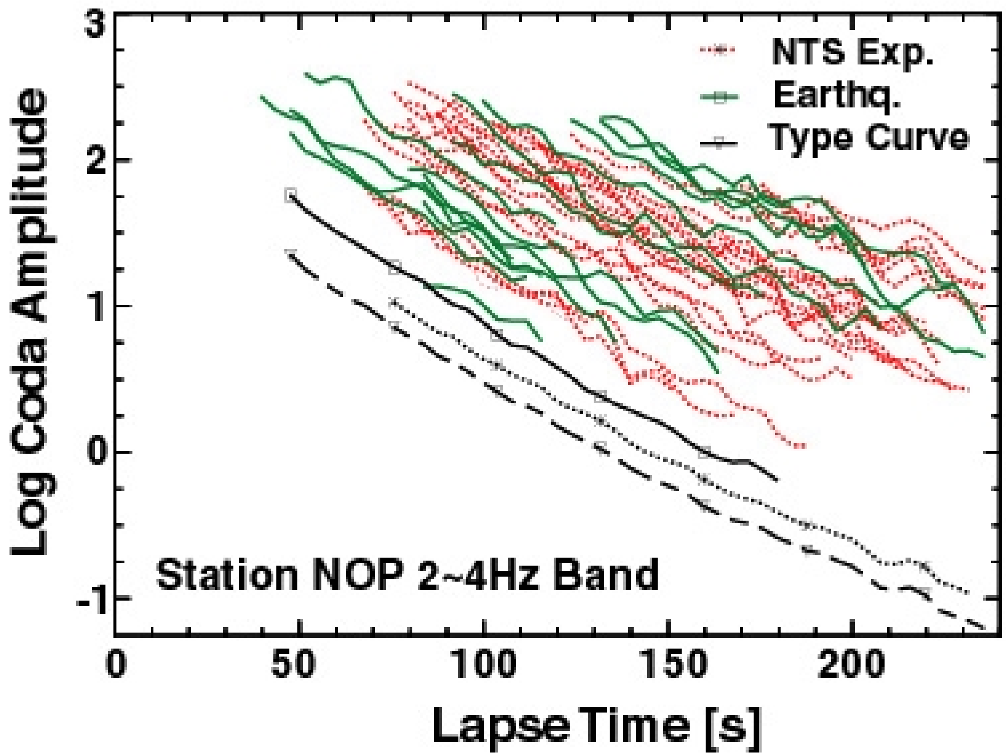
Coda Q^{-1} Values from Measurements made Worldwide

(From Sato and Fehler, 1998)



Decay Rate is Generally Stable within a Region

Coda Decay of NTS Explosions and Southern Great Basin Earthquakes



What is coda Q?

- No clear answer
- Aki originally said attenuation is dominated by scattering (in Earth) and coda Q is scattering Q
- Frankel and Wennerberg (Energy Flux Model) propose coda Q is intrinsic Q; energy removed from total wavefield

Amplitude at long lapse-time is proportional to source excitation * site amplification*path effect

$$\left\langle \left| \dot{u}_{ij}^{\text{S Coda}}(t;f) \right|^2 \right\rangle_{\text{T}} \propto W_i^S(f) \left| N_j^S(f) \right|^2 \frac{e^{-Q_C^{-1} 2\pi f t}}{t^n}$$

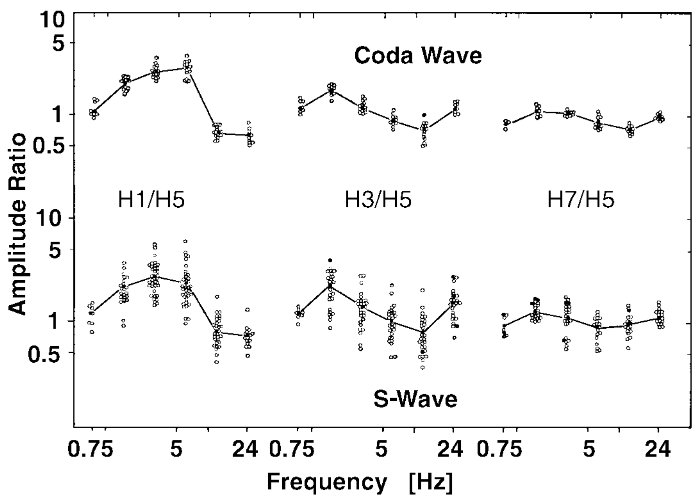
$\left\langle \left| \dot{u}_{ij}^{\text{S Coda}}(t;f) \right|^2 \right\rangle_{\text{T}}$	is moving average over a few cycles centered at time t of square of S-coda velocity wavefield in frequency band centered on f, source i, receiver j
$W_i^S(f)$	is source energy radiation from source i in frequency band centered on f
$\left| N_j^S(f) \right|$	is site amplification for receiver j

Relative Site Amplification estimate:

Divide coda amplitude for one event at one site by that for same event at another site
(Usually best to take average over many events)

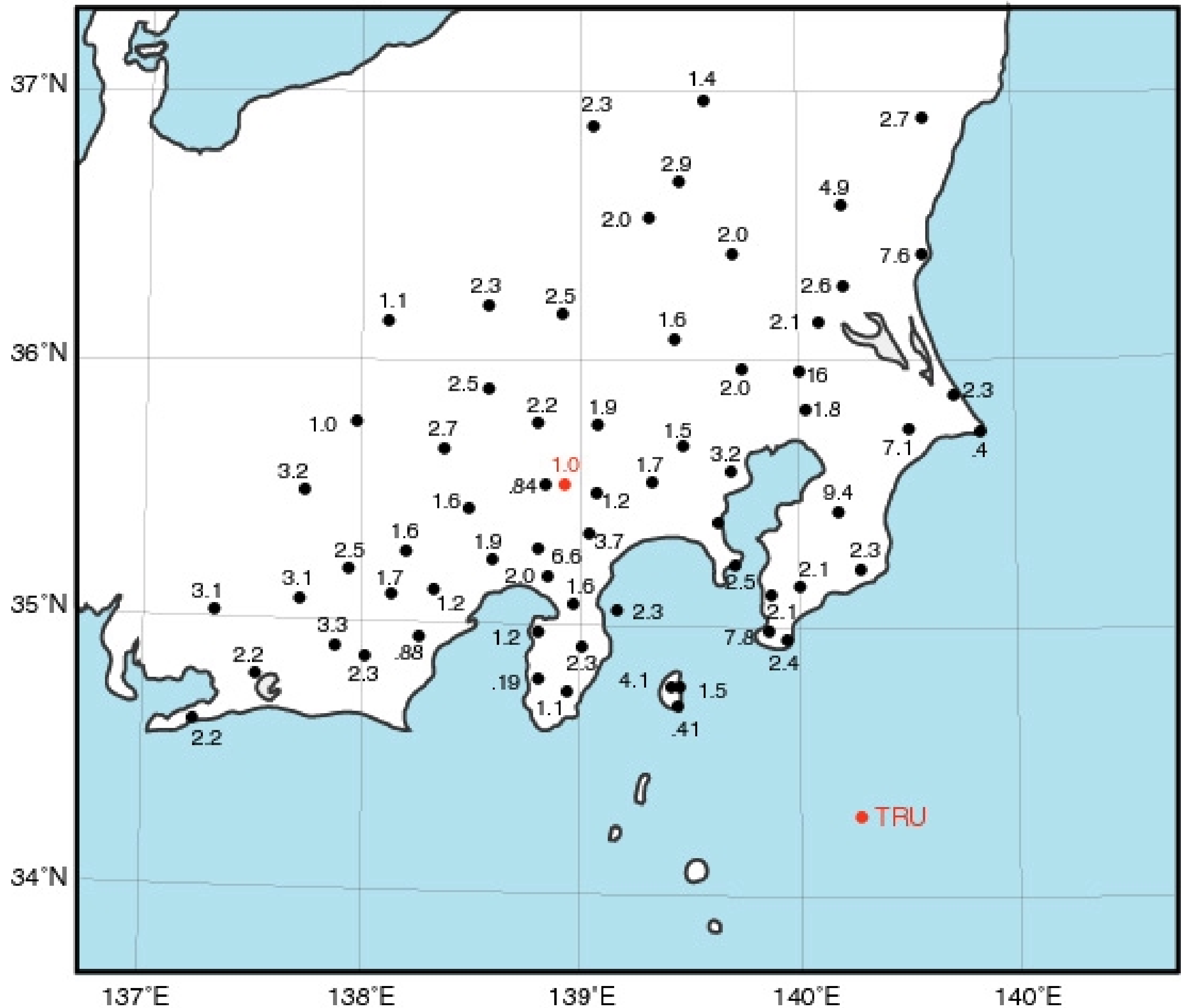
$$\frac{N_j^S(f)}{N_k^S(f)} = \sqrt{\frac{\left\langle \left| \dot{u}_{ij}^{\mathrm{S\ Coda}}(t_c; f) \right|^2 \right\rangle_{\mathrm{T}}}{\left\langle \left| \dot{u}_{ik}^{\mathrm{S\ Coda}}(t_c; f) \right|^2 \right\rangle_{\mathrm{T}}}}$$

First Quantitative Observational Verification



Comparison of direct-arrival amplification with coda amplification; found less scatter in coda measurements (Tsujiura, 1978)

Site Amplifications Relative to TRU for 3 Hz Band



Relative Source Factor Determination

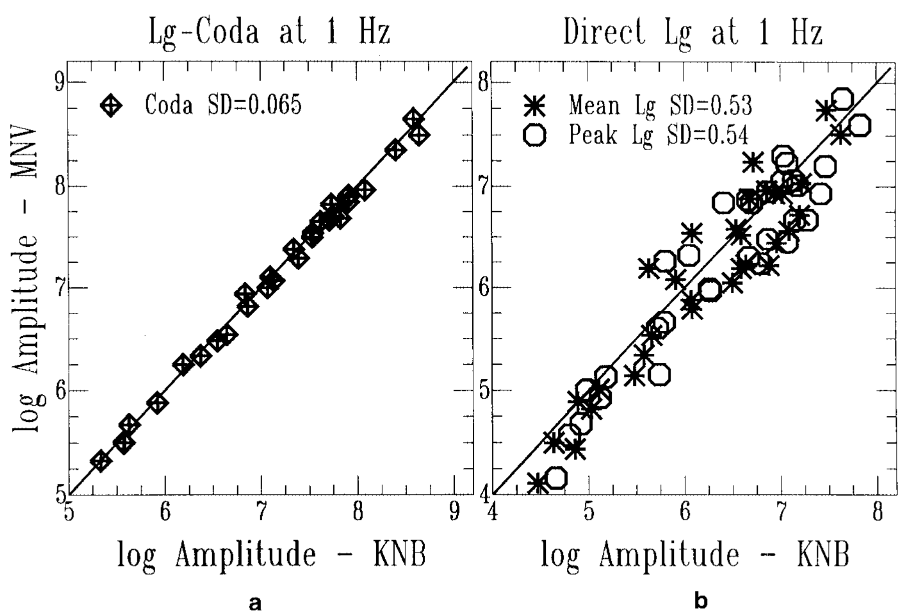
Divide coda amplitude for one event at one site by that for another event at the same site

(Usually best to take average over many sites)

$$\frac{W_i^S(f)}{W_k^S(f)} = \frac{\left\langle \left| \dot{u}_{ij}^{\mathrm{S\ coda}}(t_c; f) \right|^2 \right\rangle_{\mathrm{T}}}{\left\langle \left| \dot{u}_{kj}^{\mathrm{S\ coda}}(t_c; f) \right|^2 \right\rangle_{\mathrm{T}}}$$

Related to concept of using coda length as an estimate of magnitude

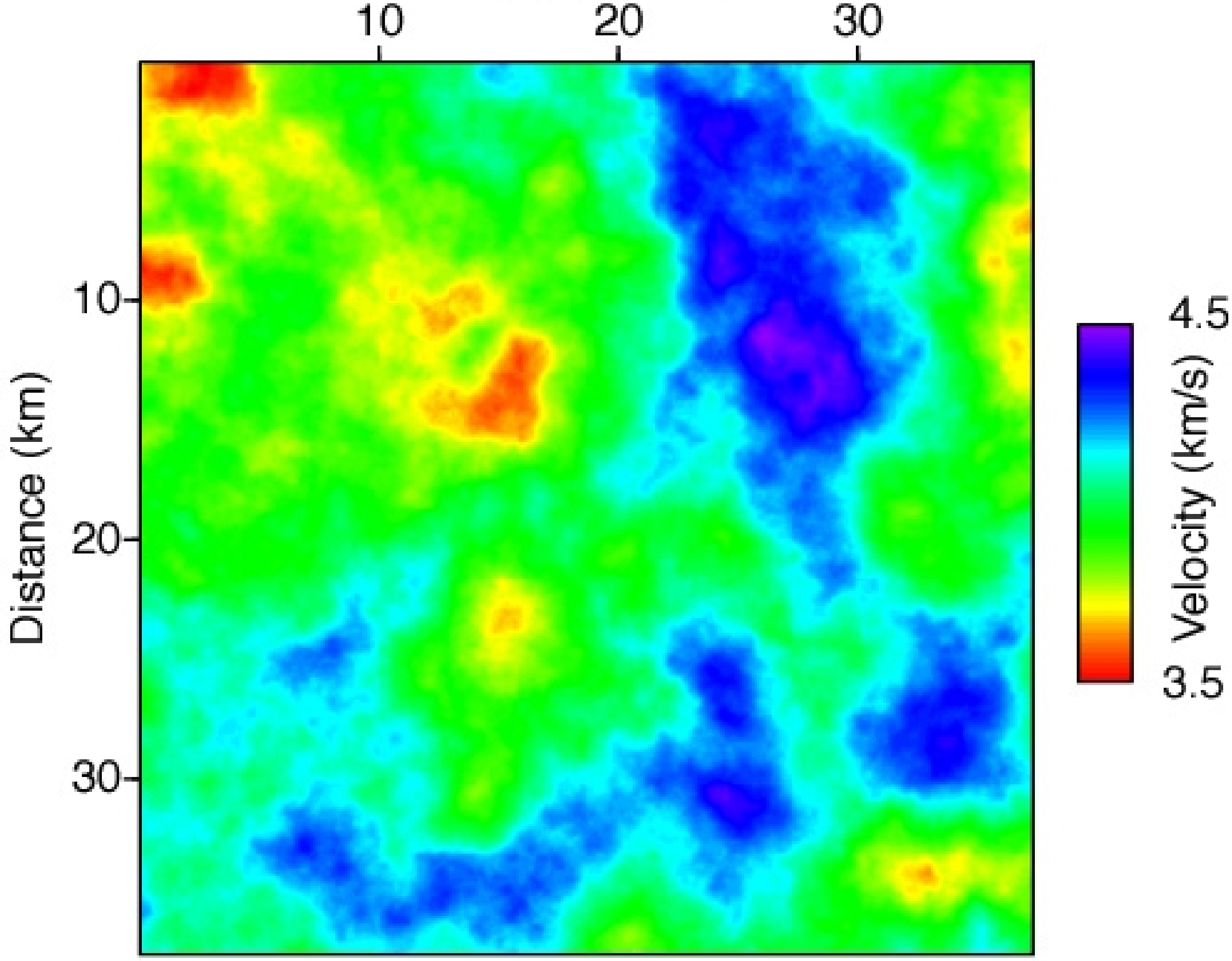
Useful to estimate magnitudes of earthquakes and sizes of nuclear explosions



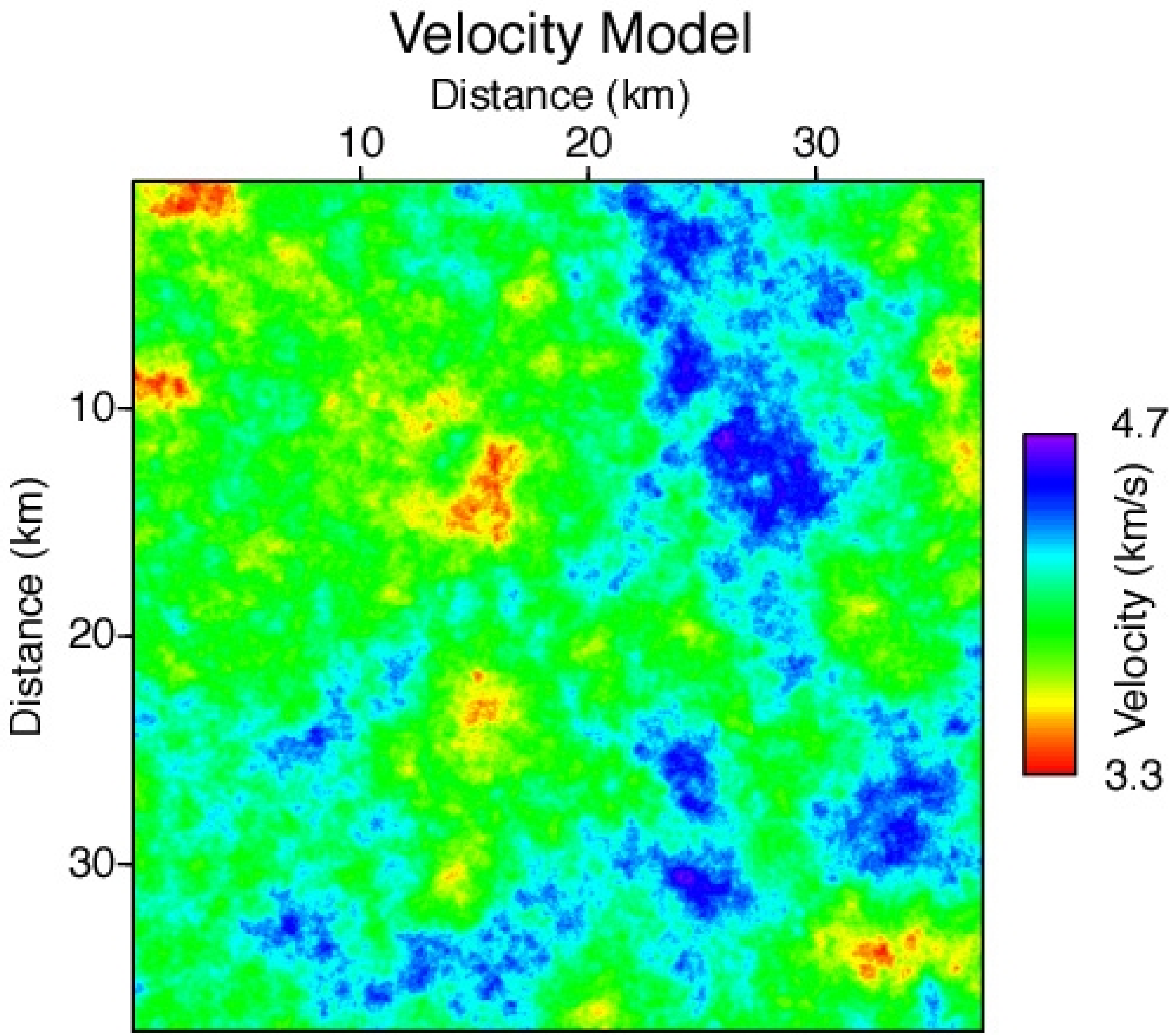
Coda Normalization Method has underlying assumption that some time after source excitation, energy is uniformly distributed in medium

- Some justification for this follows
- Basic assumption of Energy-Flux model of Frankel and Wennerberg (1987)

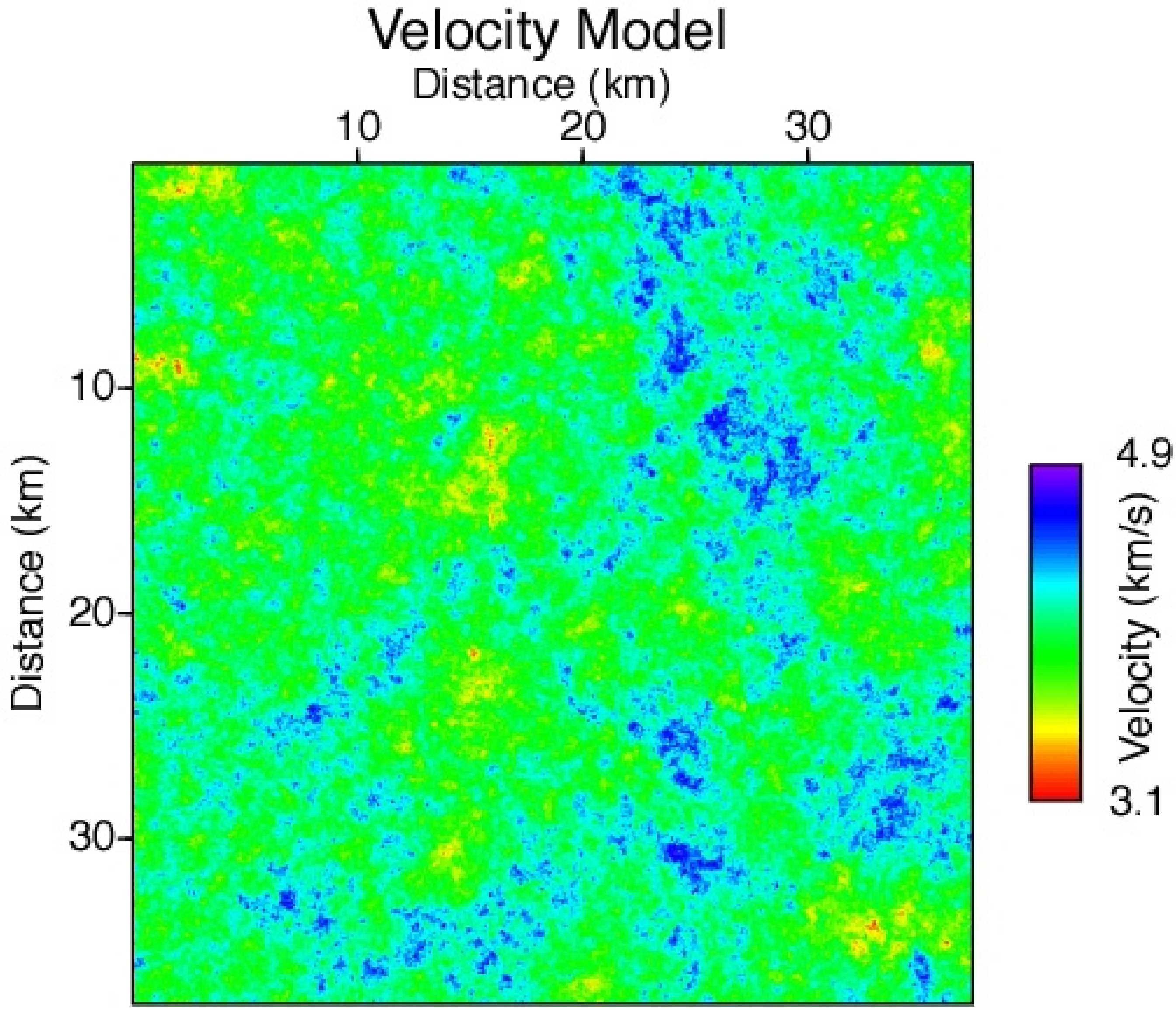
Velocity Model



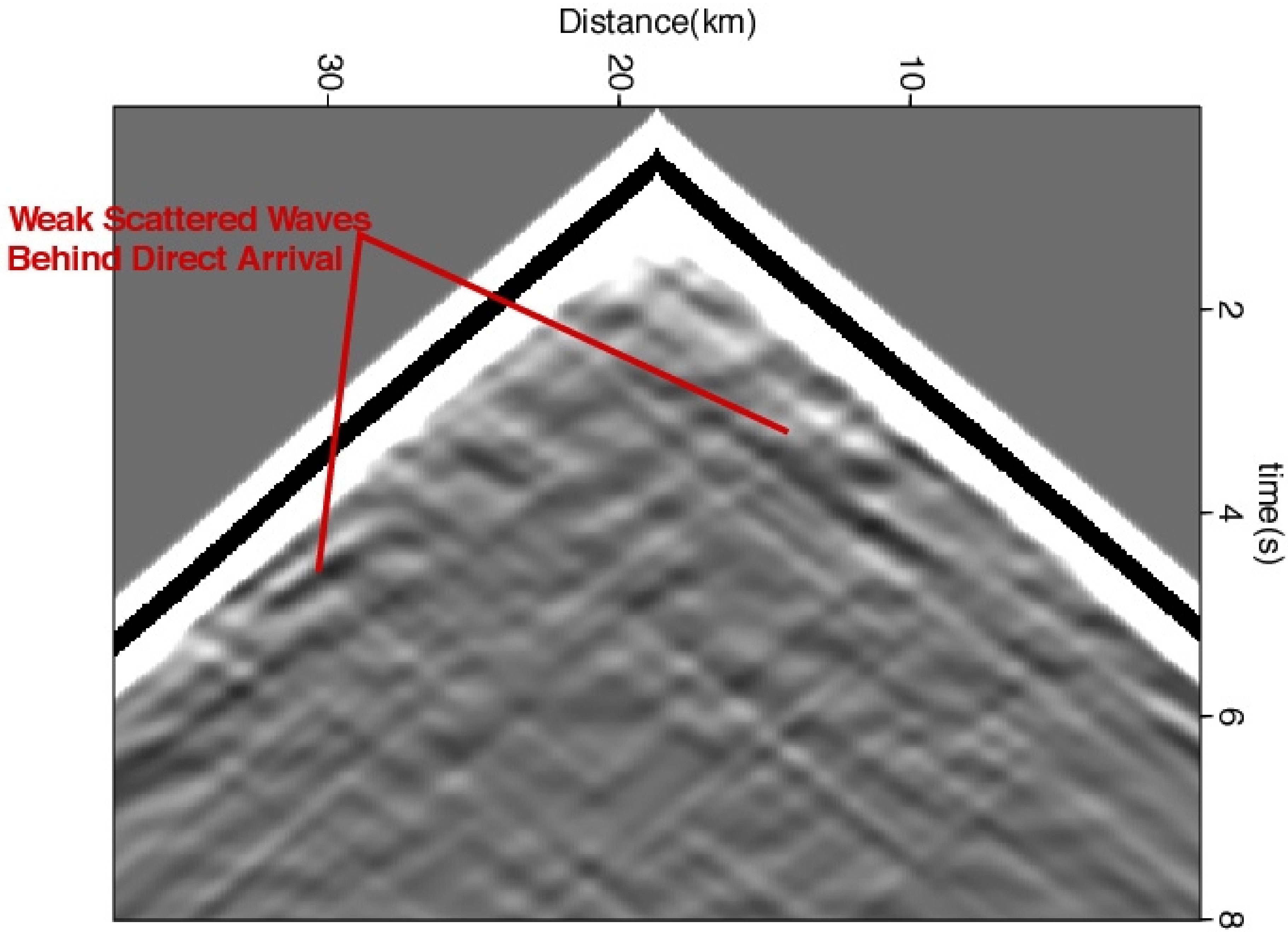
Van Karman, K=1.



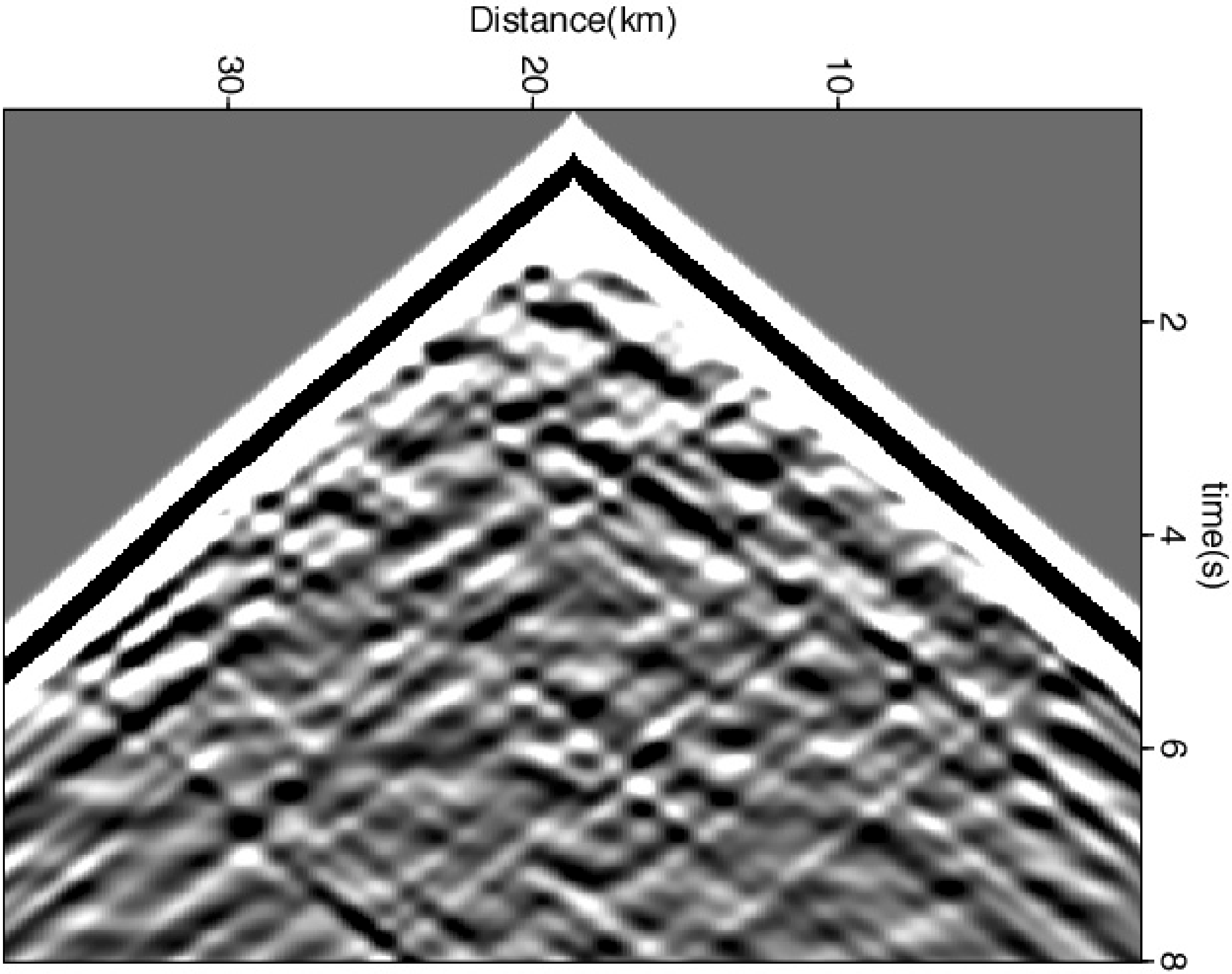
Van Karman, K=0.5



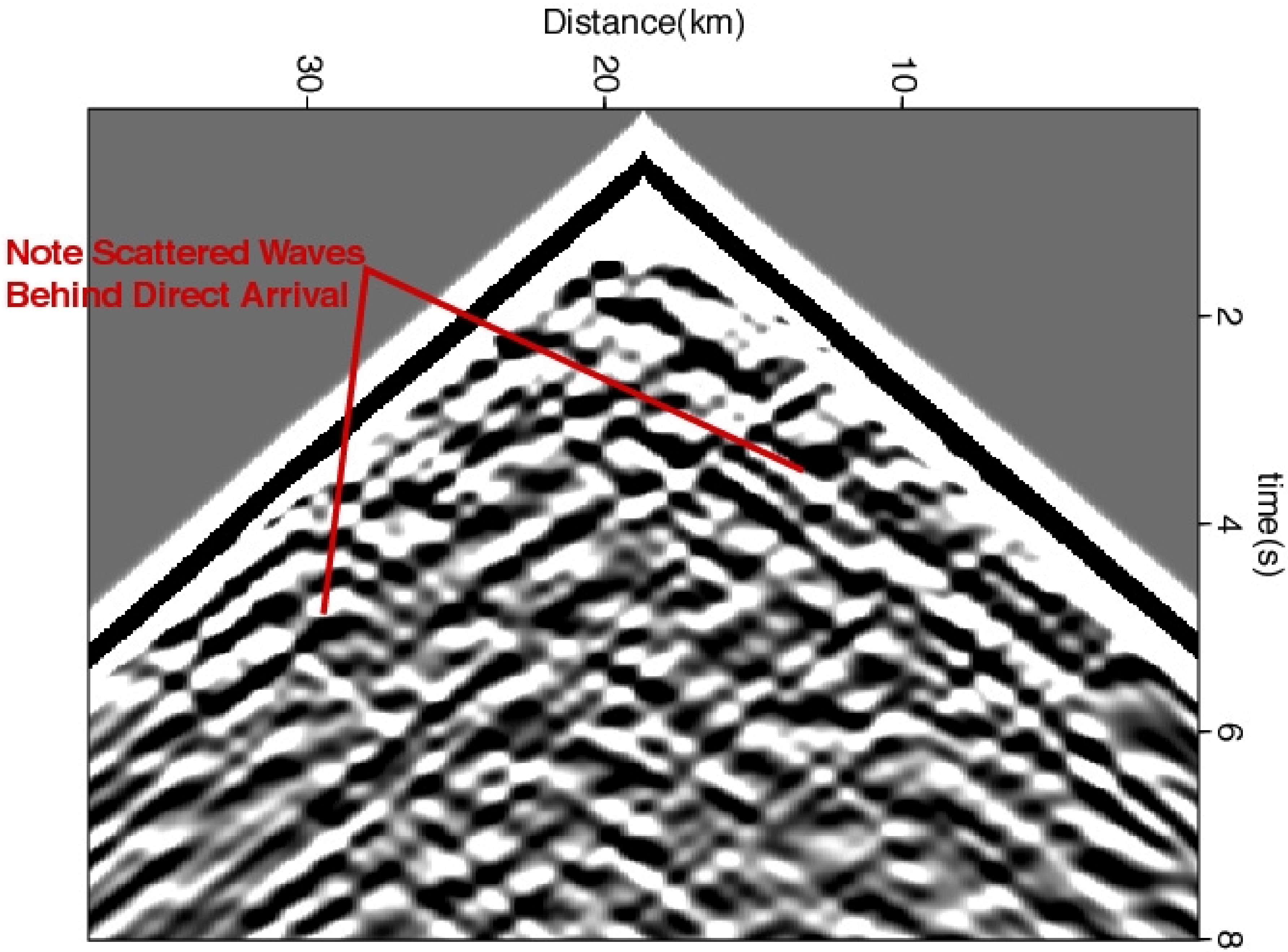
Van Karman, K=0.1



Van Karman ACF. K=1.

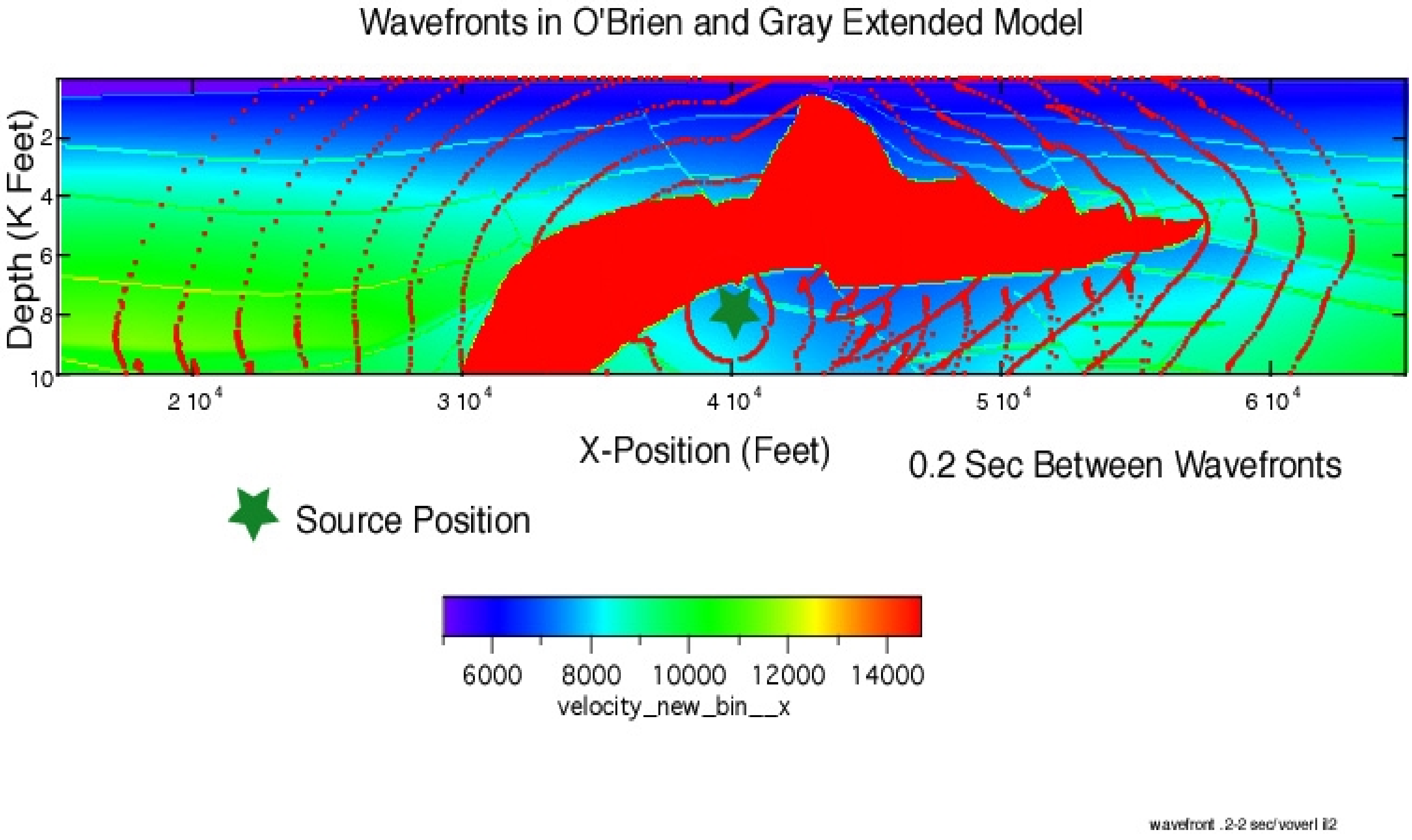


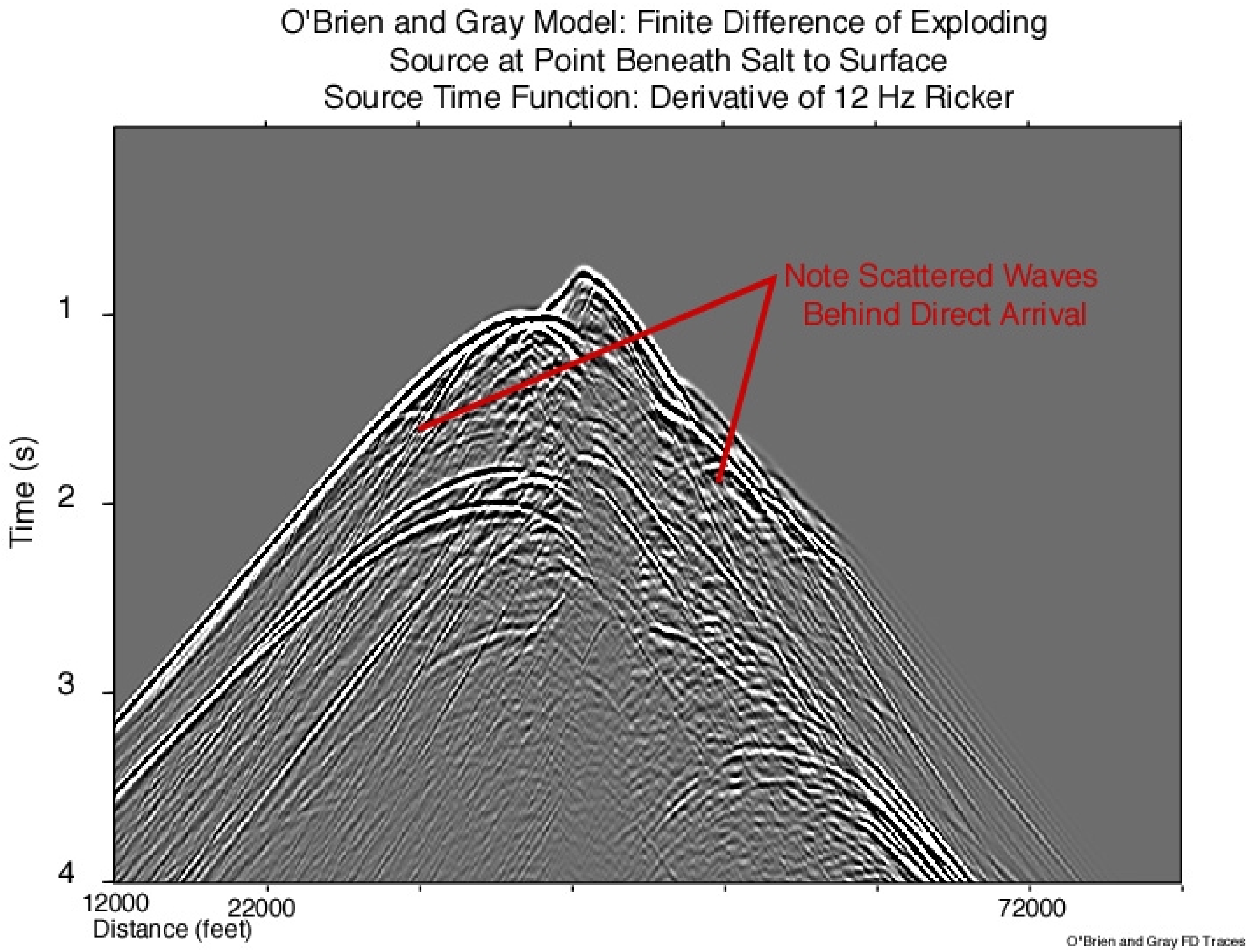
Van Karman ACF, K=.5

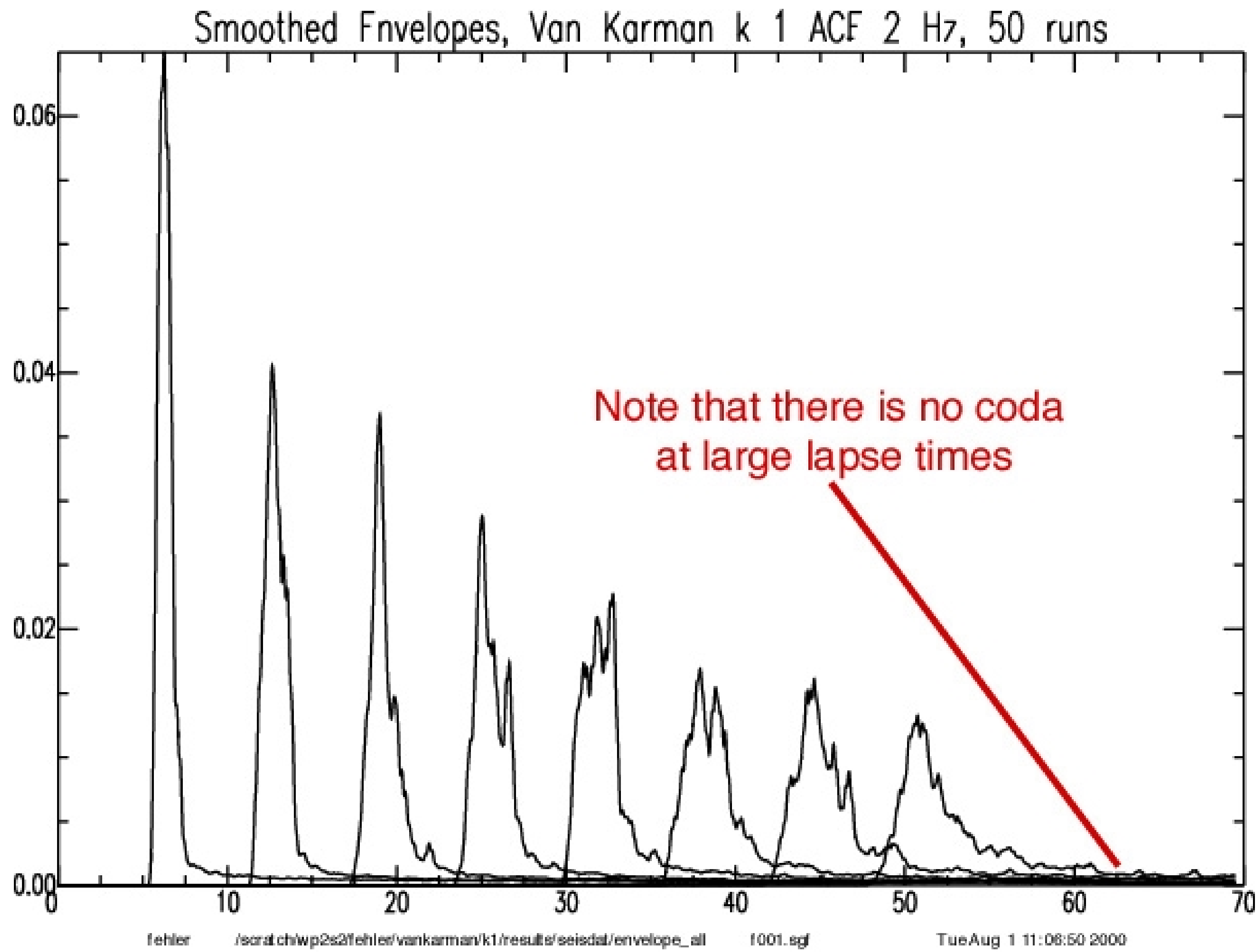


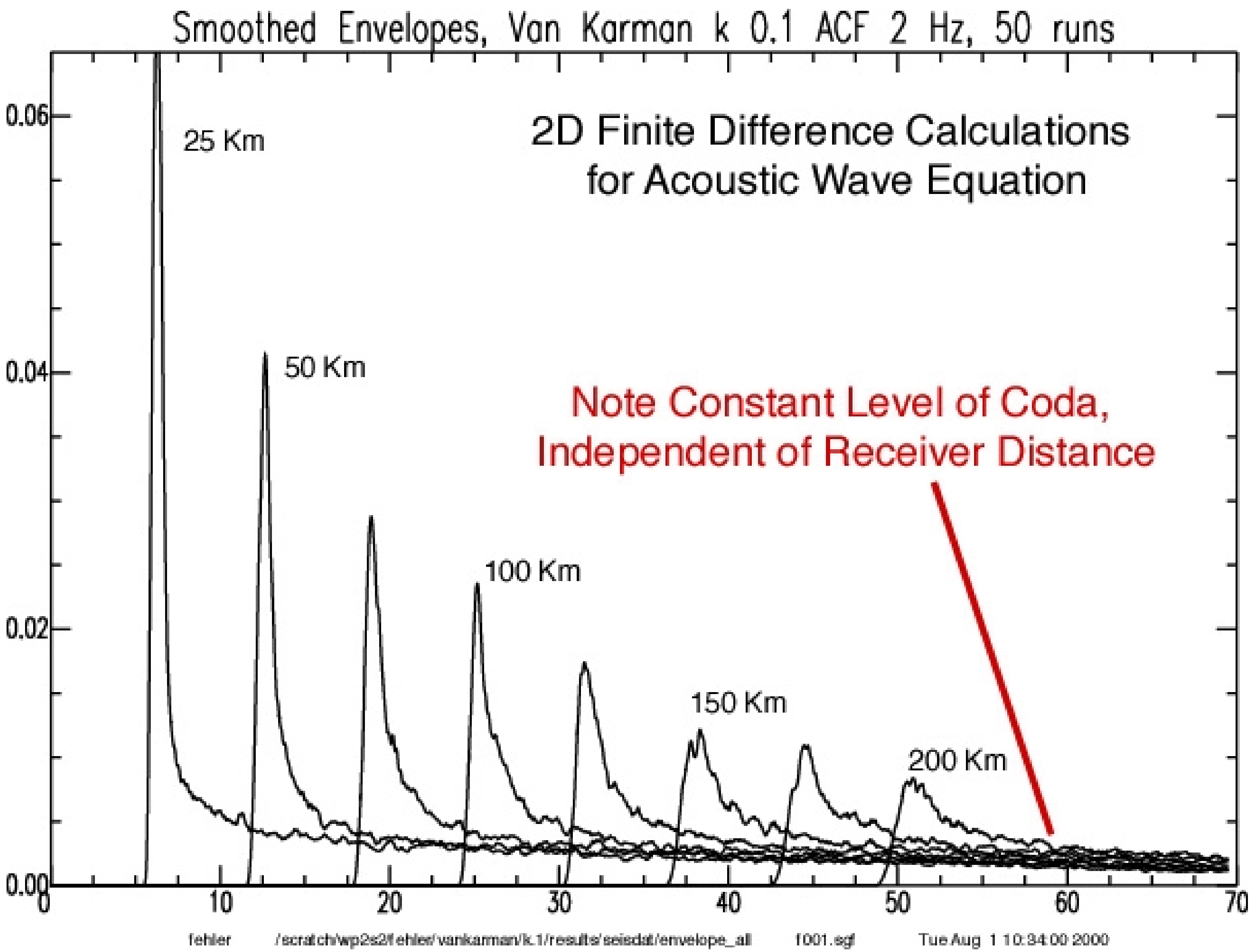
Van Karman ACF, K=.1

2D Exploration Model




O'Brien and Gray Model: Finite Difference of Exploding
Source at Point Beneath Salt to Surface
Source Time Function: Derivative of 12 Hz Ricker
Note Scattered Waves
Behind Direct Arrival
Time (s)
1
2
3
4
12000
22000
72000
Distance (feet)
O'Brien and Gray FD Traces


Smoothed Fnvelopes, Van Karman k 1 ACF 2 Hz, 50 runs
0.00
0.02
0.04
0.06
0
10
20
30
40
50
60
70
Note that there is no coda
at large lapse times
fehler /scratch/wp2s2/fehler/vankarman/k1/results/seisdat/envelope_all f001.sgf Tue Aug 1 11:08:50 2000



Smoothed Envelopes, Van Karman k 0.1 ACF 2 Hz, 50 runs
2D Finite Difference Calculations
for Acoustic Wave Equation
Note Constant Level of Coda,
Independent of Receiver Distance
25 Km
50 Km
100 Km
150 Km
200 Km
0.06
0.04
0.02
0.00
0
10
20
30
40
50
60
70
fehler
/scratch/wp2s2/fehler/vankarman/k.1/results/seisdat/envelope_all
f001.sgf
Tue Aug 1 10:34:00 2000

Radiative Transfer Theory also shows energy is uniformly distributed within some volume behind direct arrival

Note: g_0=.01 km^{-1}

Gives scaled time = 2 for t=50s

Scaled distance = 1 for 100 km